Laws of thermal radiation for hyperbolic materials

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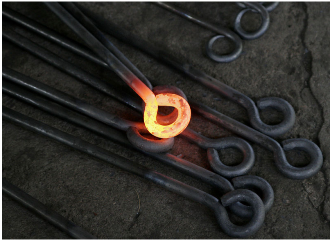
black-body radiation

- Planck's radiation law

$$u(\omega) = \frac{\omega^2}{\pi^2 c^3} \times \hbar\omega \times \frac{1}{e^{\hbar\omega/k_B T} - 1}$$



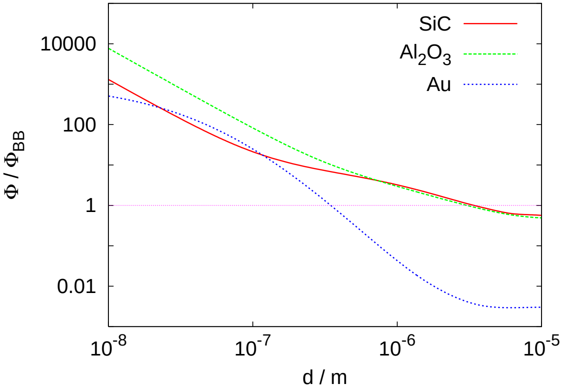
- Stefan-Boltzmann's law

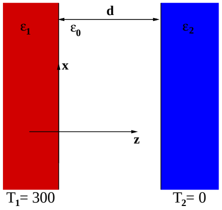
$$\Phi_{BB} = \frac{c}{4} \int d\omega\, u(\omega) = \sigma T^4$$

with
$\sigma = \frac{2\pi^5 k_B^4}{15 h^3 c^2} = 5.67 \cdot 10^{-8} W m^{-2} K^{-4}$

  - for real (grey) emitter: $\Phi \leq \Phi_{BB}$
  - Be careful, this holds only for $d \gg \lambda_{th}$!

Theoretical predictions





E.

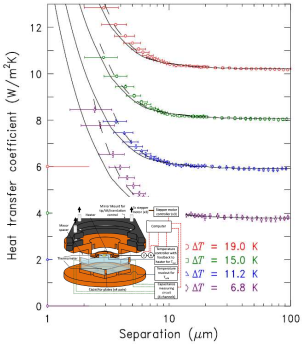
G. Cravalho, C. L. Tien, and B. P. Caren, Trans. ASME Ser. C **89**, 351 (1967)

A. Olivei, Rev. Phys. Appl. **3**, 225 (1968)

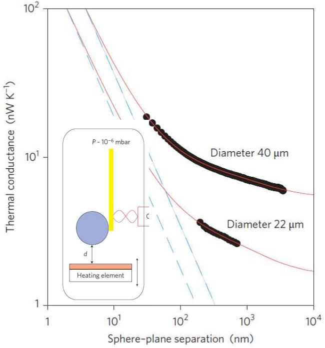
Polder and van Hove, Phys. Rev. B **4**, 3303 (1971)

Some recent experimental results

halfspace - halfspace



sphere - halfspace



- Hu et al., APL **92**, 133106 (2008)

  Ottens et al., PRL **107**, 014301 (2011)

  Kralik et al., PRL **109**, 224302 (2012)

  M. Lim et al., PRB **91**, 195136 (2015)

  ...

- Narayanaswamy et al., PRB **78**, 115303 (2008)

  Shen et al., Nano Lett. **9**, 2909 (2009)

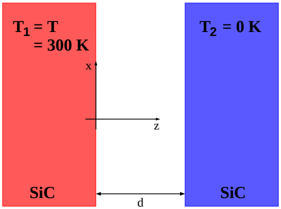
  Rousseau et al., Nature Photonics **3**, 514 (2009)

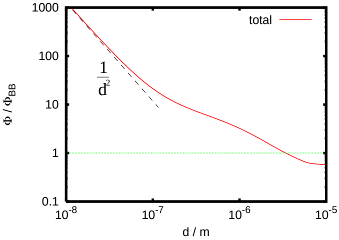
  J. Shi et al., Nano Lett. **15**, 1217 (2015).

  ...

heat flux expression

$$\Phi = \langle S_z \rangle = \int \frac{\mathrm{d}\,\omega}{2\pi} \frac{\hbar\omega}{e^{\hbar\omega/(k_B T)} - 1} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} (\mathcal{T}_s + \mathcal{T}_p)$$





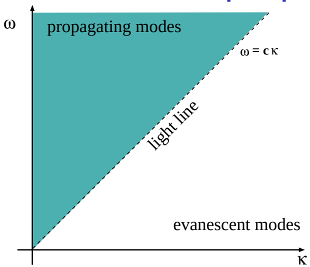
- transmission coefficient (Polder and van Hove, PRB **4**, 3303 (1971))

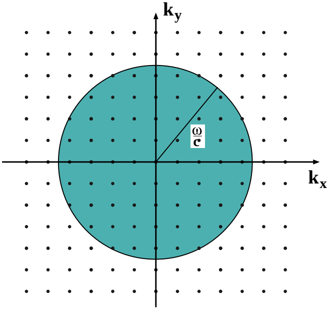
$$\mathcal{T}_i(\omega, \boldsymbol{\kappa}; d) = \begin{cases} \dfrac{(1-|r_i^{10}|^2)(1-|r_i^{20}|^2)}{|1-r_i^{10} r_i^{20} \exp(2\mathrm{i}k_z d)|^2}, & \kappa < \dfrac{\omega}{c} \\ \dfrac{\mathrm{Im}(r_i^{10})\mathrm{Im}(r_i^{20}) e^{-2|k_z|d}}{|1-r_i^{10} r_i^{20} \exp(2\mathrm{i}k_z d)|^2}, & \kappa > \dfrac{\omega}{c} \end{cases}$$

