Near-field thermal radiation energy conversion

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WE-Heraeus-Seminar:

Heat Transfer and Heat Conduction on the Nanoscale

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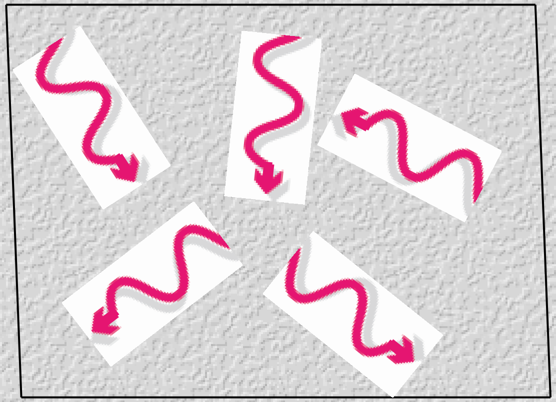
Planck's radiation law

N: photons in a box with frequency quantum states

N_ω : photons with frequency ω

g_ω : quantum states



Single configuration:

x: photon
|…|: state

xxx|x|xx|xxxx|xxx|xx

Bosons: no restriction on the numeber of particles in each level

Ω: number of microstates; combinatorial problem

$$\Omega_{BE} = \prod_\omega \frac{(N_\omega + g_\omega - 1)!}{N_\omega!(g_\omega - 1)!}$$

Planck radiation law from thermodynamics

i) Maximizing the number of microstates:

$$\boxed{\ln \Omega_{\mathrm{BE}} = \sum \ln (N_\nu + g_\nu)! - \sum \ln N_\nu! - \sum \ln g_\nu!}$$

$$\boxed{N_\nu = \frac{g_\nu}{e^{\beta h\nu} - 1}}$$

β?

ii) Condition: energy is constant

$$\boxed{\sum N_\nu h\nu = U = \mathrm{const}}$$

$$\boxed{N_\nu = \frac{8\pi V}{c^3}\frac{\nu^2}{e^{h\nu/kT} - 1}}$$

iii) Boltzmann entropy

$$\boxed{dS = k\, d \ln \Omega_{\mathrm{BE}} = k \sum \ln \left(1 + \frac{g_\nu}{N_\nu}\right) dN_\nu}$$

$$\boxed{dS = k\beta\, dU}$$

$$\boxed{dS/dU = 1/T}$$

$$\boxed{\beta = \frac{1}{kT}}$$

Photon tunneling: near-field regime

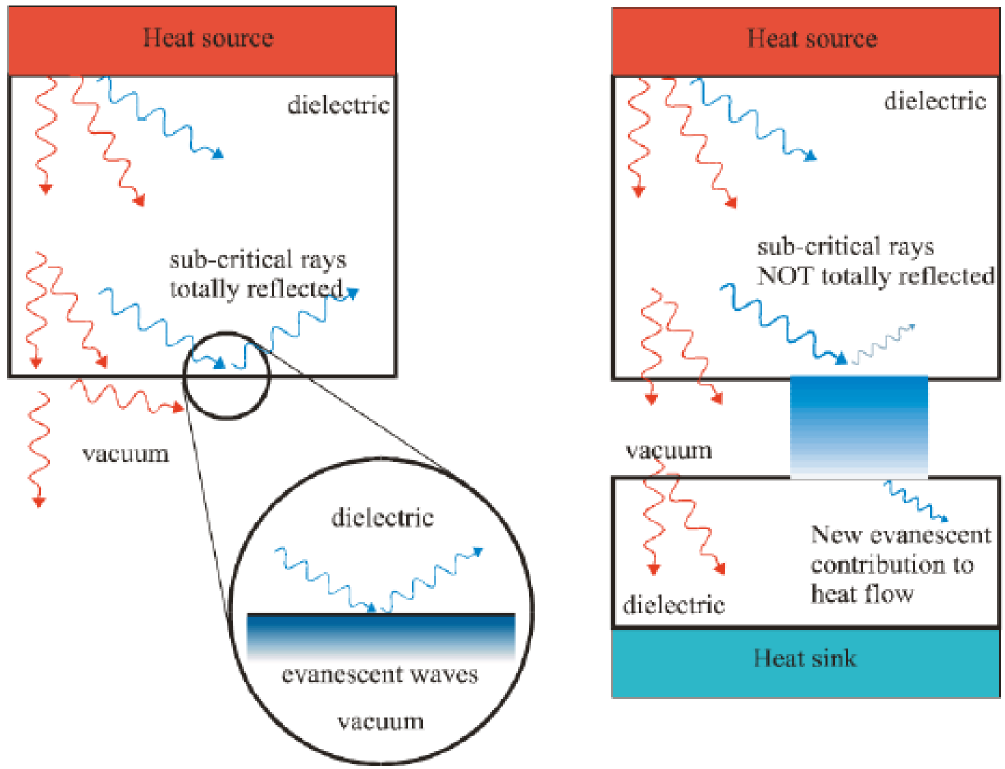
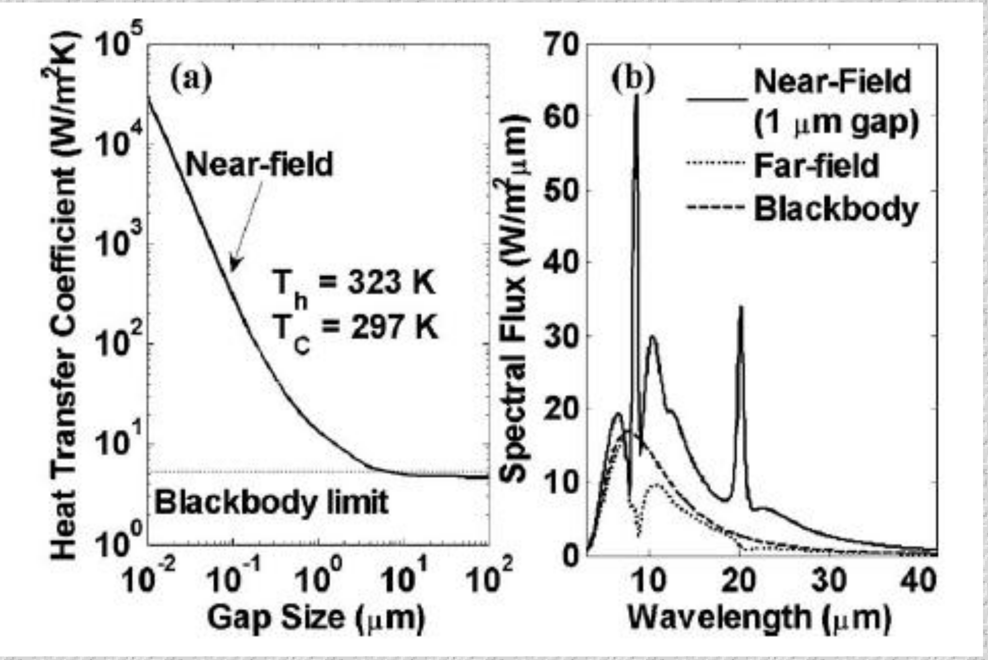


Figure: Evanescent waves play no role in heat loss from a hot dielectric surface to vacuum, left hand figure, but evanescent waves can carry heat from a hot to a cold dielectric surface, right hand figure. [J. B. Pendry, J. Phys.: Condens. Matter 11 (1999) 6621-6633]

Heat flux much larger than blackbody limit! $\Phi_{\rm bb} = \sigma T^4$

Enhancement of the heat flux in the near-field



Questions:

-How much work can we extract from the radiation?
What is the efficiency of the process?
How to compute the dissipation?

-What is the optimal structure for heat transfer?

