Introduction to macroscopic QED and its applications

Stefan Scheel
Universität Rostock, Germany

Acknowledgements

- Prof. Dirk-Gunnar Welsch (Jena)
- Dr. Christian Raabe (Jenoptik)
- Dr. Stefan Yoshi Buhmann (Freiburg)
- Dr. Simen Ådnøy Ellingsen (Trondheim)
- Dr. David T. Butcher (EY)
- Dr. John Alexander Crosse (Singapore)
- Dr. Sofia Isabel de Carvalho Ribeiro (Lisboa)
- Robin Schmidt (Rostock)

Outline

Macroscopic QED and Rytov theory

Dispersion forced from macroscopic QED

Nonequilibrium dispersion forces and heat transfer

Summary

Quantum optics of macroscopic systems

Maxwell´s equations in free space

- Maxwell's equations: $\nabla \cdot \mathbf{B}(\mathbf{r}) = 0 \quad \nabla \times \mathbf{E}(\mathbf{r}) = -\dot{\mathbf{B}}(\mathbf{r})$
  $\nabla \cdot \mathbf{E}(\mathbf{r}) = 0 \quad \nabla \times \mathbf{B}(\mathbf{r}) = \frac{1}{c^2}\dot{\mathbf{E}}(\mathbf{r})$
- introduce vector potential in Coulomb gauge $\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$ and $\mathbf{E}(\mathbf{r}) = -\dot{\mathbf{A}}(\mathbf{r})$ with $\nabla \cdot \mathbf{A}(\mathbf{r}) = 0$ (transversality condition)
- wave equation for vector potential

$$\Delta \mathbf{A}(\mathbf{r},t) - \frac{1}{c^2}\ddot{\mathbf{A}}(\mathbf{r},t) = 0$$

- separation of variables (mode decomposition)

$$\mathbf{A}(\mathbf{r},t) = \sum_\lambda \mathbf{A}_\lambda(\mathbf{r}) u_\lambda(t)$$

Maxwell's equations in free space

- Maxwell's equations: $\boldsymbol{\nabla}\cdot\mathbf{B}(\mathbf{r})=0 \qquad \boldsymbol{\nabla}\times\mathbf{E}(\mathbf{r})=-\dot{\mathbf{B}}(\mathbf{r})$
  $\boldsymbol{\nabla}\cdot\mathbf{E}(\mathbf{r})=0 \qquad \boldsymbol{\nabla}\times\mathbf{B}(\mathbf{r})=\frac{1}{c^2}\dot{\mathbf{E}}(\mathbf{r})$
- introduce vector potential in Coulomb gauge $\mathbf{B}(\mathbf{r})=\boldsymbol{\nabla}\times\mathbf{A}(\mathbf{r})$ and $\mathbf{E}(\mathbf{r})=-\dot{\mathbf{A}}(\mathbf{r})$ with $\boldsymbol{\nabla}\cdot\mathbf{A}(\mathbf{r})=0$ (transversality condition)
- wave equation for vector potential

$$\Delta\mathbf{A}(\mathbf{r},t)-\frac{1}{c^2}\ddot{\mathbf{A}}(\mathbf{r},t)=0$$

- separation of variables (mode decomposition)

$$\mathbf{A}(\mathbf{r},t)=\sum_\lambda \mathbf{A}_\lambda(\mathbf{r})u_\lambda(t)$$

Maxwell's equations in free space

- Maxwell's equations: $\boldsymbol{\nabla}\cdot\mathbf{B}(\mathbf{r})=0 \quad \boldsymbol{\nabla}\times\mathbf{E}(\mathbf{r})=-\dot{\mathbf{B}}(\mathbf{r})$
  $\boldsymbol{\nabla}\cdot\mathbf{E}(\mathbf{r})=0 \quad \boldsymbol{\nabla}\times\mathbf{B}(\mathbf{r})=\frac{1}{c^2}\dot{\mathbf{E}}(\mathbf{r})$
- introduce vector potential in Coulomb gauge $\mathbf{B}(\mathbf{r})=\boldsymbol{\nabla}\times\mathbf{A}(\mathbf{r})$ and $\mathbf{E}(\mathbf{r})=-\dot{\mathbf{A}}(\mathbf{r})$ with $\boldsymbol{\nabla}\cdot\mathbf{A}(\mathbf{r})=0$ (transversality condition)
- wave equation for vector potential

$$\Delta\mathbf{A}(\mathbf{r},t)-\frac{1}{c^2}\ddot{\mathbf{A}}(\mathbf{r},t)=0$$

- separation of variables (mode decomposition)

$$\mathbf{A}(\mathbf{r},t)=\sum_\lambda \mathbf{A}_\lambda(\mathbf{r})u_\lambda(t)$$

Maxwell's equations in free space

- Maxwell's equations: $\boldsymbol{\nabla}\cdot\mathbf{B}(\mathbf{r})=0$ $\quad \boldsymbol{\nabla}\times\mathbf{E}(\mathbf{r})=-\dot{\mathbf{B}}(\mathbf{r})$
  $\boldsymbol{\nabla}\cdot\mathbf{E}(\mathbf{r})=0$ $\quad \boldsymbol{\nabla}\times\mathbf{B}(\mathbf{r})=\frac{1}{c^2}\dot{\mathbf{E}}(\mathbf{r})$
- introduce vector potential in Coulomb gauge $\mathbf{B}(\mathbf{r})=\boldsymbol{\nabla}\times\mathbf{A}(\mathbf{r})$ and $\mathbf{E}(\mathbf{r})=-\dot{\mathbf{A}}(\mathbf{r})$ with $\boldsymbol{\nabla}\cdot\mathbf{A}(\mathbf{r})=0$ (transversality condition)
- wave equation for vector potential

$$\Delta\mathbf{A}(\mathbf{r},t)-\frac{1}{c^2}\ddot{\mathbf{A}}(\mathbf{r},t)=0$$

- separation of variables (mode decomposition)

$$\mathbf{A}(\mathbf{r},t)=\sum_\lambda \mathbf{A}_\lambda(\mathbf{r})u_\lambda(t)$$

Classical Hamiltonian

- Mode decomposition in cartesian coordinates

$$\mathbf{A}(\mathbf{r},t)=\sum_\sigma\int\frac{d^3k}{(2\pi)^{3/2}}\mathbf{e}_\sigma\left[u_{\mathbf{k}\sigma}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}+u^*_{\mathbf{k}\sigma}e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\right]$$

- classical Hamiltonian

$$H=\frac{1}{2}\int d^3r\left[\varepsilon_0\mathbf{E}^2(\mathbf{r})+\frac{1}{\mu_0}\mathbf{B}^2(\mathbf{r})\right]=2\varepsilon_0\sum_\sigma\int d^3k\,\omega^2|u_{\mathbf{k}\sigma}|^2$$

- define $q_{\mathbf{k}\sigma}=\sqrt{\varepsilon_0}(u_{\mathbf{k}\sigma}+u^*_{\mathbf{k}\sigma})$ and $p_{\mathbf{k}\sigma}=-i\omega\sqrt{\varepsilon_0}(u_{\mathbf{k}\sigma}-u^*_{\mathbf{k}\sigma})$
- Hamiltonian turns into

$$\boxed{H=\frac{1}{2}\sum_\sigma\int d^3k\,(p^2_{\mathbf{k}\sigma}+\omega^2q^2_{\mathbf{k}\sigma})}$$

$\Rightarrow$ infinite sum of uncoupled harmonic oscillators with frequencies $\omega=|\mathbf{k}|c$

Classical Hamiltonian

- Mode decomposition in cartesian coordinates

$$\mathbf{A}(\mathbf{r},t)=\sum_\sigma \int \frac{d^3k}{(2\pi)^{3/2}}\,\mathbf{e}_\sigma \left[u_{\mathbf{k}\sigma}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}+u^*_{\mathbf{k}\sigma}e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\right]$$

- classical Hamiltonian

$$H=\frac{1}{2}\int d^3r\left[\varepsilon_0\mathbf{E}^2(\mathbf{r})+\frac{1}{\mu_0}\mathbf{B}^2(\mathbf{r})\right]=2\varepsilon_0\sum_\sigma\int d^3k\,\omega^2|u_{\mathbf{k}\sigma}|^2$$

- define $q_{\mathbf{k}\sigma}=\sqrt{\varepsilon_0}(u_{\mathbf{k}\sigma}+u^*_{\mathbf{k}\sigma})$ and $p_{\mathbf{k}\sigma}=-i\omega\sqrt{\varepsilon_0}(u_{\mathbf{k}\sigma}-u^*_{\mathbf{k}\sigma})$
- Hamiltonian turns into

$$\boxed{H=\frac{1}{2}\sum_\sigma\int d^3k\,(p^2_{\mathbf{k}\sigma}+\omega^2q^2_{\mathbf{k}\sigma})}$$

$\Rightarrow$ infinite sum of uncoupled harmonic oscillators with frequencies $\omega=|\mathbf{k}|c$

Classical Hamiltonian

- Mode decomposition in cartesian coordinates

$$\mathbf{A}(\mathbf{r},t)=\sum_{\sigma}\int\frac{d^3k}{(2\pi)^{3/2}}\mathbf{e}_{\sigma}\left[u_{\mathbf{k}\sigma}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}+u^*_{\mathbf{k}\sigma}e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\right]$$

- classical Hamiltonian

$$H=\frac{1}{2}\int d^3r\left[\varepsilon_0\mathbf{E}^2(\mathbf{r})+\frac{1}{\mu_0}\mathbf{B}^2(\mathbf{r})\right]=2\varepsilon_0\sum_{\sigma}\int d^3k\,\omega^2|u_{\mathbf{k}\sigma}|^2$$