- $\mathcal{T}_i \in [0 : 1]$

Pendry, J. Phys.: Condens. **11**, 6621 (1999)

propagating modes





- plane wave

$$E_y = A(x, y; t)\mathrm{e}^{\mathrm{i}k_z z}$$

- prop. modes $\kappa < \frac{\omega}{c}$

$$k_z = \sqrt{\frac{\omega^2}{c^2} - \kappa^2} \in \mathbb{R}$$

- evan. modes $\kappa > \frac{\omega}{c}$

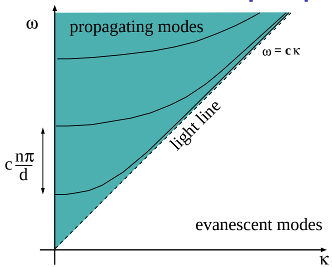
$$k_z = \mathrm{i}\sqrt{\kappa^2 - \frac{\omega^2}{c^2}} \in \mathbb{C}$$

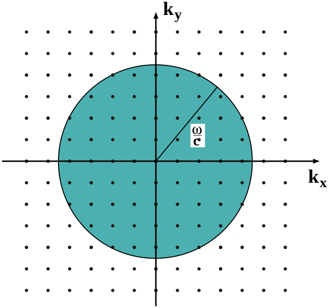
- res. transmission

$$k_z \equiv \frac{n\pi}{d}, n \in \mathbb{N}$$

Nanoscale Radiative Heat Transfer and Its Applications, ISBN: 978-953-51-0060-7, InTech (2012)

propagating modes





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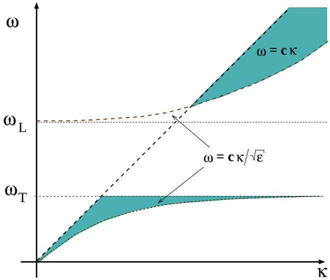
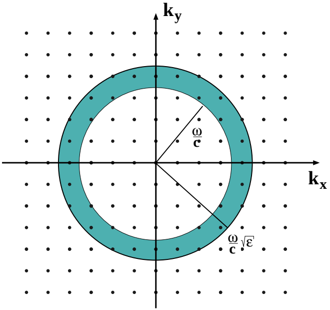
Frustrated internal reflection

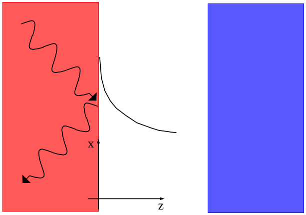


- propagating waves inside the medium

$$k_{1,z} = \sqrt{\frac{\omega^2}{c^2}\epsilon - \kappa^2} \in \mathbb{R}$$

$$\Leftrightarrow \kappa < \frac{\omega}{c}\sqrt{\epsilon}$$





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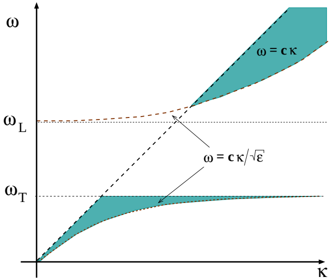

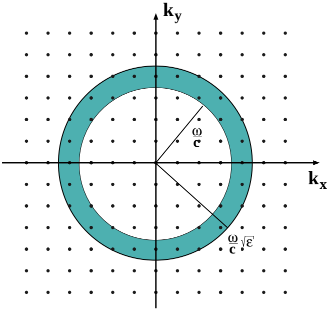

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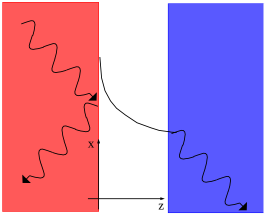
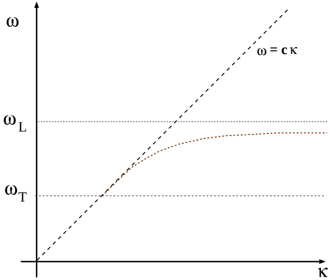


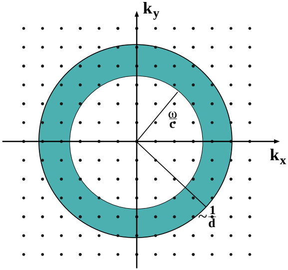
Nanoscale Radiative Heat Transfer and Its Applications, ISBN: 978-953-51-0060-7, InTech (2012)

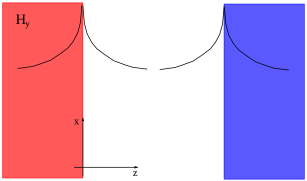
Surface modes



- 'bound' to the surface
- for p-polarisation only
- transmission coeff. $\kappa \gg \omega/c$

$$\mathcal{T}_p \approx \frac{\mathrm{Im}(r_p^{10})\mathrm{Im}(r_p^{20})e^{-2\kappa d}}{|1 - r_p^{10} r_p^{20} \exp(-2\kappa d)|^2}$$





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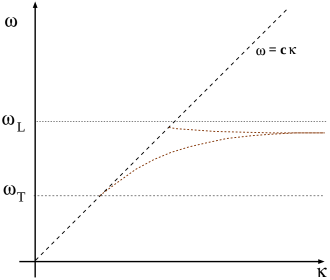