Thermodynamics for the NF

Thermodynamics of thermal radiation

Energy flux radiated:

$$\boxed{\dot{U}(T) = \int_0^\infty d\omega\, \hbar\omega n(\omega, T)\varphi(\omega)}$$

$$\boxed{n(\omega, T) = \left(e^{\hbar\omega/k_B T} - 1\right)^{-1}}$$

Spectral flux of modes

Entropy flux:

$$\frac{1}{T} = \frac{d\dot{S}}{d\dot{U}}.$$

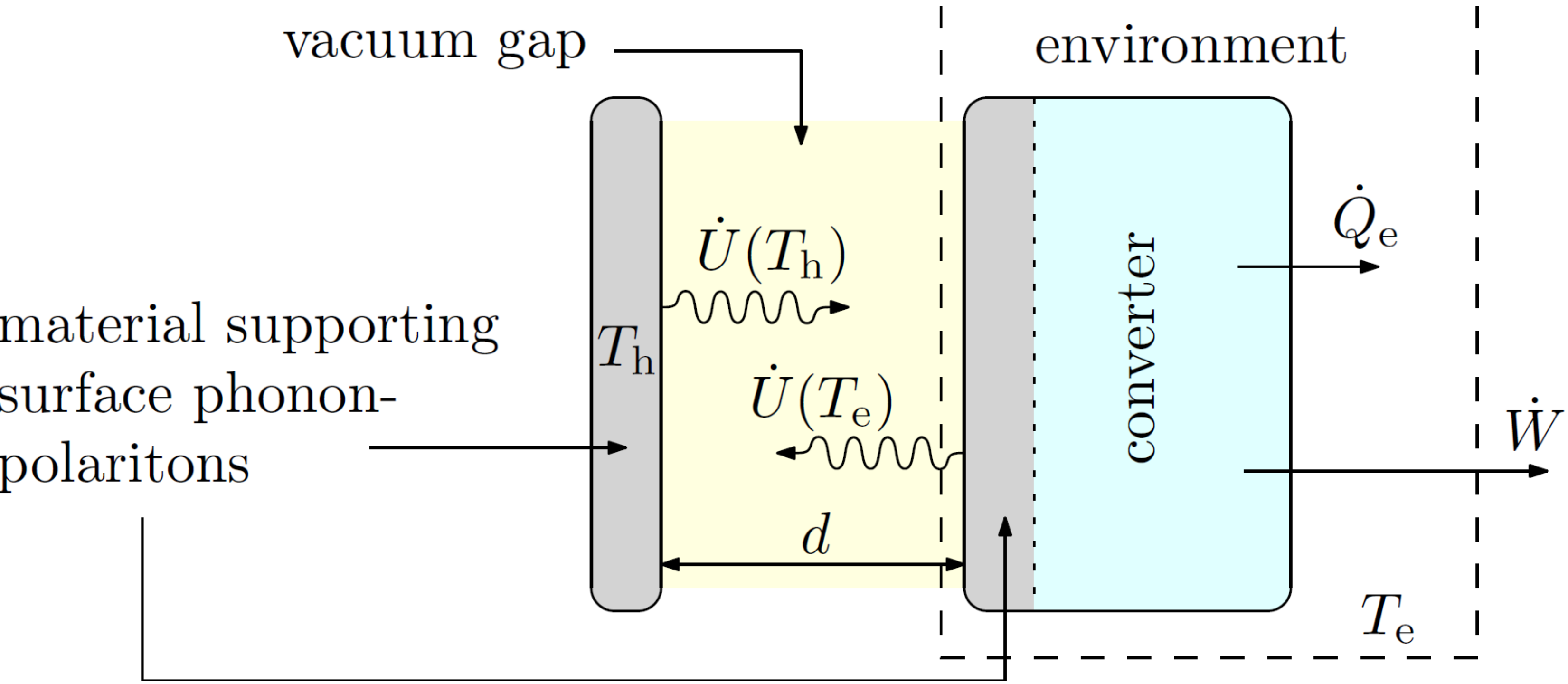
$$\dot{S}(T) = \int_0^T dT' \frac{1}{T'} \frac{d\dot{U}(T')}{dT'}$$

$$\boxed{\dot{S}(T) = \int_0^\infty d\omega\, k_B m(\omega, T)\varphi(\omega)}$$

$$m(\omega, T) = [1 + n(\omega, T)]\ln[1 + n(\omega, T)] - n(\omega, T)\ln n(\omega, T),$$

Free energy: $\boxed{G = U - TS}$

Energy conversion



1st Law:

$$\Delta\dot{U} + \dot{Q}_\mathrm{e} + \dot{W} = 0,$$

$$\Delta\dot{U} = \dot{U}(T_\mathrm{e}) - \dot{U}(T_\mathrm{h})$$

2nd Law:

$$\Delta\dot{S} + \Delta\dot{S}_\mathrm{e} = \Delta\dot{S}_\mathrm{irr} \geq 0,$$

$$\Delta\dot{S} = \dot{S}(T_\mathrm{e}) - \dot{S}(T_\mathrm{h})$$

$$\Delta\dot{S}_\mathrm{e} = \dot{Q}_\mathrm{e}/T_\mathrm{e}$$

Efficiency

Work:

$$\dot{W} = T_e \Delta \dot{S} - \Delta \dot{U} - T_e \Delta \dot{S}_{irr}$$

Ideal work:

$$\dot{\mathcal{W}} \equiv T_e \Delta \dot{S} - \Delta \dot{U}$$

Energy dissipation
Nonequilibrium thermodynamics

Gouy-Stodola theorem

$$\eta \equiv \frac{\dot{W}}{\dot{U}(T_h)} = \frac{\dot{\mathcal{W}} - T_e \Delta \dot{S}_{irr}}{\dot{U}(T_h)}$$

$$\bar{\eta} = \frac{\dot{\mathcal{W}}}{\dot{U}(T_h)} \geq \eta.$$

$$\Delta\dot{U} = \int_0^\infty d\omega\, \hbar\omega [n(\omega, T_2) - n(\omega, T_1)] \varphi(\omega)$$

Spectral flux of modes:

$$\varphi(\omega) = \sum_{\alpha=\mathrm{p,s}} \left\{ \int_0^{\omega/c} \frac{d\kappa\, \kappa}{4\pi^2} \frac{[1 - |R_\alpha(\kappa,\omega)|^2]^2}{|1 - e^{2i\gamma d} R_\alpha^2(\kappa,\omega)|^2} + \int_{\omega/c}^\infty \frac{d\kappa\, \kappa}{\pi^2} \frac{e^{-2|\gamma| d}\, \mathrm{Im}^2[R_\alpha(\kappa,\omega)]}{|1 - e^{-2|\gamma| d} R_\alpha^2(\kappa,\omega)|^2} \right\}.$$

$$\gamma = \sqrt{(\omega/c)^2 - \kappa^2}$$

Lorentz model:

$$\varepsilon(\omega) = \varepsilon_\infty \frac{\omega_\mathrm{L}^2 - \omega^2 - i\Gamma\omega}{\omega_\mathrm{T}^2 - \omega^2 - i\Gamma\omega}$$

When:

$$d \ll \lambda_T = c\hbar / k_\mathrm{B} T$$

$\lambda_T = 7.6\ \mu\mathrm{m}$ for $T = 300\,\mathrm{K}$

Emission is dominated by SPPs

Ex.: SiC, hBN

Black body radiation

The reflection coefficient vanishes:

$$\varphi_{\mathrm{bb}}(\omega) = \left(\frac{\omega}{2\pi c}\right)^2$$

$$\dot{U}_{\mathrm{bb}} = \sigma T^4$$

$$\dot{S}_{\mathrm{bb}} = 4\sigma T^3/3$$

$$\dot{W}_{\mathrm{bb}} = \sigma\left(T_{\mathrm{h}}^4 - T_{\mathrm{e}}^4\right) - \frac{4}{3}\sigma T_{\mathrm{e}}\left(T_{\mathrm{h}}^3 - T_{\mathrm{e}}^3\right)$$

$$\overline{\eta}_{\mathrm{bb}} = 1 - \frac{4}{3}\frac{T_{\mathrm{e}}}{T_{\mathrm{h}}} + \frac{1}{3}\left(\frac{T_{\mathrm{e}}}{T_{\mathrm{h}}}\right)^4$$