- define $q_{\mathbf{k}\sigma}=\sqrt{\varepsilon_0}(u_{\mathbf{k}\sigma}+u^*_{\mathbf{k}\sigma})$ and $p_{\mathbf{k}\sigma}=-i\omega\sqrt{\varepsilon_0}(u_{\mathbf{k}\sigma}-u^*_{\mathbf{k}\sigma})$
- Hamiltonian turns into

$$H=\frac{1}{2}\sum_{\sigma}\int d^3k\left(p^2_{\mathbf{k}\sigma}+\omega^2q^2_{\mathbf{k}\sigma}\right)$$

$\Rightarrow$ infinite sum of uncoupled harmonic oscillators with frequencies $\omega=|\mathbf{k}|c$

Classical Hamiltonian

- Mode decomposition in cartesian coordinates

$$\mathbf{A}(\mathbf{r},t)=\sum_\sigma\int\frac{d^3k}{(2\pi)^{3/2}}\mathbf{e}_\sigma\left[u_{\mathbf{k}\sigma}e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}+u^*_{\mathbf{k}\sigma}e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\right]$$

- classical Hamiltonian

$$H=\frac{1}{2}\int d^3r\left[\varepsilon_0\mathbf{E}^2(\mathbf{r})+\frac{1}{\mu_0}\mathbf{B}^2(\mathbf{r})\right]=2\varepsilon_0\sum_\sigma\int d^3k\,\omega^2|u_{\mathbf{k}\sigma}|^2$$

- define $q_{\mathbf{k}\sigma}=\sqrt{\varepsilon_0}(u_{\mathbf{k}\sigma}+u^*_{\mathbf{k}\sigma})$ and $p_{\mathbf{k}\sigma}=-i\omega\sqrt{\varepsilon_0}(u_{\mathbf{k}\sigma}-u^*_{\mathbf{k}\sigma})$
- Hamiltonian turns into

$$\boxed{H=\frac{1}{2}\sum_\sigma\int d^3k\,(p^2_{\mathbf{k}\sigma}+\omega^2q^2_{\mathbf{k}\sigma})}$$

$\Rightarrow$ infinite sum of uncoupled harmonic oscillators with frequencies $\omega=|\mathbf{k}|c$

Quantum optics of macroscopic systems

Quantization

- replace c-number functions $q_{\mathbf{k}\sigma}$, $p_{\mathbf{k}\sigma}$ by operators $\hat{q}_{\mathbf{k}\sigma}$, $\hat{p}_{\mathbf{k}\sigma}$
- postulate equal-time commutation relations $[\hat{q}_{\mathbf{k}\sigma}, \hat{p}_{\mathbf{k}'\sigma'}] = i\hbar\delta(\mathbf{k}-\mathbf{k}')\delta_{\sigma\sigma'}$
- define creation and annihilation operators

$$\hat{a}_\sigma(\mathbf{k}) = \sqrt{\frac{\omega}{2\hbar}}\left(\hat{q}_{\mathbf{k}\sigma} + \frac{i\hat{p}_{\mathbf{k}\sigma}}{\omega}\right), \quad \hat{a}^\dagger_\sigma(\mathbf{k}) = \sqrt{\frac{\omega}{2\hbar}}\left(\hat{q}_{\mathbf{k}\sigma} - \frac{i\hat{p}_{\mathbf{k}\sigma}}{\omega}\right)$$

  with equal-time commutation relations

$$\left[\hat{a}_\sigma(\mathbf{k}), \hat{a}^\dagger_{\sigma'}(\mathbf{k}')\right] = \delta(\mathbf{k}-\mathbf{k}')\delta_{\sigma\sigma'}$$

- shorthand notation $\lambda \equiv (\mathbf{k},\sigma) \Rightarrow \hat{\mathbf{A}}(\mathbf{r}) = \sum_\lambda [\mathbf{A}_\lambda(\mathbf{r})\hat{a}_\lambda + \text{h.c.}]$
- Hamiltonian

$$\hat{H} = \sum_\lambda \hbar\omega_\lambda\left(\hat{a}^\dagger_\lambda\hat{a}_\lambda + \frac{1}{2}\right)$$

Quantization

- replace c-number functions $q_{\mathbf{k}\sigma}$, $p_{\mathbf{k}\sigma}$ by operators $\hat{q}_{\mathbf{k}\sigma}$, $\hat{p}_{\mathbf{k}\sigma}$
- postulate equal-time commutation relations $[\hat{q}_{\mathbf{k}\sigma}, \hat{p}_{\mathbf{k}'\sigma'}] = i\hbar\delta(\mathbf{k}-\mathbf{k}')\delta_{\sigma\sigma'}$
- define creation and annihilation operators

$$\hat{a}_\sigma(\mathbf{k}) = \sqrt{\frac{\omega}{2\hbar}}\left(\hat{q}_{\mathbf{k}\sigma} + \frac{i\hat{p}_{\mathbf{k}\sigma}}{\omega}\right), \quad \hat{a}^\dagger_\sigma(\mathbf{k}) = \sqrt{\frac{\omega}{2\hbar}}\left(\hat{q}_{\mathbf{k}\sigma} - \frac{i\hat{p}_{\mathbf{k}\sigma}}{\omega}\right)$$

  with equal-time commutation relations

$$\boxed{\left[\hat{a}_\sigma(\mathbf{k}), \hat{a}^\dagger_{\sigma'}(\mathbf{k}')\right] = \delta(\mathbf{k}-\mathbf{k}')\delta_{\sigma\sigma'}}$$

- shorthand notation $\lambda \equiv (\mathbf{k},\sigma) \Rightarrow \hat{\mathbf{A}}(\mathbf{r}) = \sum_\lambda [\mathbf{A}_\lambda(\mathbf{r})\hat{a}_\lambda + \text{h.c.}]$
- Hamiltonian

$$\boxed{\hat{H} = \sum_\lambda \hbar\omega_\lambda\left(\hat{a}^\dagger_\lambda\hat{a}_\lambda + \frac{1}{2}\right)}$$

Quantization

- replace c-number functions $q_{\mathbf{k}\sigma}$, $p_{\mathbf{k}\sigma}$ by operators $\hat{q}_{\mathbf{k}\sigma}$, $\hat{p}_{\mathbf{k}\sigma}$
- postulate equal-time commutation relations $[\hat{q}_{\mathbf{k}\sigma}, \hat{p}_{\mathbf{k}'\sigma'}] = i\hbar\delta(\mathbf{k}-\mathbf{k}')\delta_{\sigma\sigma'}$
- define creation and annihilation operators

$$\hat{a}_\sigma(\mathbf{k}) = \sqrt{\frac{\omega}{2\hbar}}\left(\hat{q}_{\mathbf{k}\sigma} + \frac{i\hat{p}_{\mathbf{k}\sigma}}{\omega}\right), \quad \hat{a}_\sigma^\dagger(\mathbf{k}) = \sqrt{\frac{\omega}{2\hbar}}\left(\hat{q}_{\mathbf{k}\sigma} - \frac{i\hat{p}_{\mathbf{k}\sigma}}{\omega}\right)$$

with equal-time commutation relations

$$\left[\hat{a}_\sigma(\mathbf{k}), \hat{a}_{\sigma'}^\dagger(\mathbf{k}')\right] = \delta(\mathbf{k}-\mathbf{k}')\delta_{\sigma\sigma'}$$

- shorthand notation $\lambda \equiv (\mathbf{k},\sigma) \Rightarrow \hat{\mathbf{A}}(\mathbf{r}) = \sum_\lambda [\mathbf{A}_\lambda(\mathbf{r})\hat{a}_\lambda + \text{h.c.}]$
- Hamiltonian

$$\hat{H} = \sum_\lambda \hbar\omega_\lambda \left(\hat{a}_\lambda^\dagger \hat{a}_\lambda + \frac{1}{2}\right)$$

Quantization in absorbing media — naive approach

naive introduction of a (necessarily complex) refractive index $n(\omega)$ leads to decaying commutation rules

$$\left[\hat{a}(\omega,t),\hat{a}^\dagger(\omega',t)\right]=e^{-n_I\omega t}\delta(\omega-\omega')$$

common solution: ad hoc introduction of Langevin noise: from time evolution of expectation values $\langle\hat{a}(t)\rangle=e^{-(i\omega+\Gamma/2)t}\langle\hat{a}(0)\rangle$ and corresponding differential equation $\frac{d}{dt}\langle\hat{a}\rangle=-(i\omega+\Gamma/2)\langle\hat{a}\rangle$

it does not follow that $\dot{\hat{a}}=-(i\omega+\Gamma/2)\hat{a}$

instead:

$$\dot{\hat{a}}=-(i\omega+\Gamma/2)\hat{a}+\hat{f}\qquad \begin{aligned}\left[\hat{f}(t_1),\hat{f}^\dagger(t_2)\right]e^{i\omega(t_1-t_2)}&=\Gamma\delta(t_1-t_2)\\ \left[\hat{a}(t_1),\hat{f}^\dagger(t_2)\right]&=0\,,\quad t_2>t_1\end{aligned}$$