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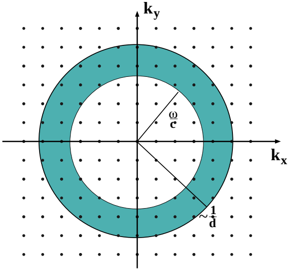
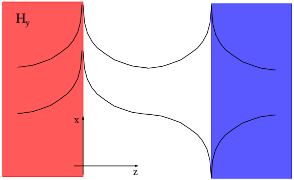
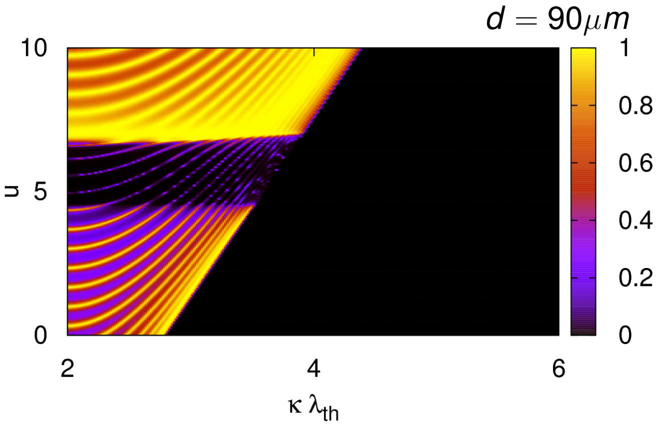




Nanoscale Radiative Heat Transfer and Its Applications, ISBN: 978-953-51-0060-7, InTech (2012)

Transmissions coefficient $\mathcal{T}_p$ (SiC, $u = \hbar\omega/k_{\rm B}T$)



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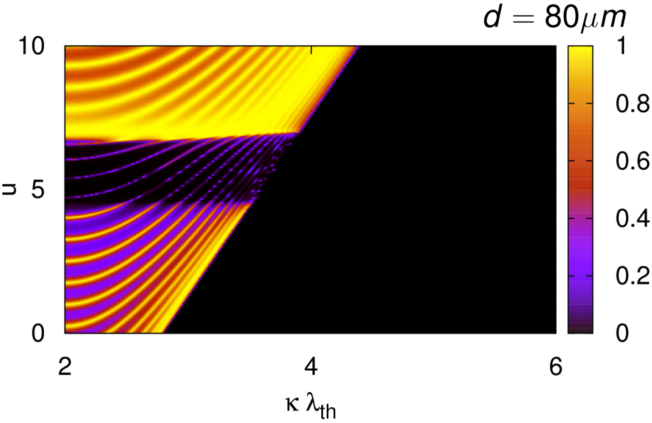

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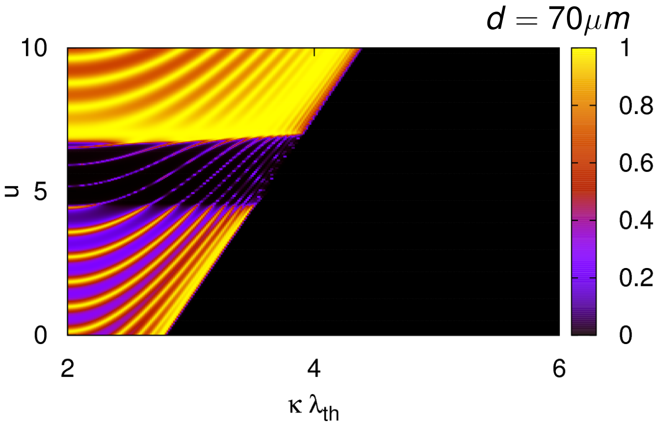

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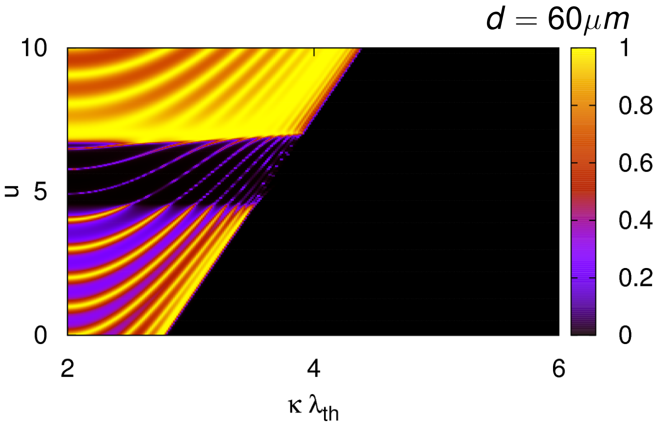

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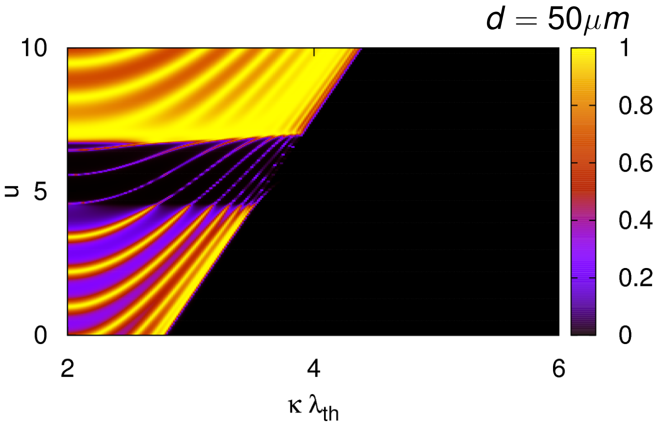