Near-field

When the surfaces are close enough the spectral flux of modes is dominated by p-polarized evanescent modes

$$\dot{U}_{\mathrm{nf}}(T) = \int_0^\infty d\omega\, \hbar\omega n(\omega,T) \frac{\mathrm{Im}[\mathrm{Li}_2(R_\mathrm{p}^2(\omega))]}{4\pi^2 d^2 f(\omega)},$$

$$\simeq \hbar\omega_0 n_0(T) \frac{\mathrm{Re}[\mathrm{Li}_2(R_\mathrm{p}^2(\omega_0))]}{4\pi d^2 f'(\omega_0)},$$

$$R_\mathrm{p}(\omega) = \frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+1}$$

$$f(\omega) = \frac{\mathrm{Im}[R_\mathrm{p}^2(\omega)]}{\mathrm{Im}^2[R_\mathrm{p}(\omega)]}$$

Radiation highly monochromatic

$$\omega_0 = \left(\frac{\varepsilon_\infty \omega_\mathrm{L}^2 + \omega_\mathrm{T}^2}{\varepsilon_\infty + 1}\right)^{1/2}$$

$$\varphi_{\mathrm{nf}}(\omega) = g_d(\omega)\delta(\omega-\omega_0)$$

$$g_d(\omega) = \frac{\mathrm{Re}[\mathrm{Li}_2(R_\mathrm{p}^2(\omega))]}{4\pi d^2 f'(\omega)}$$

$$\dot{S}_{\mathrm{nf}}(T) = \int_0^\infty d\omega\, k_\mathrm{B} m(\omega,T)\varphi_{\mathrm{nf}}(\omega) = k_\mathrm{B} m_0(T) g_d(\omega_0)$$

Work:

$$\dot{\mathcal{W}}_{\rm nf} = \hbar\omega_0 g_d(\omega_0)\left\{\frac{k_{\rm B}T_{\rm e}}{\hbar\omega_0}[m_0(T_{\rm e}) - m_0(T_{\rm h})] - [n_0(T_{\rm e}) - n_0(T_{\rm h})]\right\}.$$

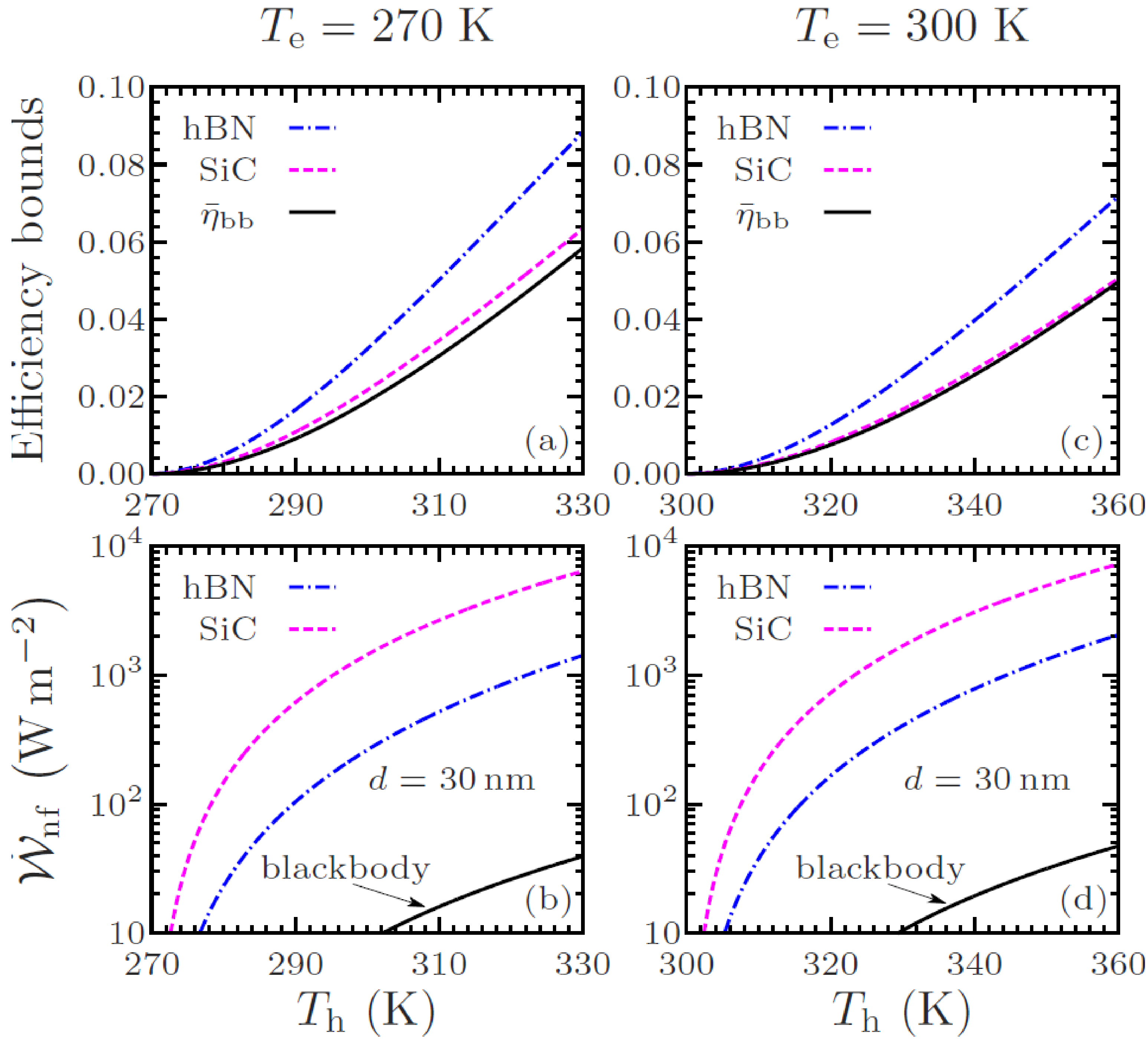
$$\dot{W}_{nf} = \zeta\left(T_h^2 - T_e^2\right) - 2\zeta\left(T_h - T_e\right)$$

Efficiency:

$$\bar{\eta}_{\rm nf} = 1 - \frac{n_0(T_{\rm e})}{n_0(T_{\rm h})} + \frac{k_{\rm B}T_{\rm e}}{\hbar\omega_0}\frac{m_0(T_{\rm e}) - m_0(T_{\rm h})}{n_0(T_{\rm h})}$$

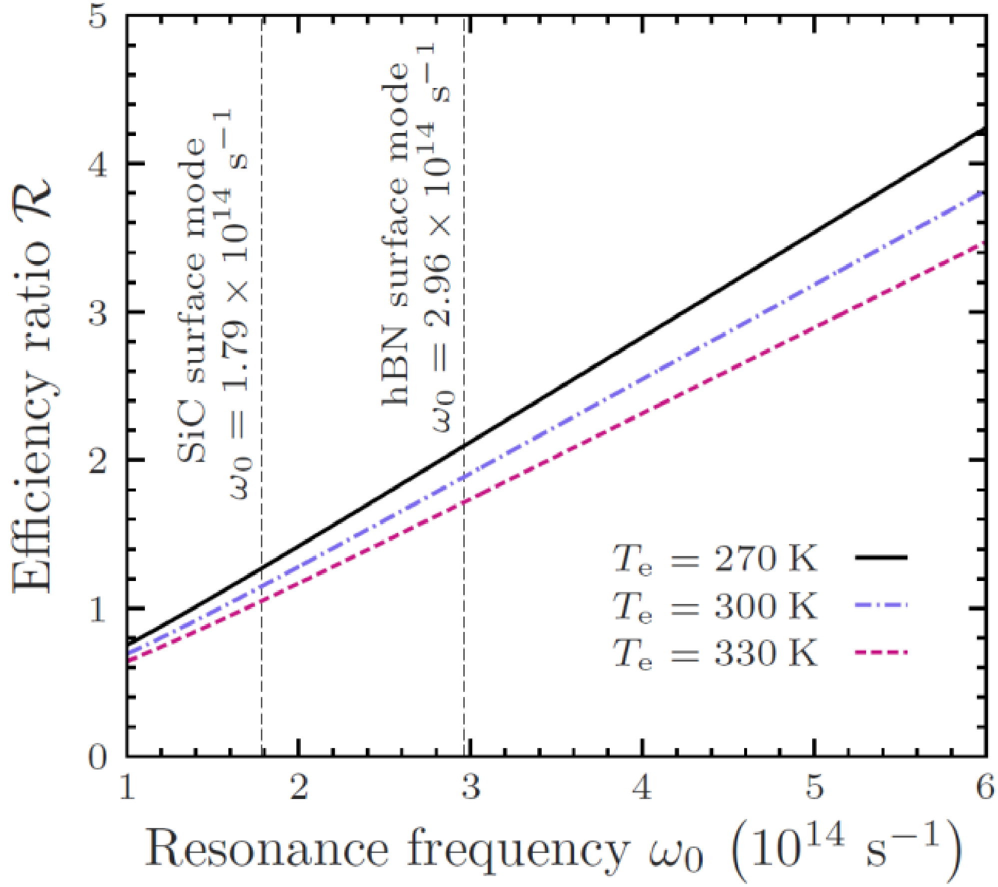
Carnot efficiency:

$$\lim_{\omega_0\to\infty}\bar{\eta}_{\rm nf} = 1 - T_{\rm e}/T_{\rm h}$$



$T_\mathrm{e} = 270$ K
$T_\mathrm{e} = 300$ K
Efficiency bounds
hBN
SiC
$\bar{\eta}_\mathrm{bb}$
(a)
(c)
0.00
0.02
0.04
0.06
0.08
0.10
270
290
310
330
300
320
340
360
$\dot{\mathcal{W}}_\mathrm{nf}$ (W m^{-2})
$d = 30$ nm
blackbody
(b)
(d)
10
10^2
10^3
10^4
T_h (K)

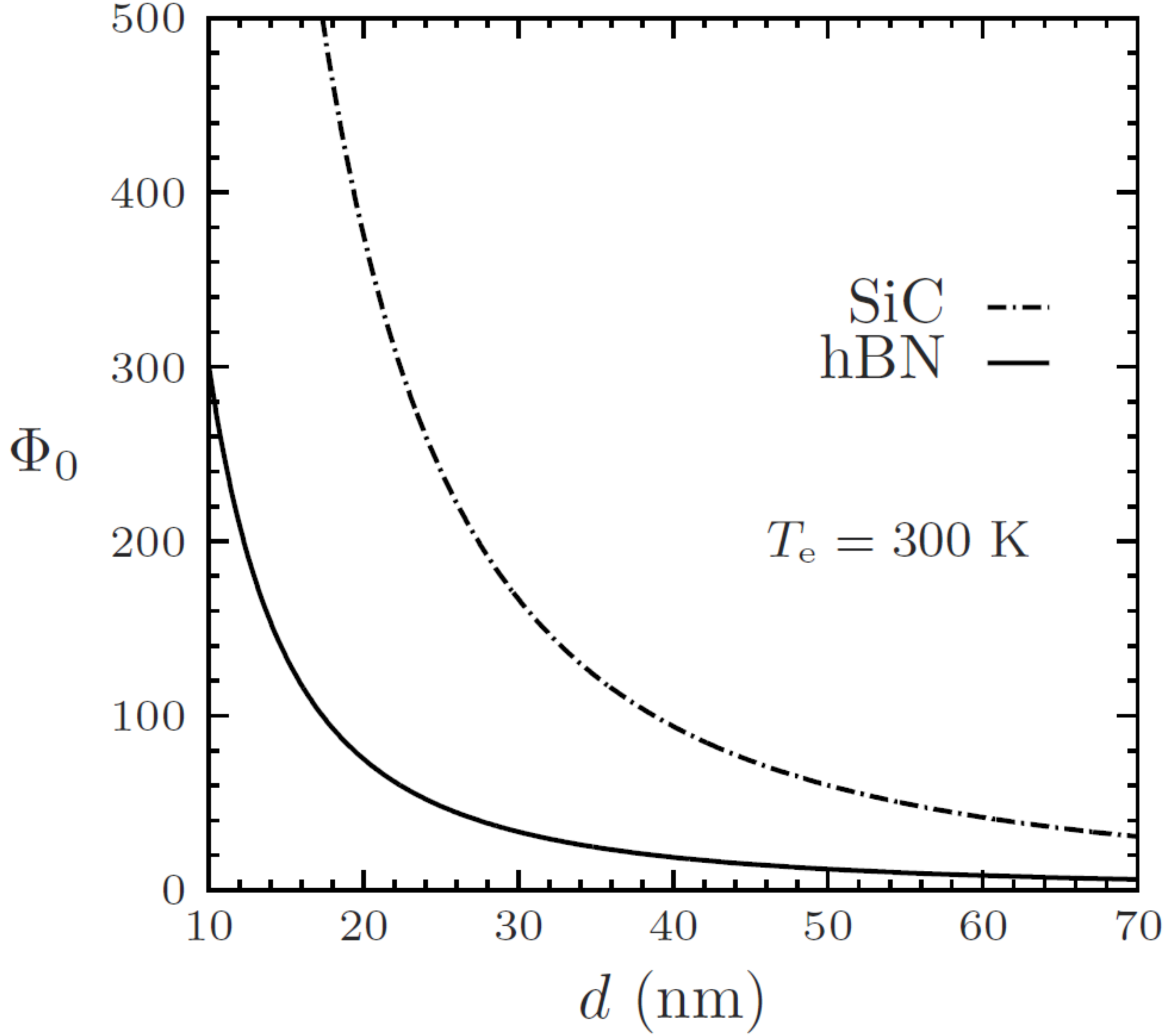
$$\bar{\eta}_{\mathrm{bb}} = 1 - \frac{4}{3}\frac{T_{\mathrm{e}}}{T_{\mathrm{h}}} + \frac{1}{3}\left(\frac{T_{\mathrm{e}}}{T_{\mathrm{h}}}\right)^4$$



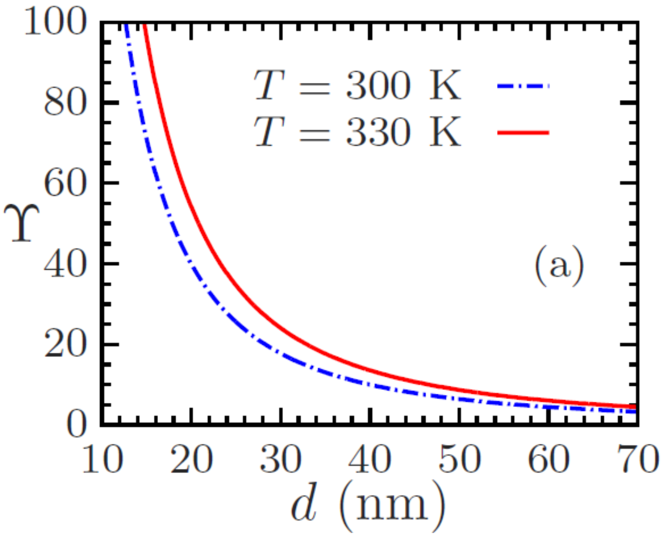
$$\mathcal{R} \equiv \lim_{T_{\mathrm{h}} \to T_{\mathrm{e}}} \frac{\bar{\eta}_{\mathrm{nf}}}{\bar{\eta}_{\mathrm{bb}}} = \frac{\hbar\omega_0}{4k_{\mathrm{B}}T_{\mathrm{e}}}\left[1 - \exp\left(-\frac{\hbar\omega_0}{k_{\mathrm{B}}T_{\mathrm{e}}}\right)\right]^{-1}$$