Quantization in absorbing media — naive approach

naive introduction of a (necessarily complex) refractive index $n(\omega)$ leads to decaying commutation rules

$$\left[\hat{a}(\omega,t),\hat{a}^\dagger(\omega',t)\right] = e^{-n_I\omega t}\delta(\omega-\omega')$$

common solution: ad hoc introduction of Langevin noise: from time evolution of expectation values $\langle\hat{a}(t)\rangle = e^{-(i\omega+\Gamma/2)t}\langle\hat{a}(0)\rangle$ and corresponding differential equation $\frac{d}{dt}\langle\hat{a}\rangle = -(i\omega+\Gamma/2)\langle\hat{a}\rangle$

it does not follow that $\dot{\hat{a}} = -(i\omega+\Gamma/2)\hat{a}$

instead:

$$\dot{\hat{a}} = -(i\omega+\Gamma/2)\hat{a}+\hat{f} \qquad \left[\hat{f}(t_1),\hat{f}^\dagger(t_2)\right]e^{i\omega(t_1-t_2)} = \Gamma\delta(t_1-t_2)$$

$$\left[\hat{a}(t_1),\hat{f}^\dagger(t_2)\right] = 0\,,\quad t_2>t_1$$

Quantization in absorbing media — naive approach

naive introduction of a (necessarily complex) refractive index $n(\omega)$ leads to decaying commutation rules

$$\left[\hat{a}(\omega,t),\hat{a}^{\dagger}(\omega',t)\right]=e^{-n_I\omega t}\delta(\omega-\omega')$$

common solution: ad hoc introduction of Langevin noise: from time evolution of expectation values $\langle\hat{a}(t)\rangle=e^{-(i\omega+\Gamma/2)t}\langle\hat{a}(0)\rangle$ and corresponding differential equation $\frac{d}{dt}\langle\hat{a}\rangle=-(i\omega+\Gamma/2)\langle\hat{a}\rangle$

it does not follow that $\dot{\hat{a}}=-(i\omega+\Gamma/2)\hat{a}$

instead:

$$\dot{\hat{a}}=-(i\omega+\Gamma/2)\hat{a}+\hat{f}\qquad \left[\hat{f}(t_1),\hat{f}^{\dagger}(t_2)\right]e^{i\omega(t_1-t_2)}=\Gamma\delta(t_1-t_2)$$

$$\left[\hat{a}(t_1),\hat{f}^{\dagger}(t_2)\right]=0\,,\quad t_2>t_1$$

Fluctuations in electromagnetism — Rytov's approach

linear response of polarisation to electric field is not complete without adding fluctuations:

$$\mathbf{P}(\mathbf{r},\omega) = \varepsilon_0\chi(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega) + \mathbf{P}_\mathrm{N}(\mathbf{r},\omega)$$

noise polarisation $\mathbf{P}_\mathrm{N}(\mathbf{r},\omega)$ with correlations (linear fluctuation-dissipation theorem)

$$\langle \mathbf{P}_\mathrm{N}(\mathbf{r},\omega)\otimes\mathbf{P}^*_\mathrm{N}(\mathbf{r}',\omega')\rangle = \frac{2k_B T\varepsilon_0}{\pi\omega}\,\mathrm{Im}\,\chi(\mathbf{r},\omega)\delta(\mathbf{r}-\mathbf{r}')\delta(\omega-\omega')\boldsymbol{I}$$

modified macroscopic Helmholtz equation:

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\mathbf{E}(\mathbf{r},\omega) - \frac{\omega^2}{c^2}\varepsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},\omega) = \frac{\omega^2}{\varepsilon_0 c^2}\mathbf{P}_\mathrm{N}(\mathbf{r},\omega)$$

S.M. Rytov, *Theory of Electrical Fluctuations and Thermal Radiation* (Acad. Sci. Press, Moscow, 1953); Sov. Phys. JETP **6**, 130 (1958).

Langevin forces and open quantum systems

simplest (Caldeira–Leggett) model of harmonic oscillator $\hat{a}$ coupled to a large number of reservoir harmonic oscillators $\hat{b}_i$

$$\hat{H} = \hat{H}_{\mathrm{sys}} + \hat{H}_{\mathrm{res}} + \hat{H}_{\mathrm{int}}\,, \quad \hat{H}_{\mathrm{res}} = \sum_i \hbar\omega_i \hat{b}_i^\dagger \hat{b}_i\,, \quad \hat{H}_{\mathrm{int}} = \hbar \sum_i g_i \hat{a}^\dagger \hat{b}_i + \text{h.c.}$$

Heisenberg's equations of motion after formal integration of $\hat{b}_i$ in Markov approximation:

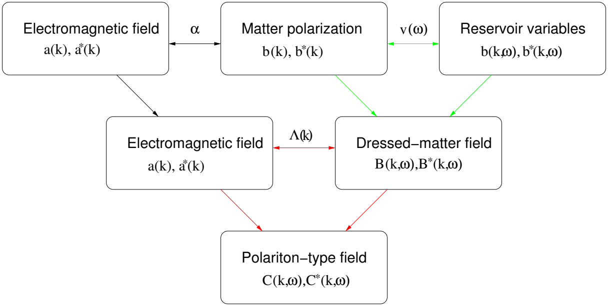
$$\dot{\hat{a}}(t) = -(i\omega + i\delta\omega + \Gamma/2)\hat{a}(t) + \hat{f}(t)$$

with

$$\Gamma = 2\pi \sum_i |g_i|^2 \delta(\omega - \omega_i)\,, \quad \delta\omega = \sum_i |g_i|^2 \mathcal{P}\frac{1}{\omega - \omega_i}\,, \quad \hat{f}(t) = -i\sum_i g_i e^{-i\omega_i t}\hat{b}_i(0)$$

G.W. Ford, M. Kac, and P. Mazur, J. Math. Phys. **6**, 504 (1965); H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002).

Huttner–Barnett model of absorbing dielectrics



two-step Fano diagonalisation provides construction of noise fields from microscopic (Hopfield) model of absorbing dielectrics ⇒ microscopic justification of the validity of noise polarisation, i.e. Rytov theory

B. Huttner and S.M. Barnett, Phys. Rev. A **46**, 4306 (1992); L.G. Suttorp and M. Wubs, Phys. Rev. A **70**, 013816 (2004).

Green function expansion of the electromagnetic field

Helmholtz equation

$$\nabla \times \nabla \times \hat{\mathbf{E}}(\mathbf{r},\omega) - \frac{\omega^2}{c^2}\varepsilon(\mathbf{r},\omega)\hat{\mathbf{E}}(\mathbf{r},\omega) = \frac{\omega^2}{c^2\varepsilon_0}\hat{\mathbf{P}}_N(\mathbf{r},\omega)$$

polariton field connected to electromagnetic fields via Green function of the Helmholtz equation

$$\hat{\mathbf{E}}(\mathbf{r},\omega) = \frac{\omega^2}{c^2\varepsilon_0}\int d^3s\, \boldsymbol{G}(\mathbf{r},\mathbf{s},\omega)\cdot\hat{\mathbf{P}}_N(\mathbf{s},\omega)\,, \quad \hat{\mathbf{P}}_N(\mathbf{r},\omega) = i\frac{\omega^2}{c^2}\sqrt{\frac{\hbar}{\pi\varepsilon_0}\varepsilon_I(\mathbf{r},\omega)}\hat{\mathbf{f}}(\mathbf{r},\omega)$$

read as linear response relation between electric field and noise polarisation with Green function acting as response function

$$\langle\hat{\mathbf{E}}(\mathbf{r},\omega)\otimes\hat{\mathbf{E}}^\dagger(\mathbf{r}',\omega')\rangle \propto \mathrm{Im}\,\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\delta(\omega-\omega')$$

T. Gruner and D.-G. Welsch, Phys. Rev. A **53**, 1818 (1996); T.D. Ho, L. Knöll, and D.-G. Welsch, Phys. Rev. A **57**, 3931 (1998); S. Scheel and D.-G. Welsch, Phys. Rev. A **58**, 700 (1998); T.D. Ho, S.Y. Buhmann, L. Knöll, D.-G. Welsch, S. Scheel, and J. Kästel, Phys. Rev. A **68**, 043816 (2003); S. Scheel and S.Y. Buhmann, Acta Physica Slovaca **58**, 675 (2008).

Green function expansion of the electromagnetic field

Helmholtz equation

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\hat{\mathbf{E}}(\mathbf{r},\omega)-\frac{\omega^2}{c^2}\varepsilon(\mathbf{r},\omega)\hat{\mathbf{E}}(\mathbf{r},\omega)=\frac{\omega^2}{c^2\varepsilon_0}\hat{\mathbf{P}}_N(\mathbf{r},\omega)$$

polariton field connected to electromagnetic fields via Green function of the Helmholtz equation

$$\hat{\mathbf{E}}(\mathbf{r},\omega)=\frac{\omega^2}{c^2\varepsilon_0}\int d^3s\,\boldsymbol{G}(\mathbf{r},\mathbf{s},\omega)\cdot\hat{\mathbf{P}}_N(\mathbf{s},\omega)\,,\quad \hat{\mathbf{P}}_N(\mathbf{r},\omega)=i\frac{\omega^2}{c^2}\sqrt{\frac{\hbar}{\pi\varepsilon_0}\varepsilon_I(\mathbf{r},\omega)}\hat{\mathbf{f}}(\mathbf{r},\omega)$$

read as linear response relation between electric field and noise polarisation with Green function acting as response function

$$\langle\hat{\mathbf{E}}(\mathbf{r},\omega)\otimes\hat{\mathbf{E}}^\dagger(\mathbf{r}',\omega')\rangle\propto\operatorname{Im}\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\delta(\omega-\omega')$$

T. Gruner and D.-G. Welsch, Phys. Rev. A **53**, 1818 (1996); T.D. Ho, L. Knöll, and D.-G. Welsch, Phys. Rev. A **57**, 3931 (1998); S. Scheel and D.-G. Welsch, Phys. Rev. A **58**, 700 (1998); T.D. Ho, S.Y. Buhmann, L. Knöll, D.-G. Welsch, S. Scheel, and J. Kästel, Phys. Rev. A **68**, 043816 (2003); S. Scheel and S.Y. Buhmann, Acta Physica Slovaca **58**, 675 (2008).