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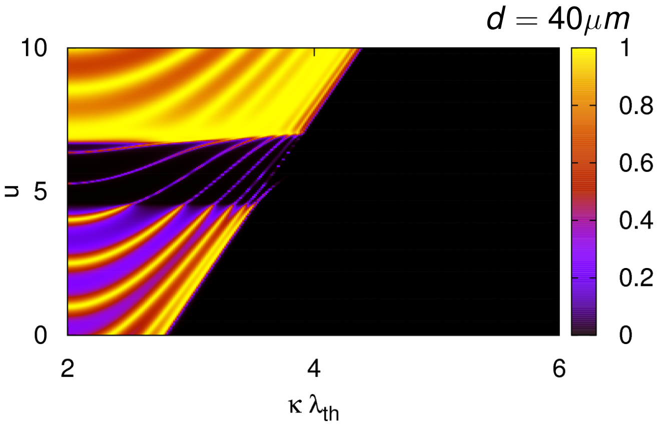

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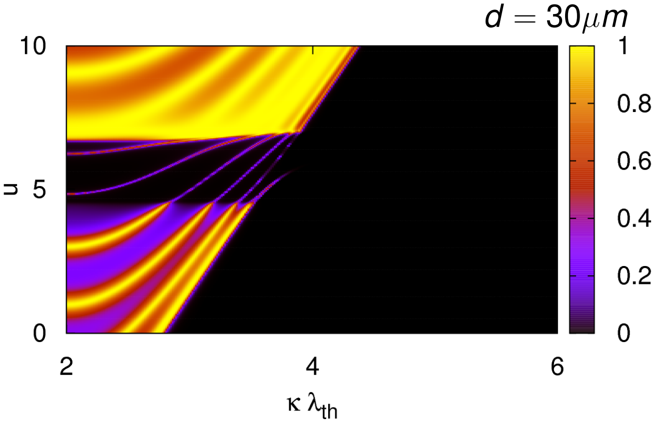

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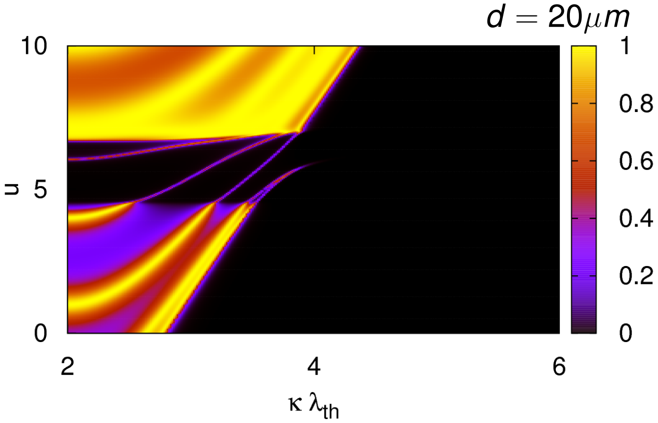

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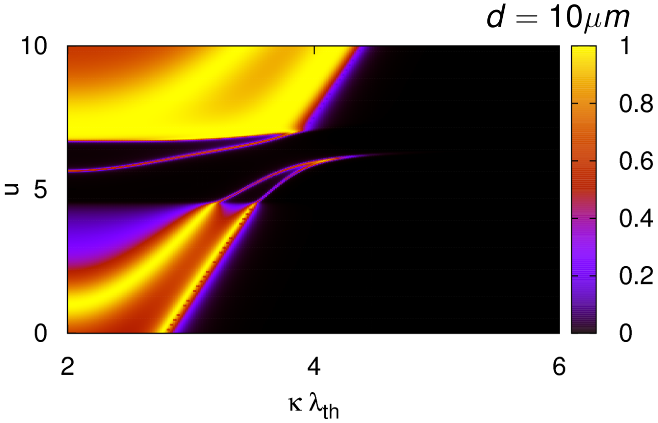

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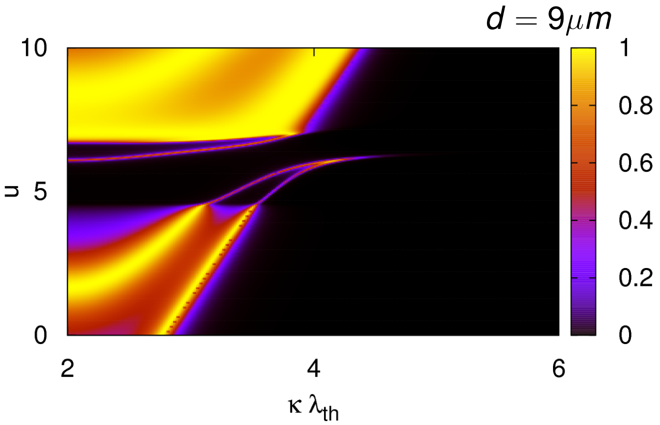

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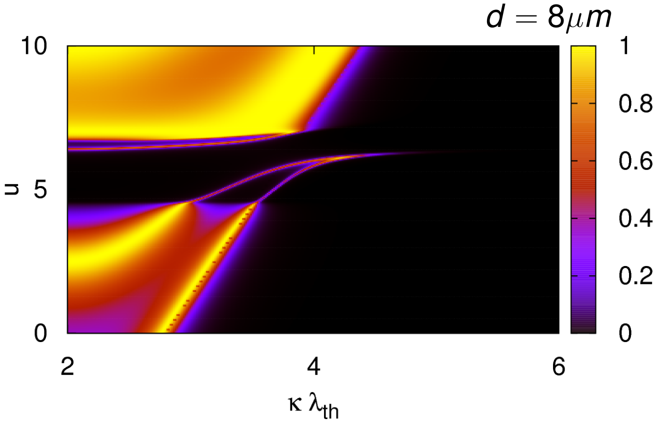