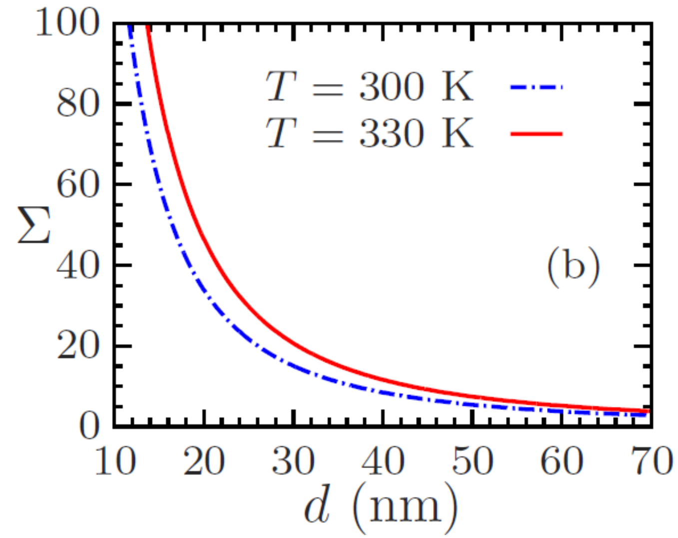
$$\bar{\eta}_{\mathrm{nf}} > \bar{\eta}_{\mathrm{bb}} \qquad \omega_0 > 3.921\,\frac{k_{\mathrm{B}}T_{\mathrm{e}}}{\hbar}$$

$\Phi_0 = \dot{\mathcal{W}}_{\rm nf}/\dot{\mathcal{W}}_{\rm bb}$



Comparison with blackbody



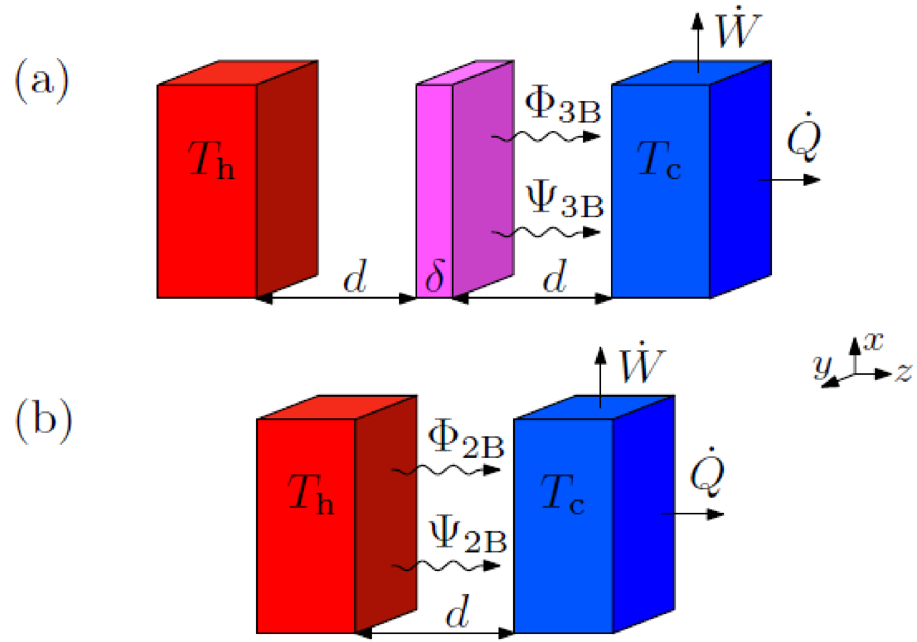


$\Upsilon(d,T) \equiv \dot{U}_{\mathrm{nf}}(d,T)/\dot{U}_{\mathrm{bb}}(T)$

$\Sigma(d,T) \equiv \dot{S}_{\mathrm{nf}}(d,T)/\dot{S}_{\mathrm{bb}}(T)$.

What is the optimal structure for heat transmission?

- two $5\,\mu$m-thick SiC samples: SPP at $\omega_{\rm spp} \simeq 1.79 \times 10^{14}\,$rad/s
- metal-like medium: surface mode (a plasmon) at $\omega_{\rm spp}$



Near-field:

$d \ll \lambda_T = \hbar c/(k_{\rm B}T)$, $\lambda_T = 7.6\,\mu$m for $T = 300\,$K

Net energy flux on the cold body

$$\Phi_{i\mathrm{B}} = \int_0^{\infty} \frac{\mathrm{d}\omega}{2\pi} \phi_{i\mathrm{B}}(\omega, d, \delta) \qquad (i = 2, 3)$$

$$\phi_{3\mathrm{B}}(\omega, d, \delta) = \hbar\omega \sum_j \int_{c\kappa>\omega} \frac{\mathrm{d}^2\boldsymbol{\kappa}}{(2\pi)^2} \left[n_{\mathrm{hi}}(\omega)\mathcal{T}_j^{(\mathrm{hi})}(\omega, \kappa, d, \delta) + n_{\mathrm{ic}}(\omega)\mathcal{T}_j^{(\mathrm{ic})}(\omega, \kappa, d, \delta) \right]$$

$$\phi_{2\mathrm{B}}(\omega, d) = \hbar\omega \sum_j \int_{c\kappa>\omega} \frac{\mathrm{d}^2\boldsymbol{\kappa}}{(2\pi)^2} n_{\mathrm{hc}}(\omega)\mathcal{T}_j^{(\mathrm{hc})}(\omega, \kappa, d)$$

with $n_{\alpha\beta}(\omega) = n_\alpha(\omega) - n_\beta(\omega)$, where $n_\alpha(\omega) = \left(e^{\hbar\omega/k_{\mathrm{B}}T_\alpha} - 1\right)^{-1}$, $\alpha = \mathrm{h}, \mathrm{i}, \mathrm{c}$

- $\mathcal{T}_j^{(\mathrm{hi})}$, $\mathcal{T}_j^{(\mathrm{ic})}$, and $\mathcal{T}_j^{(\mathrm{hc})}$ are the transmission coefficients for polarization j that depend on optical properties of materials

Net entropy flux on the cold body

$\Phi_{i\mathrm{B}} = \sum_\alpha \Phi_\alpha^{(i\mathrm{B})}(T_\alpha)$,

$$\Psi_\alpha^{(i\mathrm{B})}(T_\alpha) = \int_0^{T_\alpha} \mathrm{d}T' \, \frac{1}{T'} \frac{\mathrm{d}}{\mathrm{d}T'} \Phi_\alpha^{(i\mathrm{B})}(T').$$

Therefore,

$$\Psi_{i\mathrm{B}} = \sum_\alpha \Psi_\alpha^{(i\mathrm{B})}(T_\alpha) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \psi_{i\mathrm{B}}(\omega, d, \delta),$$