Consistency checks

- ETCR between electric field and magnetic induction $\surd$

$$\left[\hat{\mathbf{E}}(\mathbf{r}),\hat{\mathbf{B}}(\mathbf{r}')\right] = -\frac{i\hbar}{\varepsilon_0}\boldsymbol{\nabla}\times\delta(\mathbf{r}-\mathbf{r}')\boldsymbol{U}$$

- consistent with fluctuation-dissipation theorem $\surd$

$$\langle 0|\hat{\mathbf{E}}(\mathbf{r},\omega)\hat{\mathbf{E}}^{\dagger}(\mathbf{r}',\omega')|0\rangle = \frac{\hbar\omega^2}{\pi\varepsilon_0 c^2}\operatorname{Im}\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\delta(\omega-\omega')$$

- Maxwell's equations follow from bilinear Hamiltonian $\surd$

$$\hat{H} = \int d^3\mathbf{r}\int_0^\infty d\omega\,\hbar\omega\,\hat{\mathbf{f}}^{\dagger}(\mathbf{r},\omega)\cdot\hat{\mathbf{f}}(\mathbf{r},\omega)$$

Consistency checks

- ETCR between electric field and magnetic induction $\surd$

$$\left[\hat{\mathbf{E}}(\mathbf{r}),\hat{\mathbf{B}}(\mathbf{r}')\right]=-\frac{i\hbar}{\varepsilon_0}\boldsymbol{\nabla}\times\delta(\mathbf{r}-\mathbf{r}')\boldsymbol{U}$$

- consistent with fluctuation-dissipation theorem $\surd$

$$\langle 0|\hat{\mathbf{E}}(\mathbf{r},\omega)\hat{\mathbf{E}}^\dagger(\mathbf{r}',\omega')|0\rangle=\frac{\hbar\omega^2}{\pi\varepsilon_0 c^2}\operatorname{Im}\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\delta(\omega-\omega')$$

- Maxwell's equations follow from bilinear Hamiltonian $\surd$

$$\hat{H}=\int d^3\mathbf{r}\int_0^\infty d\omega\,\hbar\omega\,\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)\cdot\hat{\mathbf{f}}(\mathbf{r},\omega)$$

Consistency checks

- ETCR between electric field and magnetic induction √

$$\left[\hat{\mathbf{E}}(\mathbf{r}), \hat{\mathbf{B}}(\mathbf{r}')\right] = -\frac{i\hbar}{\varepsilon_0} \boldsymbol{\nabla} \times \delta(\mathbf{r} - \mathbf{r}') \boldsymbol{U}$$

- consistent with fluctuation-dissipation theorem √

$$\langle 0|\hat{\mathbf{E}}(\mathbf{r},\omega)\hat{\mathbf{E}}^\dagger(\mathbf{r}',\omega')|0\rangle = \frac{\hbar\omega^2}{\pi\varepsilon_0 c^2} \mathrm{Im}\, \boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\delta(\omega-\omega')$$

- Maxwell's equations follow from bilinear Hamiltonian √

$$\hat{H} = \int d^3\mathbf{r} \int_0^\infty d\omega\, \hbar\omega\, \hat{\mathbf{f}}^\dagger(\mathbf{r},\omega) \cdot \hat{\mathbf{f}}(\mathbf{r},\omega)$$

General linear dielectrics

Most general linear constitutive relation:

$$\hat{\mathbf{j}}_{\mathrm{in}}(\mathbf{r},t)=\int_{-\infty}^{\infty}d\tau\int d^3r'\,\boldsymbol{Q}(\mathbf{r},\mathbf{r}',\tau)\cdot\hat{\mathbf{E}}(\mathbf{r}',t-\tau)+\hat{\mathbf{j}}_{\mathrm{N}}(\mathbf{r},t)$$

with causal conductivity tensor $\boldsymbol{Q}(\mathbf{r},\mathbf{r}',\tau)=0$ for $c\tau<|\mathbf{r}-\mathbf{r}'|$

generalised inhomogeneous Helmholtz equation:

$$\left[\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times-\frac{\omega^2}{c^2}\right]\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)-i\mu_0\omega\int d^3s\,\boldsymbol{Q}(\mathbf{r},\mathbf{s},\omega)\cdot\boldsymbol{G}(\mathbf{s},\mathbf{r}',\omega)=\boldsymbol{\delta}(\mathbf{r}-\mathbf{r}')$$

linear Hamiltonian with bosonic amplitude operators: $\hat{H}=\int d^3r\int_0^{\infty}d\omega\,\hbar\omega\,\hat{\mathbf{f}}^{\dagger}(\mathbf{r},\omega)\cdot\hat{\mathbf{f}}(\mathbf{r},\omega)$

from decomposition of current $\hat{\mathbf{j}}_{\mathrm{N}}(\mathbf{r},\omega)=\sqrt{\frac{\hbar\omega}{\pi}}\int d^3r'\,\boldsymbol{R}(\mathbf{r},\mathbf{r}',\omega)\cdot\hat{\mathbf{f}}(\mathbf{r}',\omega)$ with $\boldsymbol{R}$ a square root of the positive definite tensor field $\mathcal{R}e\,\boldsymbol{Q}$

C. Raabe, S. Scheel, and D.-G. Welsch, Phys. Rev. A **75**, 053813 (2007); S.Y. Buhmann, D.T. Butcher, and S. Scheel, New J. Phys. **14**, 083034 (2012).

General linear dielectrics

Most general linear constitutive relation:

$$\hat{\mathbf{j}}_{\mathrm{in}}(\mathbf{r},t) = \int_{-\infty}^{\infty} d\tau \int d^3r'\, \boldsymbol{Q}(\mathbf{r},\mathbf{r}',\tau)\cdot\hat{\mathbf{E}}(\mathbf{r}',t-\tau) + \hat{\mathbf{j}}_{\mathrm{N}}(\mathbf{r},t)$$

with causal conductivity tensor $\boldsymbol{Q}(\mathbf{r},\mathbf{r}',\tau)=0$ for $c\tau<|\mathbf{r}-\mathbf{r}'|$

generalised inhomogeneous Helmholtz equation:

$$\left[\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times-\frac{\omega^2}{c^2}\right]\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega) - i\mu_0\omega\int d^3s\, \boldsymbol{Q}(\mathbf{r},\mathbf{s},\omega)\cdot\boldsymbol{G}(\mathbf{s},\mathbf{r}',\omega) = \boldsymbol{\delta}(\mathbf{r}-\mathbf{r}')$$

linear Hamiltonian with bosonic amplitude operators: $\hat{H}=\int d^3r\int_0^\infty d\omega\,\hbar\omega\,\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)\cdot\hat{\mathbf{f}}(\mathbf{r},\omega)$

from decomposition of current $\hat{\mathbf{j}}_{\mathrm{N}}(\mathbf{r},\omega)=\sqrt{\frac{\hbar\omega}{\pi}}\int d^3r'\,\boldsymbol{R}(\mathbf{r},\mathbf{r}',\omega)\cdot\hat{\mathbf{f}}(\mathbf{r}',\omega)$ with $\boldsymbol{R}$ a square root of the positive definite tensor field $\mathcal{R}e\,\boldsymbol{Q}$

C. Raabe, S. Scheel, and D.-G. Welsch, Phys. Rev. A **75**, 053813 (2007); S.Y. Buhmann, D.T. Butcher, and S. Scheel, New J. Phys. **14**, 083034 (2012).

General linear dielectrics

Most general linear constitutive relation:

$$\hat{\mathbf{j}}_{\mathrm{in}}(\mathbf{r},t) = \int_{-\infty}^{\infty} d\tau \int d^3r'\, \boldsymbol{Q}(\mathbf{r},\mathbf{r}',\tau)\cdot\hat{\mathbf{E}}(\mathbf{r}',t-\tau) + \hat{\mathbf{j}}_{\mathrm{N}}(\mathbf{r},t)$$

with causal conductivity tensor $\boldsymbol{Q}(\mathbf{r},\mathbf{r}',\tau)=0$ for $c\tau<|\mathbf{r}-\mathbf{r}'|$

generalised inhomogeneous Helmholtz equation:

$$\left[\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times-\frac{\omega^2}{c^2}\right]\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega) - i\mu_0\omega\int d^3s\, \boldsymbol{Q}(\mathbf{r},\mathbf{s},\omega)\cdot\boldsymbol{G}(\mathbf{s},\mathbf{r}',\omega) = \boldsymbol{\delta}(\mathbf{r}-\mathbf{r}')$$

linear Hamiltonian with bosonic amplitude operators: $\hat{H} = \int d^3r \int_0^\infty d\omega\, \hbar\omega\, \hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)\cdot\hat{\mathbf{f}}(\mathbf{r},\omega)$

from decomposition of current $\hat{\mathbf{j}}_{\mathrm{N}}(\mathbf{r},\omega) = \sqrt{\frac{\hbar\omega}{\pi}}\int d^3r'\, \boldsymbol{R}(\mathbf{r},\mathbf{r}',\omega)\cdot\hat{\mathbf{f}}(\mathbf{r}',\omega)$ with $\boldsymbol{R}$ a square root of the positive definite tensor field $\mathcal{R}\mathrm{e}\,\boldsymbol{Q}$

C. Raabe, S. Scheel, and D.-G. Welsch, Phys. Rev. A **75**, 053813 (2007); S.Y. Buhmann, D.T. Butcher, and S. Scheel, New J. Phys. **14**, 083034 (2012).