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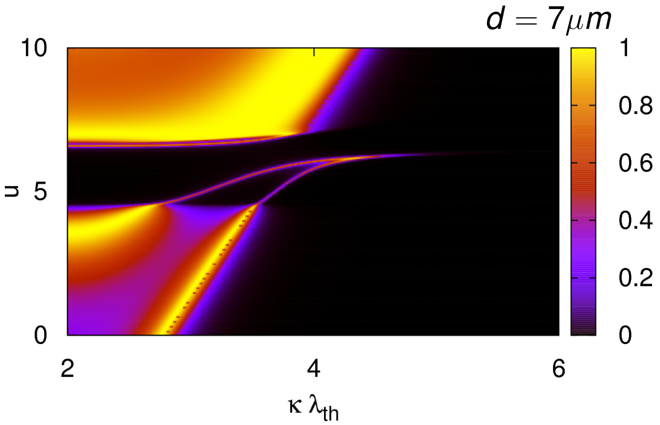

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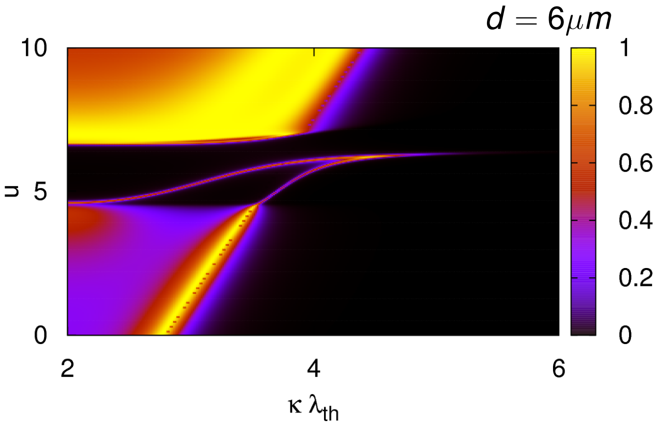

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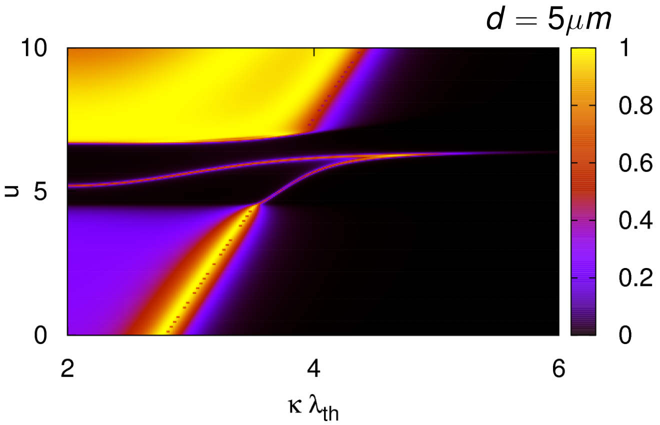

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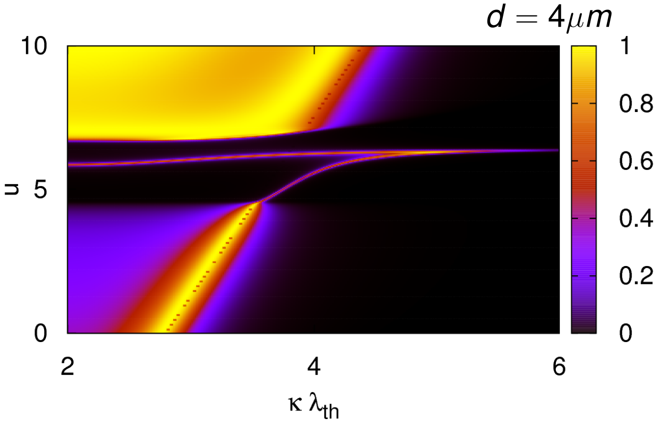

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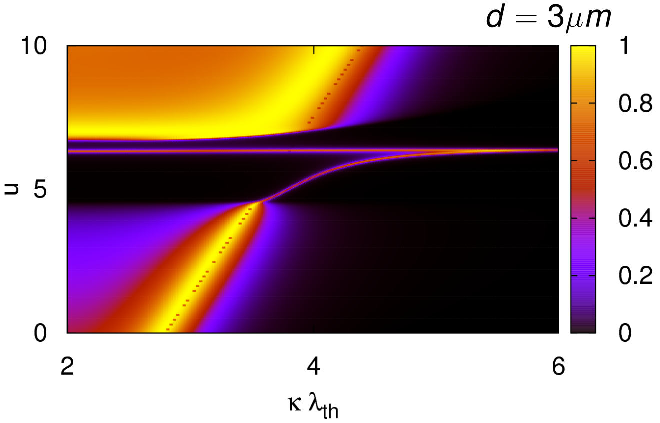
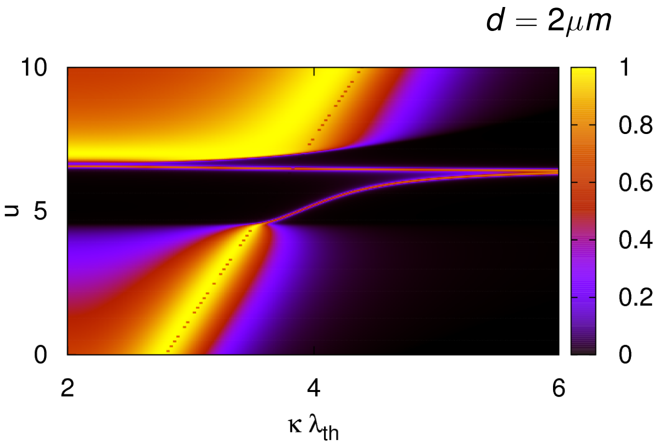