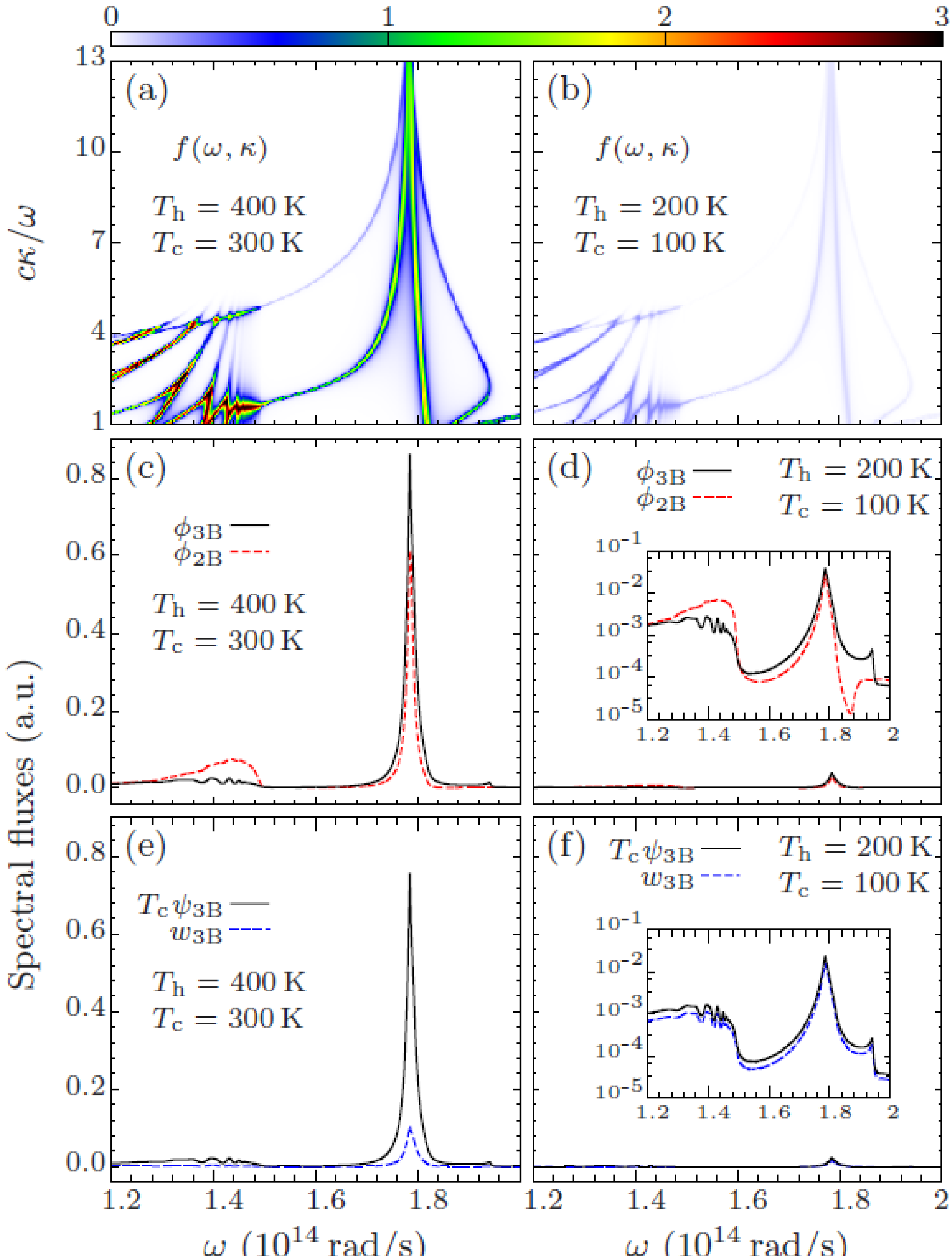
where the spectral entropy fluxes take the form

$$\psi_{3\mathrm{B}}(\omega, d, \delta) = k_\mathrm{B} \sum_j \int_{c\kappa>\omega} \frac{\mathrm{d}^2\boldsymbol{\kappa}}{(2\pi)^2} \left[m_{\mathrm{hi}}(\omega) \mathcal{T}_j^{(\mathrm{hi})}(\omega, \kappa, d, \delta) + m_{\mathrm{ic}}(\omega) \mathcal{T}_j^{(\mathrm{ic})}(\omega, \kappa, d, \delta) \right],$$

$$\psi_{2\mathrm{B}}(\omega, d) = k_\mathrm{B} \sum_j \int_{c\kappa>\omega} \frac{\mathrm{d}^2\boldsymbol{\kappa}}{(2\pi)^2} m_{\mathrm{hc}}(\omega) \mathcal{T}_j^{(\mathrm{hc})}(\omega, \kappa, d),$$

with $m_{\alpha\beta}(\omega) = m_\alpha(\omega) - m_\beta(\omega)$ and

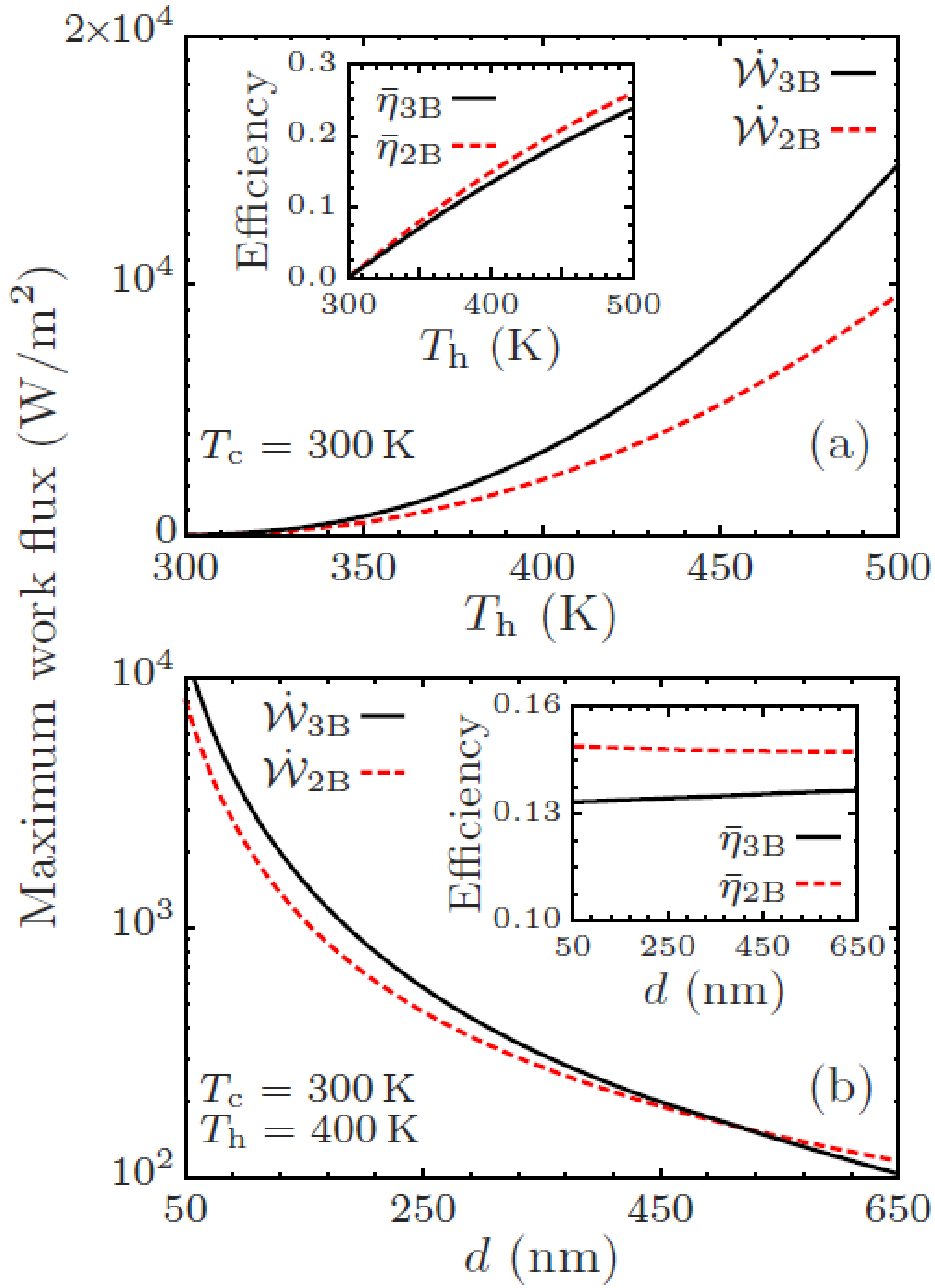
$$m_\alpha(\omega) = [1 + n_\alpha(\omega)] \ln [1 + n_\alpha(\omega)] - n_\alpha(\omega) \ln n_\alpha(\omega)$$