Properties of the Green tensor

- Schwarz reflection principle $\boldsymbol{G}^*(\mathbf{r},\mathbf{r}',\omega) = \boldsymbol{G}(\mathbf{r},\mathbf{r}',-\omega^*)$
- do not require reciprocity, $\boldsymbol{G}^{\mathsf{T}}(\mathbf{r}',\mathbf{r},\omega) \neq \boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)$
- integral relation for Green tensor:

$$\mu_0\omega \int d^3s \int d^3s'\, \boldsymbol{G}(\mathbf{r},\mathbf{s},\omega)\cdot \mathcal{R}\mathrm{e}\,\boldsymbol{Q}(\mathbf{s},\mathbf{s}',\omega)\cdot \boldsymbol{G}^\dagger(\mathbf{r}',\mathbf{s}',\omega) = \mathcal{I}\mathrm{m}\,\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)$$

  with generalised real and imaginary parts of a tensor field:

$$\mathcal{R}\mathrm{e}\,\boldsymbol{T}(\mathbf{r},\mathbf{r}') = \tfrac{1}{2}\left[\boldsymbol{T}(\mathbf{r},\mathbf{r}') + \boldsymbol{T}^\dagger(\mathbf{r}',\mathbf{r})\right],$$
$$\mathcal{I}\mathrm{m}\,\boldsymbol{T}(\mathbf{r},\mathbf{r}') = \tfrac{1}{2i}\left[\boldsymbol{T}(\mathbf{r},\mathbf{r}') - \boldsymbol{T}^\dagger(\mathbf{r}',\mathbf{r})\right]$$

- field fluctuations (generalised linear fluctuation-dissipation relation):

$$\left\langle\left\{\Delta\hat{\mathbf{E}}(\mathbf{r},\omega),\Delta\hat{\mathbf{E}}^\dagger(\mathbf{r}',\omega')\right\}\right\rangle = \frac{\hbar}{\pi}\,\mathcal{I}\mathrm{m}\left[\mu_0\omega^2\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\right]\delta(\omega-\omega')$$

General linear media and duality

Maxwell's equations in dual-pair notation:

$$\boldsymbol{\nabla}\cdot\begin{pmatrix} Z_0\hat{\mathbf{D}} \\ \hat{\mathbf{B}} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{\nabla}\times\begin{pmatrix} \hat{\mathbf{E}} \\ Z_0\hat{\mathbf{H}} \end{pmatrix} - i\omega\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} Z_0\hat{\mathbf{D}} \\ \hat{\mathbf{B}} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

constitutive relations:

$$\begin{pmatrix} Z_0\hat{\mathbf{D}} \\ \hat{\mathbf{B}} \end{pmatrix} = \frac{1}{c}\begin{pmatrix} \varepsilon & \xi \\ \zeta & \mu \end{pmatrix}\star\begin{pmatrix} \hat{\mathbf{E}} \\ Z_0\hat{\mathbf{H}} \end{pmatrix} + \begin{pmatrix} \boldsymbol{I} & \xi \\ \mathbf{0} & \mu \end{pmatrix}\star\begin{pmatrix} Z_0\hat{\mathbf{P}}_N \\ \mu_0\hat{\mathbf{M}}_N \end{pmatrix}$$

Maxwell's equations invariant under duality transformation

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix}^{\circledast} = D(\Theta)\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix}, \quad D(\Theta) = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix}$$

reciprocal media: discrete symmetry with $\Theta = n\pi/2$
nonreciprocal media: continuous symmetry!

S.Y. Buhmann and S. Scheel, Phys. Rev. Lett. **102**, 140404 (2010); S.Y. Buhmann, D.T. Butcher, and S. Scheel, New J. Phys. **14**, 083034 (2012).

Classification of media

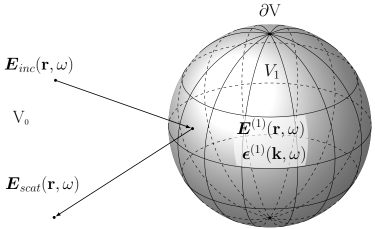
- *Isotropic media* ($\boldsymbol{\varepsilon}=\varepsilon\boldsymbol{I}$, $\boldsymbol{\mu}=\mu\boldsymbol{I}$, $\boldsymbol{\xi}=\boldsymbol{\zeta}=\boldsymbol{0}$): Onsager reciprocity holds; generalised real and imaginary parts reduce to ordinary ones; discrete duality symmetry.
- *Biisotropic media* ($\boldsymbol{\varepsilon}=\varepsilon\boldsymbol{I}$, $\boldsymbol{\mu}=\mu\boldsymbol{I}$, $\boldsymbol{\xi}=\xi\boldsymbol{I}$, $\boldsymbol{\zeta}=\zeta\boldsymbol{I}$): generalised real and imaginary parts reduce to ordinary ones; continuous duality symmetry.
- *Anisotropic media* ($\boldsymbol{\xi}=\boldsymbol{\zeta}=\boldsymbol{0}$): discrete duality symmetry.

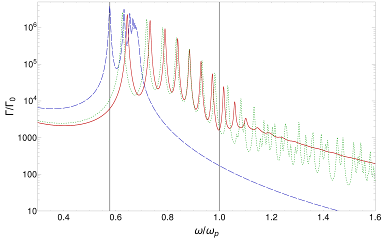
important examples:

- *Non-reciprocal media*: $\boldsymbol{\chi}=(\boldsymbol{\zeta}+\boldsymbol{\xi}^{\mathsf{T}})/2\neq\boldsymbol{0}$
- *Chiral media*: $\boldsymbol{\kappa}=(\boldsymbol{\zeta}-\boldsymbol{\xi}^{\mathsf{T}})/(2i)\neq\boldsymbol{0}$

Example: LDOS near a spatially dispersive nanosphere

dyadic Green function from Huygens' principle and extinction theorem with Maxwell boundary conditions





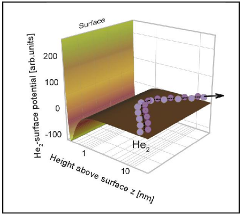
radial LDOS near Na nanosphere with 2nm radius from local and nonlocal Mie theories

R. Schmidt and S. Scheel, Phys. Rev. A **93**, 033804 (2016).

Quantum optics of macroscopic systems

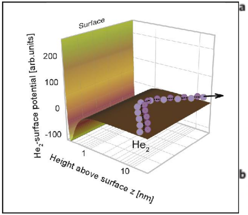
Quantum optics of macroscopic systems

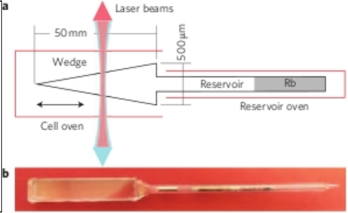
Examples of dispersion forces



Quantum optics of macroscopic systems

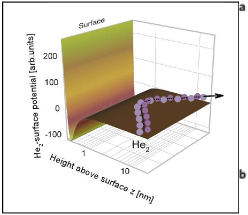
Examples of dispersion forces

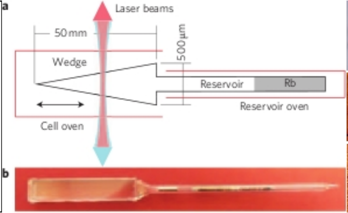


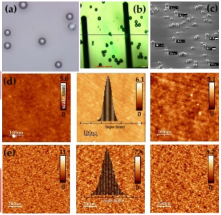


Quantum optics of macroscopic systems

Examples of dispersion forces

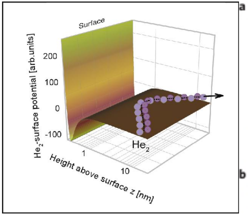


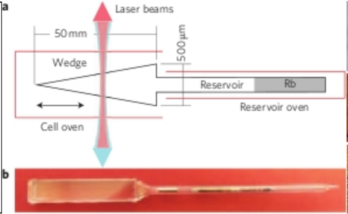


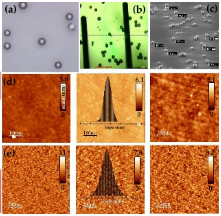


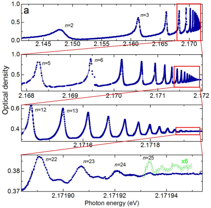
Quantum optics of macroscopic systems

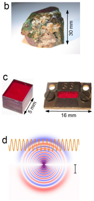
Examples of dispersion forces





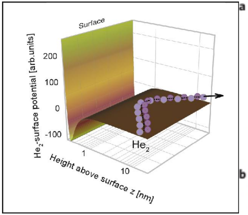


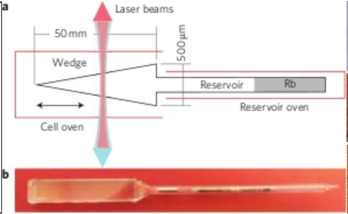


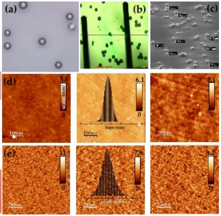


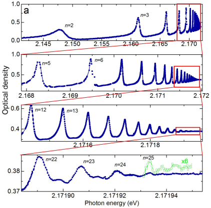
Quantum optics of macroscopic systems

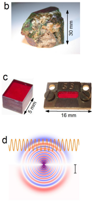
Examples of dispersion forces





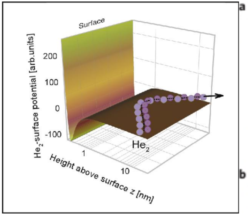


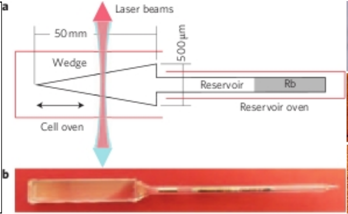


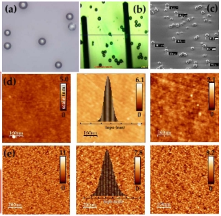


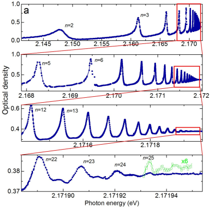
Quantum optics of macroscopic systems

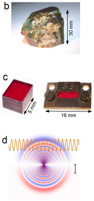
Examples of dispersion forces

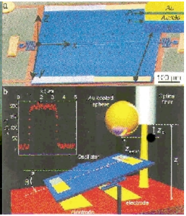












Dispersion forces as average quantum Lorentz forces

average of quantum Lorentz force

$$\hat{\mathbf{F}} = \int_V d^3r \left[\hat{\varrho}(\mathbf{r})\hat{\mathbf{E}}(\mathbf{r}) + \hat{\mathbf{j}}(\mathbf{r}) \times \hat{\mathbf{B}}(\mathbf{r})\right]$$

seems to vanish in the ground state because $\langle 0|\hat{\mathbf{E}}(\mathbf{r})|0\rangle = \langle 0|\hat{\mathbf{B}}(\mathbf{r})|0\rangle = 0$ as well as $\langle 0|\hat{\varrho}(\mathbf{r})|0\rangle = \langle 0|\hat{\mathbf{j}}(\mathbf{r})|0\rangle = 0$