Transmissions coefficient $\mathcal{T}_p$ (SiC, $u = \hbar\omega / k_\mathrm{B} T$)



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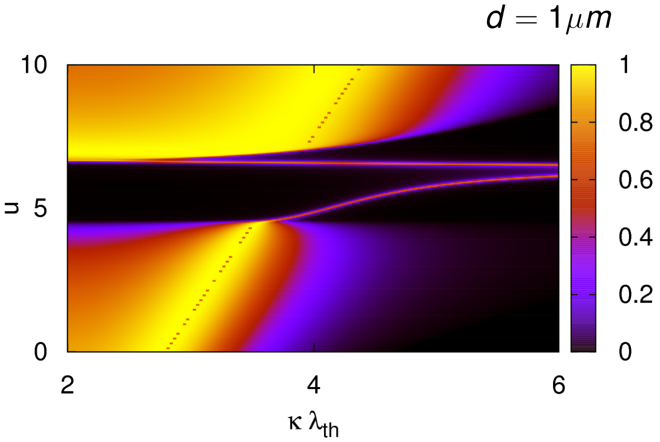
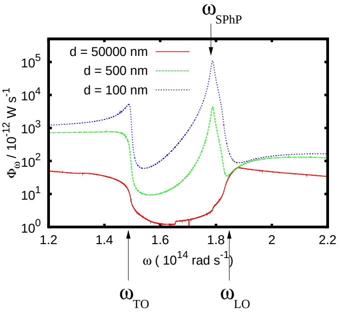
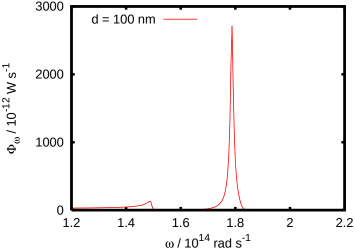
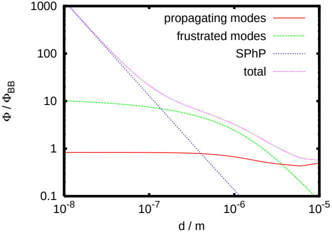


heat flux (SiC, $T_1 = 300\,\mathrm{K}$, $T_2 = 0\,\mathrm{K}$)







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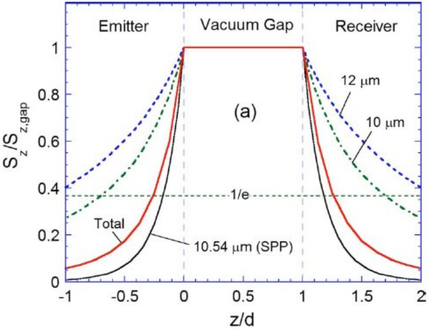
Landauer: Biehs, Rousseau, Greffet, PRL **105**, 234301 (2010)

Limits:Ben-Abdallah, Joulain, PRB **82**, 121419 (R)(2010)

Penetration depth of surface modes in SiC

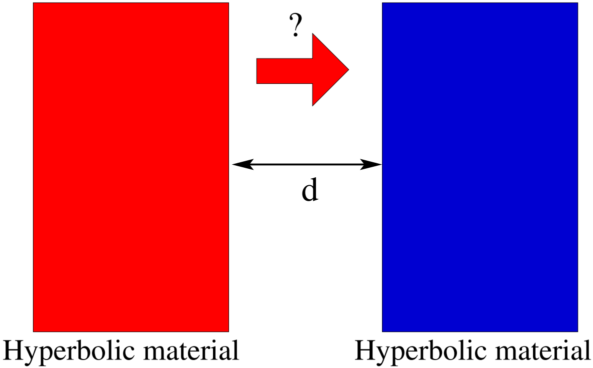


Basu and Zhang, Appl. Phys. Lett. **95**, 133104 (2009)

- Ultrasmall penetration depth ($d \in [1\,\mathrm{nm}, 100\,\mathrm{nm}]$)

$$\delta = 0.25d$$

heat flux between hyperbolic materials?



Nefedov and Simovski, PRB **84**, 195459 (2011)

Biehs, Tschikin and Ben-Abdallah, PRL **109**, 104301 (2012)

Guo et al., APL **101**, 131106 (2012)

indefinite/hyperbolic media I

- uni-axial materials

$$\boldsymbol{\epsilon} = \begin{pmatrix} \epsilon_\perp & 0 & 0 \\ 0 & \epsilon_\perp & 0 \\ 0 & 0 & \epsilon_\parallel \end{pmatrix}$$

- ordinary modes (OM) and extra-ordinary modes (EM)

$$\text{OM: } \frac{k_\perp^2}{\epsilon_\perp} + \frac{k_\parallel^2}{\epsilon_\perp} = \frac{\omega^2}{c^2} \qquad \text{EM: } \frac{k_\perp^2}{\epsilon_\parallel} + \frac{k_\parallel^2}{\epsilon_\perp} = \frac{\omega^2}{c^2}$$