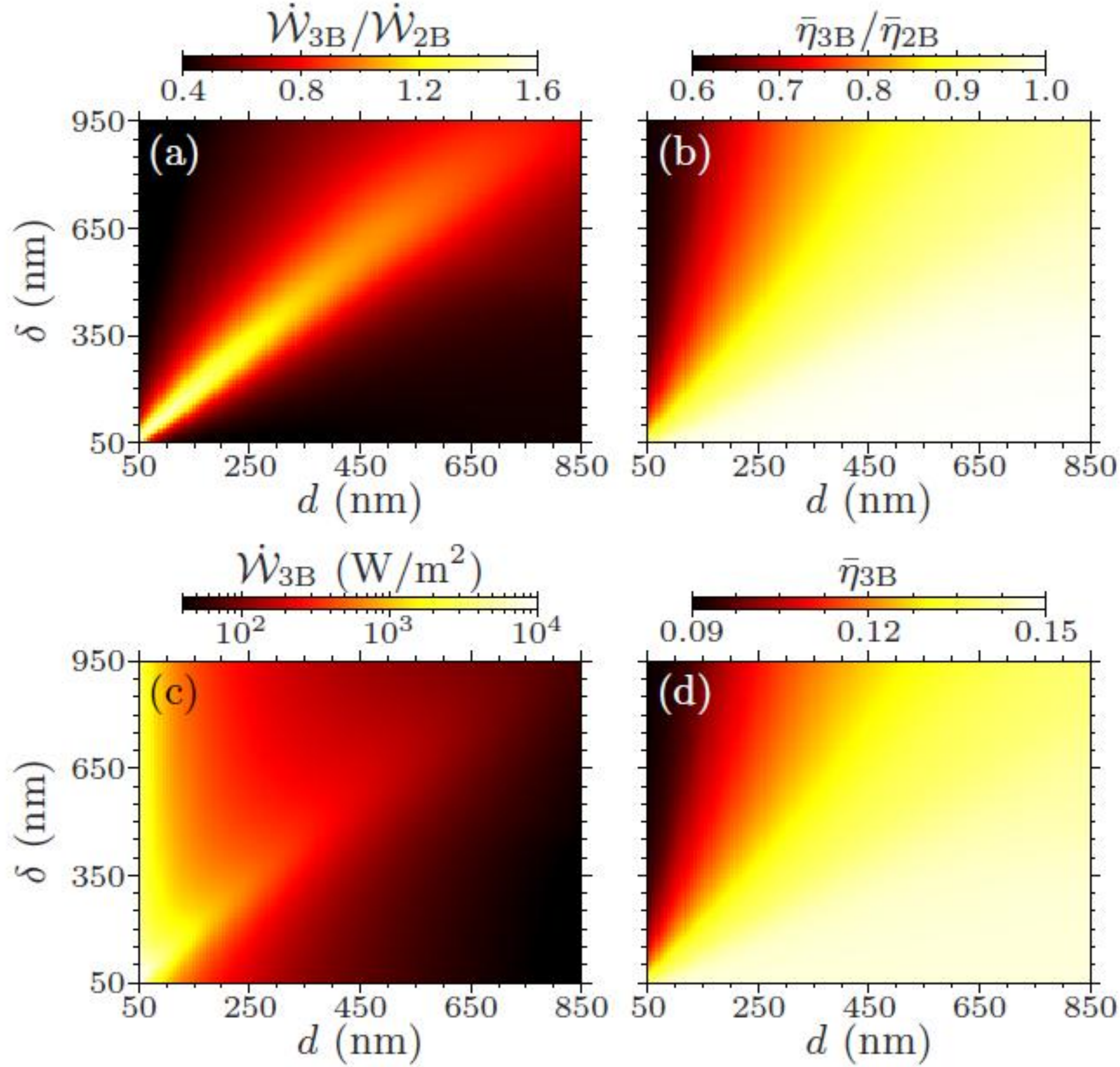
$d = 500$ nm and $\delta = 667$ nm

$$f(\omega,\kappa) = 10^{22} \times \left(n_{hi}\mathcal{T}_p^{(hi)} + n_{ic}\mathcal{T}_p^{(ic)} \right)$$

$$w_{3B} = \phi_{3B} - T_c\psi_{3B}$$



Maximum work flux (W/m^2)
2×10^4
10^4
0
300
350
400
450
500
T_h (K)
$\dot{\mathcal{W}}_{3\mathrm{B}}$
$\dot{\mathcal{W}}_{2\mathrm{B}}$
$T_\mathrm{c} = 300\,\mathrm{K}$
(a)
Efficiency
0.3
0.2
0.1
0.0
300
400
500
T_h (K)
$\bar{\eta}_{3\mathrm{B}}$
$\bar{\eta}_{2\mathrm{B}}$
10^4
10^3
10^2
50
250
450
650
d (nm)
$\dot{\mathcal{W}}_{3\mathrm{B}}$
$\dot{\mathcal{W}}_{2\mathrm{B}}$
$T_\mathrm{c} = 300\,\mathrm{K}$
$T_\mathrm{h} = 400\,\mathrm{K}$
(b)
Efficiency
0.16
0.13
0.10
50
250
450
650
d (nm)
$\bar{\eta}_{3\mathrm{B}}$
$\bar{\eta}_{2\mathrm{B}}$



$\dot{\mathcal{W}}_{3B}/\dot{\mathcal{W}}_{2B}$
0.4
0.8
1.2
1.6
$\bar{\eta}_{3B}/\bar{\eta}_{2B}$
0.6
0.7
0.8
0.9
1.0
(a)
(b)
(c)
(d)
δ (nm)
d (nm)
950
650
350
50
50
250
450
650
850
$\dot{\mathcal{W}}_{3B}$ (W/m^2)
10^2
10^3
10^4
$\bar{\eta}_{3B}$
0.09
0.12
0.15

Results

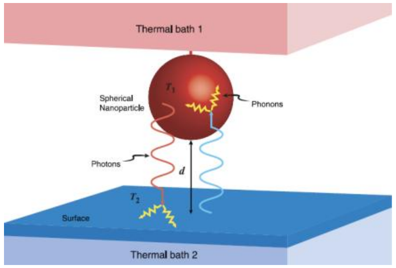
- Near-field thermodynamics, **NFT**
- The maximum work that can be obtained from the thermal radiation in the **near-field** regime is much larger than that corresponding to the **blackbody limit**.
- Thermal radiation energy conversion can be **more efficient** in the near-field regime.
- **Optimal configuration**: Three-body photon tunneling can produce more work than in two body systems
- New possibilities for the design of **energy converters** that can be used to harvest energy from sources of moderate temperature at the nanoscale

I. Latella et al., J. Appl. Phys., 115, 124307 (2014)
I. Latella et al., Phys. Rev. Applied, 4, 011001 (2015)

Casimir forces from dissipation

Casimir-Lifshitz force between a sphere of radius R and a plate separated by a distance

$$d << R$$



Proximity-force approximation (Derjaguin):

$$F_{\mathrm{PFA}} = 2\pi R \varepsilon^{p-p}$$

$$\varepsilon^{p-p} = -\pi^2 \hbar c / (720 d^3)$$

Deviations from PFA arise from curvature:

$$F_{\mathrm{PFA}} = \alpha \frac{2\pi^3 R \hbar c}{720 d^3} \left[1 + \beta \frac{d}{R} + \mathcal{O}\left(\frac{d^2}{R^2} \right) \right]$$

α, β : Correction coefficients

Kinetic model

$$\dot{Q}_1(\omega) = a_1(\omega, T_1, T_2) J_{2\to 1}(\omega, T_2)$$

Rate of change of the energy of the rdiation field

$$J_{2\to 1}(\omega, T_2) = e_2(\omega, T_2)\dot{u}(\omega, T_2)$$

i) Dynamics of the system: **collective excitations or vibration modes**

ii) **Overall relaxation** (Matthiessen rule)

$$\tau^{-1} = \tau_1^{-1} + \cdots + \tau_n^{-1}$$

iii) **Relaxation times**

$$\tau_\ell^{-1} = \tau_{\ell-1}^{-1} + \xi_\ell \tau_{\ell-1}^{-1}$$

ξ_ℓ small random input

$$\tau_\ell^{-1} = \tau_0^{-1}(1 + \textstyle\sum_{i=1}^{\ell} \xi_i)$$

For large l, $\sum \xi_i$ is asymptotically **normally distributed**

$$\tau^{-1}(\omega) = \frac{1}{\sqrt{2\pi}\sigma\tau_0} \exp\left[-\frac{\ln^2(\omega/\omega_0)}{2\sigma^2}\right]$$

iv) Adiabatic approximation

$$\dot{u}(\omega, T_2) = u(\omega, T_2)/\tau(\omega)$$

$$u(\omega, T_2) = \frac{\hbar\omega}{2} \coth\left(\frac{\hbar\omega}{2k_B T_2}\right)$$

Energy of a harmonic oscillator

v) The relaxation process entails **dissipation**. Represents the asymptotic of the diffusion coefficient

vi) The relaxation time leads to a **new model for the dielectric permetivity** (generalized Drude model)

$$\epsilon(\omega) = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + i\omega\tau(\omega)}$$

and to a new fluctuation-dissipation relation.