But: it acquires nonzero average even in the absence of external electromagnetic fields due to correlated zero-point fluctuations $\langle 0|\hat{\mathbf{E}}^2(\mathbf{r})|0\rangle \neq 0$

$\Rightarrow$ relate charge and current densities to electromagnetic fields and use field correlation functions to obtain nonzero average force:

$$\langle\hat{\mathbf{E}}(\mathbf{r},\omega)\hat{\mathbf{E}}^\dagger(\mathbf{r}',\omega')\rangle \propto \omega^2 \mathrm{Im}\,\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\delta(\omega-\omega')[\bar{n}_{\mathrm{th}}(\omega)+1]$$

Dispersion forces as average quantum Lorentz forces

average of quantum Lorentz force

$$\hat{\mathbf{F}} = \int_V d^3r \left[\hat{\varrho}(\mathbf{r})\hat{\mathbf{E}}(\mathbf{r}) + \hat{\mathbf{j}}(\mathbf{r}) \times \hat{\mathbf{B}}(\mathbf{r})\right]$$

seems to vanish in the ground state because $\langle 0|\hat{\mathbf{E}}(\mathbf{r})|0\rangle = \langle 0|\hat{\mathbf{B}}(\mathbf{r})|0\rangle = 0$ as well as $\langle 0|\hat{\varrho}(\mathbf{r})|0\rangle = \langle 0|\hat{\mathbf{j}}(\mathbf{r})|0\rangle = 0$

But: it acquires nonzero average even in the absence of external electromagnetic fields due to correlated zero-point fluctuations $\langle 0|\hat{\mathbf{E}}^2(\mathbf{r})|0\rangle \neq 0$

$\Rightarrow$ relate charge and current densities to electromagnetic fields and use field correlation functions to obtain nonzero average force:

$$\langle\hat{\mathbf{E}}(\mathbf{r},\omega)\hat{\mathbf{E}}^\dagger(\mathbf{r}',\omega')\rangle \propto \omega^2 \mathrm{Im}\, \boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\delta(\omega-\omega')[\bar{n}_{\mathrm{th}}(\omega)+1]$$

Dispersion forces as average quantum Lorentz forces

average of quantum Lorentz force

$$\hat{\mathbf{F}} = \int_V d^3r \left[\hat{\varrho}(\mathbf{r})\hat{\mathbf{E}}(\mathbf{r}) + \hat{\mathbf{j}}(\mathbf{r}) \times \hat{\mathbf{B}}(\mathbf{r})\right]$$

seems to vanish in the ground state because $\langle 0|\hat{\mathbf{E}}(\mathbf{r})|0\rangle = \langle 0|\hat{\mathbf{B}}(\mathbf{r})|0\rangle = 0$ as well as $\langle 0|\hat{\varrho}(\mathbf{r})|0\rangle = \langle 0|\hat{\mathbf{j}}(\mathbf{r})|0\rangle = 0$

But: it acquires nonzero average even in the absence of external electromagnetic fields due to correlated zero-point fluctuations $\langle 0|\hat{\mathbf{E}}^2(\mathbf{r})|0\rangle \neq 0$

$\Rightarrow$ relate charge and current densities to electromagnetic fields and use field correlation functions to obtain nonzero average force:

$$\langle\hat{\mathbf{E}}(\mathbf{r},\omega)\hat{\mathbf{E}}^\dagger(\mathbf{r}',\omega')\rangle \propto \omega^2 \mathrm{Im}\,\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\delta(\omega-\omega')[\bar{n}_{\mathrm{th}}(\omega)+1]$$

Universal scaling laws for dispersion interactions

- Casimir stress: $\hat{\boldsymbol{T}}(\mathbf{r},\mathbf{r}) = \lim_{\mathbf{r}'\to\mathbf{r}} \left[\boldsymbol{\Theta}(\mathbf{r},\mathbf{r}') - \frac{1}{2}\mathrm{Tr}\boldsymbol{\Theta}(\mathbf{r},\mathbf{r}')\right]$

$$\boldsymbol{\Theta}(\mathbf{r},\mathbf{r}') = \frac{\hbar}{\pi}\int_0^\infty d\omega \left[\frac{\omega^2}{c^2}\,\mathrm{Im}\,\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega) - \boldsymbol{\nabla}\times\mathrm{Im}\,\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\times\overleftarrow{\boldsymbol{\nabla}}\right]$$

- Casimir–Polder potential: $U(\mathbf{r}_A) = \frac{\hbar\mu_0}{2\pi}\int_0^\infty d\xi\,\xi^2\alpha_A(i\xi)\mathrm{Tr}\,\boldsymbol{G}^{(S)}(\mathbf{r}_A,\mathbf{r}_A,i\xi)$

- van der Waals potential:

$$U(\mathbf{r}_A,\mathbf{r}_B) = -\frac{\hbar\mu_0^2}{2\pi}\int_0^\infty d\xi\,\xi^4\alpha_A(i\xi)\alpha_B(i\xi)\mathrm{Tr}\left[\boldsymbol{G}^{(S)}(\mathbf{r}_A,\mathbf{r}_B,i\xi)\cdot\boldsymbol{G}^{(S)}(\mathbf{r}_B,\mathbf{r}_A,i\xi)\right]$$

Universal scaling laws for dispersion interactions

- Casimir stress: $\hat{\boldsymbol{T}}(\mathbf{r},\mathbf{r}) = \lim_{\mathbf{r}'\to\mathbf{r}} \left[\boldsymbol{\Theta}(\mathbf{r},\mathbf{r}') - \frac{1}{2}\mathrm{Tr}\boldsymbol{\Theta}(\mathbf{r},\mathbf{r}')\right]$

$$\boldsymbol{\Theta}(\mathbf{r},\mathbf{r}') = \frac{\hbar}{\pi}\int_0^\infty d\omega \left[\frac{\omega^2}{c^2}\,\mathrm{Im}\,\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega) - \boldsymbol{\nabla}\times\mathrm{Im}\,\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\times\overleftarrow{\boldsymbol{\nabla}}'\right]$$

- Casimir–Polder potential: $U(\mathbf{r}_A) = \frac{\hbar\mu_0}{2\pi}\int_0^\infty d\xi\,\xi^2\alpha_A(i\xi)\mathrm{Tr}\,\boldsymbol{G}^{(S)}(\mathbf{r}_A,\mathbf{r}_A,i\xi)$

- van der Waals potential:

$$U(\mathbf{r}_A,\mathbf{r}_B) = -\frac{\hbar\mu_0^2}{2\pi}\int_0^\infty d\xi\,\xi^4\alpha_A(i\xi)\alpha_B(i\xi)\mathrm{Tr}\left[\boldsymbol{G}^{(S)}(\mathbf{r}_A,\mathbf{r}_B,i\xi)\cdot\boldsymbol{G}^{(S)}(\mathbf{r}_B,\mathbf{r}_A,i\xi)\right]$$

Universal scaling laws for dispersion interactions

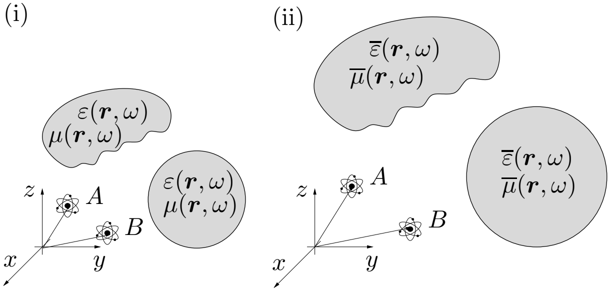
- Casimir stress: $\hat{\boldsymbol{T}}(\mathbf{r},\mathbf{r}) = \lim_{\mathbf{r}'\to\mathbf{r}} \left[\boldsymbol{\Theta}(\mathbf{r},\mathbf{r}') - \frac{1}{2}\mathrm{Tr}\,\boldsymbol{\Theta}(\mathbf{r},\mathbf{r}')\right]$

$$\boldsymbol{\Theta}(\mathbf{r},\mathbf{r}') = \frac{\hbar}{\pi}\int_0^\infty d\omega\,\left[\frac{\omega^2}{c^2}\,\mathrm{Im}\,\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega) - \boldsymbol{\nabla}\times\mathrm{Im}\,\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)\times\overleftarrow{\boldsymbol{\nabla}}\right]$$

- Casimir–Polder potential: $U(\mathbf{r}_A) = \frac{\hbar\mu_0}{2\pi}\int_0^\infty d\xi\,\xi^2\alpha_A(i\xi)\mathrm{Tr}\,\boldsymbol{G}^{(S)}(\mathbf{r}_A,\mathbf{r}_A,i\xi)$
- van der Waals potential:

$$U(\mathbf{r}_A,\mathbf{r}_B) = -\frac{\hbar\mu_0^2}{2\pi}\int_0^\infty d\xi\,\xi^4\alpha_A(i\xi)\alpha_B(i\xi)\mathrm{Tr}\left[\boldsymbol{G}^{(S)}(\mathbf{r}_A,\mathbf{r}_B,i\xi)\cdot\boldsymbol{G}^{(S)}(\mathbf{r}_B,\mathbf{r}_A,i\xi)\right]$$