- dielectrics

$$\epsilon_\perp > 0 \quad \text{and} \quad \epsilon_\parallel > 0$$

- hyperbolic/indefinite media

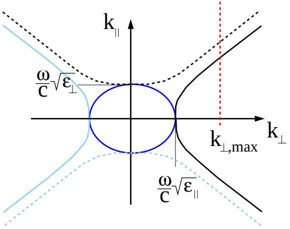
$$\epsilon_\perp \epsilon_\parallel < 0$$

Smith and Schurig, PRL **90**, 077405 (2003)

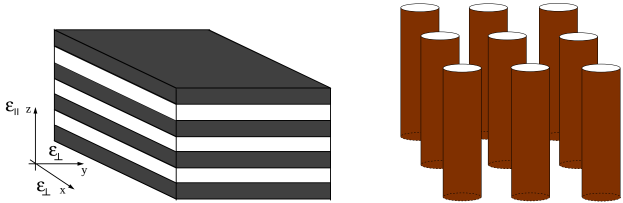
indefinite/hyperbolic media II

- isofrequency lines
  extra-ordinary modes

$$\frac{k_\perp^2}{\epsilon_\parallel} + \frac{k_\parallel^2}{\epsilon_\perp} = \frac{\omega^2}{c^2}$$



- hyperbolic media → nano-structuration ($k_{\max} = \frac{\pi}{\Lambda}$)



- natural hyperbolic media -> Bi_2 Se_3, Sr_2RuO_4, etc.

Narimanov and Kildishev, Nat. Photonics **9**, 214 (2015)

Heat flux for anisotropic materials

- heat flux ($T_1 = T$ und $T_2 = 0$)

$$\Phi = \langle S_z \rangle = \int \frac{\mathrm{d}\,\omega}{2\pi} \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/(k_\mathrm{B} T)} - 1} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \mathcal{T}(\omega, \boldsymbol{\kappa}; d)$$

- Transmission coefficient

$$\mathcal{T}(\omega, \boldsymbol{\kappa}; d) = \begin{cases} \mathrm{Tr}\left[(1 - \mathbb{R}_2^\dagger \mathbb{R}_2)\mathbb{D}^{12}(1 - \mathbb{R}_1^\dagger \mathbb{R}_1)\mathbb{D}^{12\dagger}\right], & \kappa < \frac{\omega}{c} \\ \mathrm{Tr}\left[(\mathbb{R}_2^\dagger - \mathbb{R}_2)\mathbb{D}^{12}(\mathbb{R}_1 - \mathbb{R}_1^\dagger)\mathbb{D}^{12\dagger}\right] e^{-2|k_z|d}, & \kappa > \frac{\omega}{c} \end{cases}$$

- Reflection matrix ($i = 1, 2$)

$$\mathbb{R}_i = \begin{bmatrix} r_i^{\mathrm{s,s}}(\omega, \kappa) & r_i^{\mathrm{s,p}}(\omega, \kappa) \\ r_i^{\mathrm{p,s}}(\omega, \kappa) & r_i^{\mathrm{p,p}}(\omega, \kappa) \end{bmatrix},$$

- 'Fabry-Pérot denominator'

$$\mathbb{D}^{12} = [\mathbb{1} - \mathbb{R}_1 \mathbb{R}_2 \exp(2\mathrm{i}k_z d)]^{-1}$$

Biehs, Rosa, Ben-Abdallah, Joulain, Greffet, Opt. Expr. **19**, A1088-A1103 (2011)

Bimonte and Santamato, Phys. Rev. A **76**, 013810 (2007)

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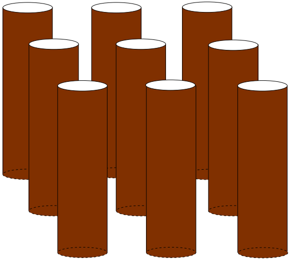
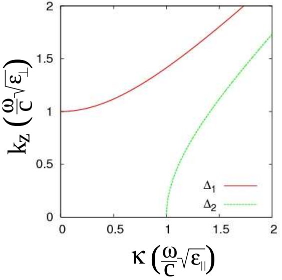
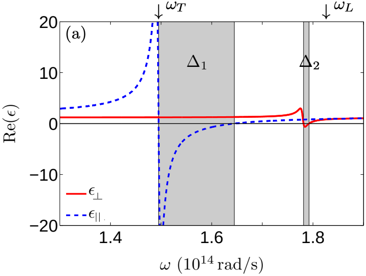
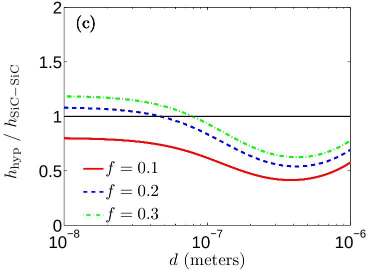
SiC nanowire phononic-polaritonic hyperbolic medium









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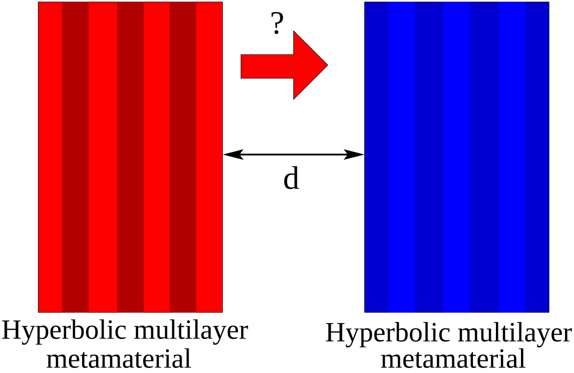
B. Liu and S. Shen, PRB **87**, 115403 (2013); M. S. Mirmoosa et al., JAP **115**, 234905 (2014)

heat flux between hyperbolic materials?