Total power: $\mathcal{P}_{i\to j} = \int_{\omega_m}^{\infty} \dot{Q}_j(\omega) g_j(\omega) d\omega$

$$g_j(\omega) = V_j \omega^2/\pi^2 c^3$$

Density of modes of material j

Force: $F_{i\to j} = \mathcal{P}_{i\to j}/c$

Isothermal case, with $a_j(\omega, T) = e_i(\omega, T) = 1$

$$F_{i\to j} \approx \frac{2^{5/2}\pi^{3/2}}{d^4}\frac{\hbar\varepsilon^5 V_j}{\sigma\tau_0'}\exp\left[-\frac{\ln^2(2\pi\varepsilon c/\omega_0 d)}{2\sigma^2}\right] \times \coth\left(\frac{h\varepsilon c}{2k_B T d}\right),$$

L. Lapas, A. Perez, J.M. Rubi, Phys. Rev. Lett. 116, 110601 (2016)

i) Unlike the PFA formula, it does not diverge as d goes to zero

ii) For large separation, the dominant term goes as

$$\boxed{k_B T/d^3}$$

iii) At zero T, it behaves as

$$\boxed{1/d^4}$$

Casimir-Lifshitz theory

iv) The PFA is not consistent with Heisenberg's uncertainty principle

H. Gies, K. Klingmüller, Phys. Rev. Lett. 96,220401 (2006)

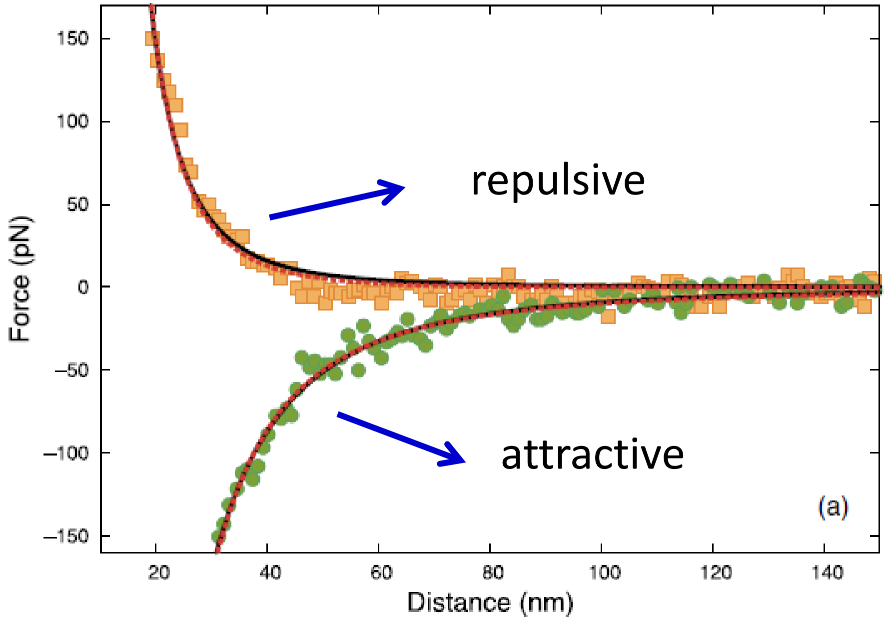
Heisenberg's principle

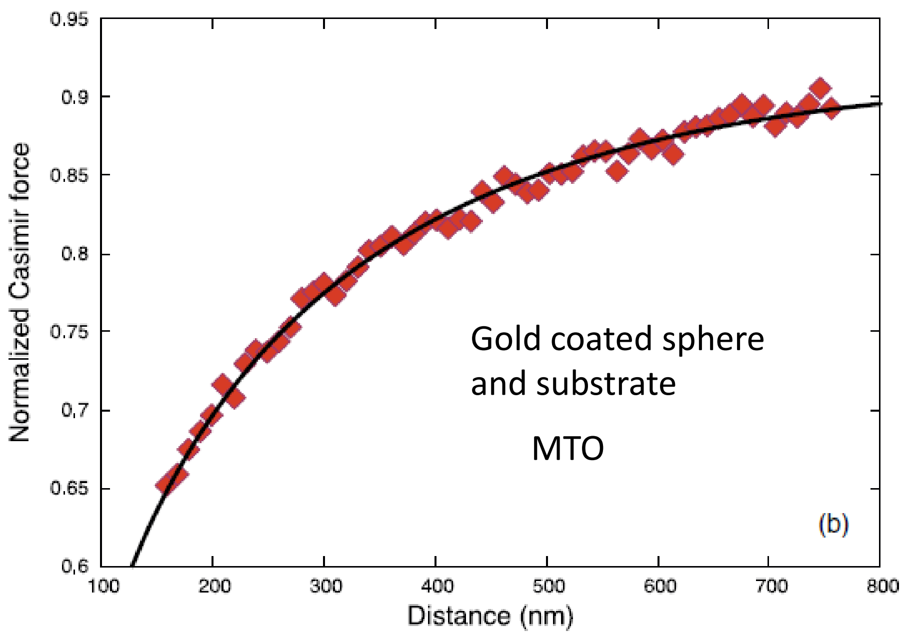
$$\Delta x \Delta p \geq \hbar / 2$$

$$\Delta x < d$$

$$\Delta p \geq \hbar / 2d$$

$$\Delta E \geq \frac{\hbar \nu c}{2d} \rightarrow \Delta \omega \geq \frac{2\pi \nu c}{d} \equiv \omega_m$$





Experiments:

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- G. L. Klimchitskaya, U. Mohideen, and V. M. Mostepanenko, Rev. Mod. Phys. **81**, 1827 (2009).
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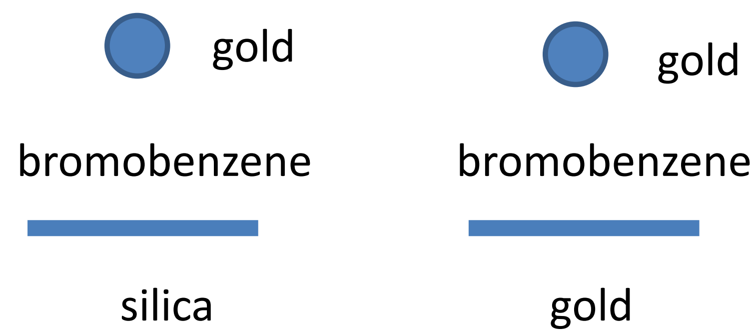

Interaction: $-(\varepsilon_1 - \varepsilon_3)(\varepsilon_2 - \varepsilon_3)$

Atractive: $\varepsilon_1 = \varepsilon_2$

Repulsive: $\varepsilon_1 > \varepsilon_3 > \varepsilon_2$

-The corrected PFA may fit experiments of Murray et al. for the attractive and repulsive forces with fitting parameters different from those predicted

repulsive (attractive)

$$\alpha = 2.82x10^{-2}(4.44x10^{-2})$$

$$\beta = -2.3x10^{2}(6.3x10^{2})$$

$$\varepsilon = 2.57(8.21)$$

$$\sigma = 3.22(2.37)$$

-The corrected PFA is not able to adjust Krause et al. experiments

$$\varepsilon = 10.23$$

$$\sigma = 2.46$$

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P.O. Chapuis; R. Vaillon