Universal scaling laws for dispersion interactions



- scaled arrangement with $\bar{\varepsilon}(\mathbf{r},\omega) = \varepsilon(\mathbf{r}/a,\omega)$ and $\bar{\mu}(\mathbf{r},\omega) = \mu(\mathbf{r}/a,\omega)$
- atomic positions $\bar{\mathbf{r}}_A = a\mathbf{r}_A$ and $\bar{\mathbf{r}}_B = a\mathbf{r}_B$

Long-distance (retarded) limit

approximate response functions by static values
$\alpha(\omega) \simeq \alpha, \varepsilon(\mathbf{r},\omega) \simeq \varepsilon(\mathbf{r})$

Green tensor of the scaled arrangement obeys

$$\left[\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times - \frac{\omega^2}{c^2}\,\overline{\varepsilon}(\mathbf{r})\right] \overline{\boldsymbol{G}}(\mathbf{r},\mathbf{r}',\omega) = \boldsymbol{\delta}(\mathbf{r}-\mathbf{r}')$$

renaming $\mathbf{r} \mapsto a\mathbf{r}$, $\omega \mapsto \omega/a$:

$$\left[\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times - \frac{\omega^2}{c^2}\,\varepsilon(\mathbf{r})\right] a\overline{\boldsymbol{G}}(a\mathbf{r},a\mathbf{r}',\omega/a) = \boldsymbol{\delta}(\mathbf{r}-\mathbf{r}') \Rightarrow \overline{\boldsymbol{G}}(a\mathbf{r},a\mathbf{r}',\omega/a) = \frac{1}{a}\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)$$

- $\overline{\boldsymbol{T}}(a\mathbf{r}) = (1/a^4)\boldsymbol{T}(\mathbf{r})$
- $\overline{U}(a\mathbf{r}_A) = (1/a^4)U(\mathbf{r}_A)$
- $\overline{U}(a\mathbf{r}_A, a\mathbf{r}_B) = (1/a^7)U(\mathbf{r}_A,\mathbf{r}_B)$

Long-distance (retarded) limit

approximate response functions by static values
$\alpha(\omega) \simeq \alpha, \varepsilon(\mathbf{r},\omega) \simeq \varepsilon(\mathbf{r})$

Green tensor of the scaled arrangement obeys

$$\left[\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times -\frac{\omega^2}{c^2}\,\overline{\varepsilon}(\mathbf{r})\right] \overline{\boldsymbol{G}}(\mathbf{r},\mathbf{r}',\omega) = \boldsymbol{\delta}(\mathbf{r}-\mathbf{r}')$$

renaming $\mathbf{r} \mapsto a\mathbf{r}, \omega \mapsto \omega/a$:

$$\left[\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times -\frac{\omega^2}{c^2}\,\varepsilon(\mathbf{r})\right] a\overline{\boldsymbol{G}}(a\mathbf{r},a\mathbf{r}',\omega/a) = \boldsymbol{\delta}(\mathbf{r}-\mathbf{r}') \Rightarrow \overline{\boldsymbol{G}}(a\mathbf{r},a\mathbf{r}',\omega/a) = \frac{1}{a}\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)$$

- $\overline{\boldsymbol{T}}(a\mathbf{r}) = (1/a^4)\boldsymbol{T}(\mathbf{r})$
- $\overline{U}(a\mathbf{r}_A) = (1/a^4)U(\mathbf{r}_A)$
- $\overline{U}(a\mathbf{r}_A, a\mathbf{r}_B) = (1/a^7)U(\mathbf{r}_A,\mathbf{r}_B)$

Short-distance (nonretarded) limit

distance shorter than all characteristic wavelengths

Born series for dyadic Green function for electric bodies:

$$\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega) = \boldsymbol{G}^{(0)}(\mathbf{r},\mathbf{r}',\omega) + \frac{\omega^2}{c^2}\int d^3s\,\chi(\mathbf{s},\omega)\boldsymbol{G}^{(0)}(\mathbf{r},\mathbf{s},\omega)\cdot\boldsymbol{G}(\mathbf{s},\mathbf{r}',\omega)$$

in short-distance limit: $\boldsymbol{G}^{(0)}(\mathbf{r},\mathbf{r}',\omega) = -\frac{c^2[\boldsymbol{I}-3\mathbf{e}_\rho\otimes\mathbf{e}_\rho]}{4\pi\omega^2\rho^3} - \frac{c^2}{3\omega^2}\delta(\boldsymbol{\rho})$

$$\Rightarrow \overline{\boldsymbol{G}}(a\mathbf{r},a\mathbf{r}',\omega) = \frac{1}{a^3}\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)$$

- $\overline{\boldsymbol{T}}(a\mathbf{r}) = (1/a^3)\boldsymbol{T}(\mathbf{r})$
- $\overline{U}(a\mathbf{r}_A) = (1/a^3)U(\mathbf{r}_A)$
- $\overline{U}(a\mathbf{r}_A,a\mathbf{r}_B) = (1/a^6)U(\mathbf{r}_A,\mathbf{r}_B)$

Short-distance (nonretarded) limit

distance shorter than all characteristic wavelengths

Born series for dyadic Green function for electric bodies:

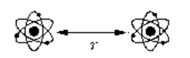
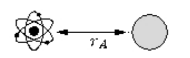
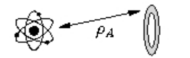
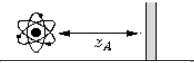
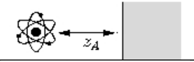
$$\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)=\boldsymbol{G}^{(0)}(\mathbf{r},\mathbf{r}',\omega)+\frac{\omega^2}{c^2}\int d^3s\,\chi(\mathbf{s},\omega)\boldsymbol{G}^{(0)}(\mathbf{r},\mathbf{s},\omega)\cdot\boldsymbol{G}(\mathbf{s},\mathbf{r}',\omega)$$

in short-distance limit: $\boldsymbol{G}^{(0)}(\mathbf{r},\mathbf{r}',\omega)=-\dfrac{c^2[\boldsymbol{I}-3\mathbf{e}_\rho\otimes\mathbf{e}_\rho]}{4\pi\omega^2\rho^3}-\dfrac{c^2}{3\omega^2}\delta(\boldsymbol{\rho})$

$$\Rightarrow\overline{\boldsymbol{G}}(a\mathbf{r},a\mathbf{r}',\omega)=\frac{1}{a^3}\boldsymbol{G}(\mathbf{r},\mathbf{r}',\omega)$$

- $\overline{T}(a\mathbf{r})=(1/a^3)T(\mathbf{r})$
- $\overline{U}(a\mathbf{r}_A)=(1/a^3)U(\mathbf{r}_A)$
- $\overline{U}(a\mathbf{r}_A,a\mathbf{r}_B)=(1/a^6)U(\mathbf{r}_A,\mathbf{r}_B)$

Universal scaling laws from the Helmholtz equation

Distance →		Retarded		Nonretarded	
Object combination →		$e \leftrightarrow e$	$e \leftrightarrow m$	$e \leftrightarrow e$	$e \leftrightarrow m$
Dual object combination →		$m \leftrightarrow m$	$m \leftrightarrow e$	$m \leftrightarrow m$	$m \leftrightarrow e$
(a)		$-\frac{1}{r^8}$	$+\frac{1}{r^8}$	$-\frac{1}{r^7}$	$+\frac{1}{r^5}$
(b)		$-\frac{1}{r_A^8}$	$+\frac{1}{r_A^8}$	$-\frac{1}{r_A^7}$	$+\frac{1}{r_A^5}$
(c)		$-\frac{1}{\rho_A^8}$	$+\frac{1}{\rho_A^8}$	$-\frac{1}{\rho_A^7}$	$+\frac{1}{\rho_A^5}$
(d)		$-\frac{1}{z_A^6}$	$+\frac{1}{z_A^6}$	$-\frac{1}{z_A^5}$	$+\frac{1}{z_A^3}$
(e)		$-\frac{1}{z_A^5}$	$+\frac{1}{z_A^5}$	$-\frac{1}{z_A^4}$	$+\frac{1}{z_A^2}$
(f)		$-\frac{1}{z^4}$	$+\frac{1}{z^4}$	$-\frac{1}{z^3}$	$+\frac{1}{z}$

S. Scheel and S.Y. Buhmann, Acta Physica Slovaca **58**, 675 (2008).

Quantum optics of macroscopic systems

Nonequilibrium dispersion forces

Casimir–Polder force in thermal nonequlibrium from dynamical theory

Casimir–Polder force in perturbative limit [$\xi_N = 2\pi k_B T N/\hbar$: Matsubara freq.]

$$\mathbf{F}_n(\mathbf{r}_A) = -\mu_0 k_B T \sum_{N=0}^{\infty} \left(1 - \tfrac{1}{2}\delta_{N0}\right) \xi_N^2 \alpha_n(i\xi_N) \nabla_A \mathrm{Tr}\, \boldsymbol{G}(\mathbf{r}_A, \mathbf{r}_A, i\xi_N) +$$
$$\frac{\mu_0}{3} \sum_k \omega_{nk}^2 \left\{\Theta(\omega_{nk})\left[\bar{n}_{\mathrm{th}}(\omega_{nk}) + 1\right] - \Theta(\omega_{kn})\bar{n}_{\mathrm{th}}(\omega_{kn})\right\} |\mathbf{d}_{nk}|^2 \nabla_A \mathrm{Tr}\,\mathrm{Re}\, \boldsymbol{G}(\mathbf{r}_A, \mathbf{r}_A, \omega_{nk})$$

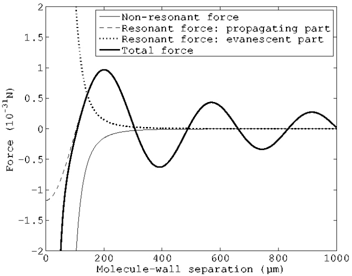
- first term: nonresonant (Lifshitz-like) force component, all (Matsubara) frequencies involved
- second term: resonant force components at atomic transition frequencies
  - further division into evanescent and propagating parts
  - due to absorption and emission of thermal photons
  - for ground-state atoms: resonant force components visible on time scales smaller than inverse ground-state heating rate Γ_{0k}^{-1}

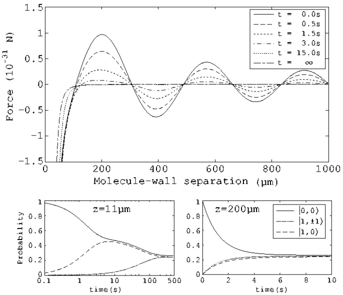
S.Y. Buhmann and S. Scheel, Phys. Rev. Lett. **100**, 253201 (2008).

Quantum optics of macroscopic systems

Dispersion forces in thermal nonequilibrium

ground-state LiH near a gold surface at room temperature:





S.Å. Ellingsen, S.Y. Buhmann, and S. Scheel, Phys. Rev. A **79**, 052903 (2009).

Temperature invariance despite large photon numbers

total potential becomes temperature-independent

in the spectroscopic high-temperature limit $T \gg T_\omega = \hbar|\omega_{kn}|/k_{\mathrm{B}}$ (temperature of radiation whose wavelength is of order z_A): lowest term in Matsubara sum dominates

and the geometric low-temperature limit $T \ll T_z = \hbar c/(z_A k_{\mathrm{B}})$ (temperature necessary to noticably populate the upper level): exponential ≈ 1, sum can be performed

S.Å. Ellingsen, S.Y. Buhmann, and S. Scheel, Phys. Rev. Lett. **104**, 223003 (2010).

Temperature invariance despite large photon numbers

total potential becomes temperature-independent

in the spectroscopic high-temperature limit $T \gg T_\omega = \hbar|\omega_{kn}|/k_\mathrm{B}$ (temperature of radiation whose wavelength is of order z_A): lowest term in Matsubara sum dominates

and the geometric low-temperature limit $T \ll T_z = \hbar c/(z_A k_\mathrm{B})$ (temperature necessary to noticably populate the upper level): exponential ≈ 1, sum can be performed

S.Å. Ellingsen, S.Y. Buhmann, and S. Scheel, Phys. Rev. Lett. **104**, 223003 (2010).

Temperature invariance despite large photon numbers

total potential becomes temperature-independent

in the spectroscopic high-temperature limit $T \gg T_\omega = \hbar|\omega_{kn}|/k_\mathrm{B}$ (temperature of radiation whose wavelength is of order z_A): lowest term in Matsubara sum dominates

and the geometric low-temperature limit $T \ll T_z = \hbar c/(z_A k_\mathrm{B})$ (temperature necessary to noticably populate the upper level): exponential ≈ 1, sum can be performed

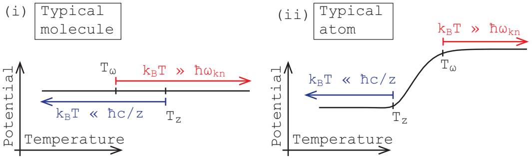
S.Å. Ellingsen, S.Y. Buhmann, and S. Scheel, Phys. Rev. Lett. **104**, 223003 (2010).

Temperature invariance despite large photon numbers

total potential becomes temperature-independent

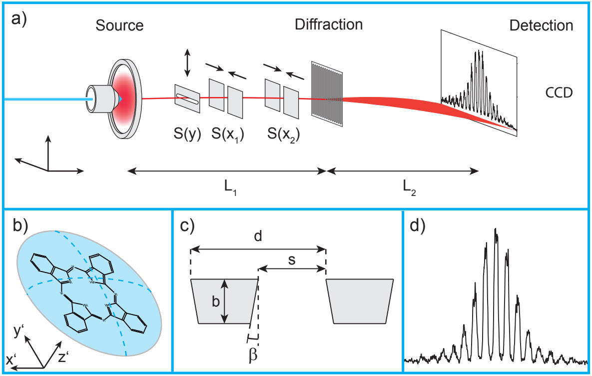
in the spectroscopic high-temperature limit $T \gg T_\omega = \hbar|\omega_{kn}|/k_{\mathrm{B}}$ (temperature of radiation whose wavelength is of order z_A): lowest term in Matsubara sum dominates

and the geometric low-temperature limit $T \ll T_z = \hbar c/(z_A k_{\mathrm{B}})$ (temperature necessary to noticably populate the upper level): exponential ≈ 1, sum can be performed



S.Å. Ellingsen, S.Y. Buhmann, and S. Scheel, Phys. Rev. Lett. **104**, 223003 (2010).

Casimir–Polder interaction with large molecules



C. Brand, J. Fiedler, T. Juffmann, M. Sclafani, C. Knobloch, S. Scheel, Y. Lilach, O. Cheshnovsky, and M. Arndt, Annals of Physics **527**, 580 (2015); J. Fiedler and S. Scheel, Annals of Physics **527**, 570 (2015).

Spectral energy density of the electromagnetic field

spectral energy density from fluctuation-dissipation theorem (Rytov):

$$u(\mathbf{r},\omega)=\frac{\varepsilon_0}{2}\langle\hat{\mathbf{E}}^\dagger(\mathbf{r},\omega)\cdot\hat{\mathbf{E}}(\mathbf{r},\omega)\rangle_T=\frac{\hbar\omega^2}{2\pi c^2}\bar{n}_{\mathrm{th}}(\omega)\,\mathrm{Tr}\,\mathrm{Im}\,\boldsymbol{G}(\mathbf{r},\mathbf{r},\omega)$$

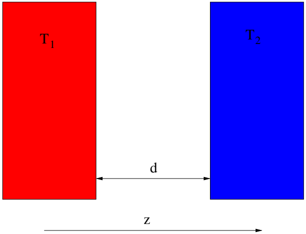
integrated density ($\boldsymbol{G}=\boldsymbol{G}^{(0)}+\boldsymbol{G}^{(S)}$):

$$\int_0^\infty d\omega\,u(\mathbf{r},\omega)=\underbrace{\frac{\hbar}{4\pi^2c^3}\int_0^\infty d\omega\,\omega^3\bar{n}_{\mathrm{th}}(\omega)}_{=\frac{1}{c}\sigma T^4}+\underbrace{\frac{\hbar}{2\pi c^2}\int_0^\infty d\omega\,\omega^2\bar{n}_{\mathrm{th}}(\omega)\,\mathrm{Tr}\,\mathrm{Im}\,\boldsymbol{G}^{(S)}(\mathbf{r},\mathbf{r},\omega)}_{\simeq\frac{\hbar}{16\pi^2z^3}\int d\omega\,\bar{n}_{\mathrm{th}}(\omega)\,\mathrm{Im}\,r_p(\omega)}$$

near-field LDOS dominates free-space LDOS, scaling laws equivalent to dispersion forces

Heat transfer between dielectric objects

heat flux between bodies at different local temperatures (justification from microscopic theories)



heat flux $\Phi = \langle S_z^{1\to 2}\rangle_T - \langle S_z^{2\to 1}\rangle_T$

average Poynting vector $\langle \hat{\mathbf{S}}(\mathbf{r})\rangle_T = \langle \hat{\mathbf{E}}(\mathbf{r}) \times \hat{\mathbf{H}}(\mathbf{r})\rangle_T$

fluctuations of individual bodies

$$\langle \hat{\mathbf{E}}(\mathbf{r},\omega) \otimes \hat{\mathbf{E}}(\mathbf{r}',\omega)\rangle_T = \int_0^\infty d\omega\, \bar{n}_{\mathrm{th}}(\omega) \frac{\hbar\omega^4}{\pi\varepsilon_0 c^4} \int_V d^3s\, \varepsilon_I(\mathbf{s},\omega) \boldsymbol{G}(\mathbf{r},\mathbf{s},\omega) \cdot \boldsymbol{G}^*(\mathbf{s},\mathbf{r}',\omega)$$

$\Rightarrow$ heat flux from local integrals of LDOS (i.e. dyadic Green function)

A.I. Volokitin and B.N.J. Persson, Rev. Mod. Phys. **79**, 1291 (2007).

Quantum optics of macroscopic systems

Macroscopic QED and Rytov theory

Dispersion forced from macroscopic QED

Nonequilibrium dispersion forces and heat transfer

Summary

Summary

- macroscopic quantum electrodynamics from microscopic Hopfield models equivalent to statistical theories (Rytov theory) in thermal equilibrium
- Green function formalism provides unified approach to dispersion forces and heat transfer
- dispersion forces in thermal nonequilibrium require dynamical theory provides by macroscopic QED
- near-field heat transfer follows the same scaling laws as dispersion forces (both related to LDOS)

- extensions to nonequilibrium (stationary) fluctuation-dissipation theorems possible (e.g. for quantum friction)
- theory only applicable for macroscopic bodies (definition of permittivities possible, tricky for particles smaller than 1nm)

Quantum optics of macroscopic systems

Summary

- macroscopic quantum electrodynamics from microscopic Hopfield models equivalent to statistical theories (Rytov theory) in thermal equilibrium
- Green function formalism provides unified approach to dispersion forces and heat transfer
- dispersion forces in thermal nonequilibrium require dynamical theory provides by macroscopic QED
- near-field heat transfer follows the same scaling laws as dispersion forces (both related to LDOS)
- extensions to nonequilibrium (stationary) fluctuation-dissipation theorems possible (e.g. for quantum friction)
- theory only applicable for macroscopic bodies (definition of permittivities possible, tricky for particles smaller than 1nm)