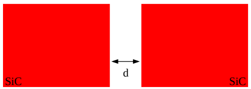
Radiative heat flux in multilayer systems

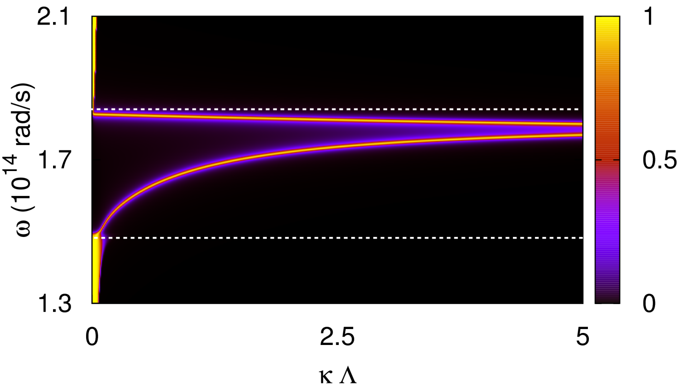
- Volokitin and Persson, Phys. Rev. B **63**, 205404 (2001)
- Narayanaswamy and Chen, JQSRT **93**, 175 (2005)
- Biehs, EPJ B **58**, 423 (2007)
- Francoeur et al., APL **93**, 043109 (2008)
- Lau et al. APL **92**, 103106 (2008)
- Ben-Abdallah et al., JAP **106**, 044306 (2009)
- Francoeur et al., JQSRT **110**, 2002 (2009)
- Pryamikov et al., JQSRT **112**, 1314 (2011)
- Tschikin et al., PLA **376**, 3462 (2012)
- Maslovski et al., PRB **87**, 155124 (2013)
- ...

Building up hyperbolic bands

Transmission coefficient $\mathcal{T}_\mathrm{p}$ ($l_1 = l_2 = 10\,\mathrm{nm}$; d = 10nm)

N = 1

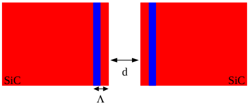


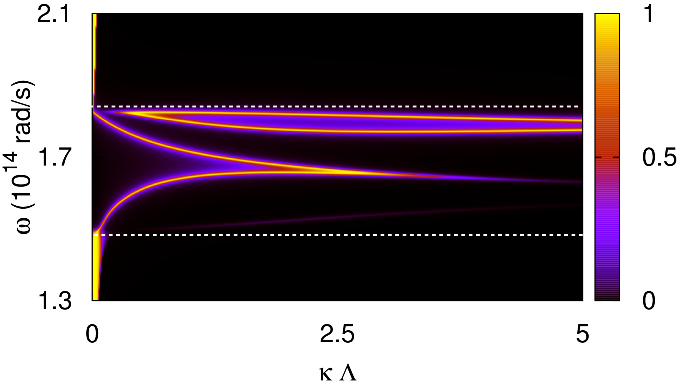


Building up hyperbolic bands

Transmission coefficient $\mathcal{T}_{\rm p}$ ($l_1 = l_2 = 10\,\rm nm$; d = 10nm)

N = 3

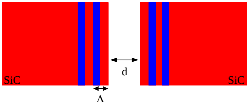


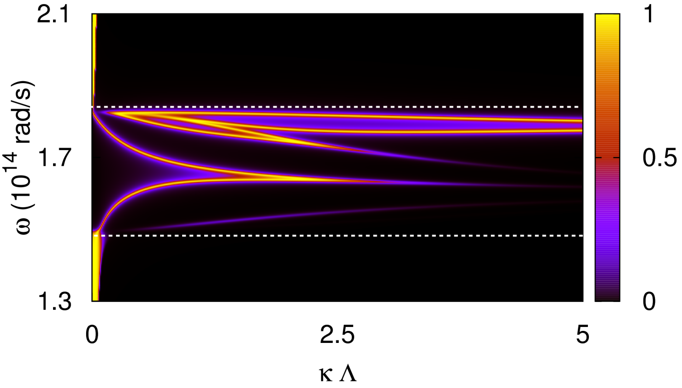


Building up hyperbolic bands

Transmission coefficient $\mathcal{T}_p$ ($l_1 = l_2 = 10\,\mathrm{nm}$; d = 10nm)

N = 5

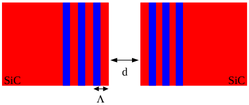


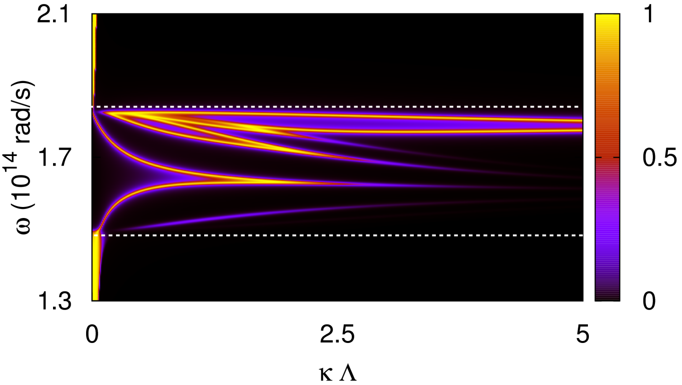


Building up hyperbolic bands

Transmission coefficient $\mathcal{T}_p$ ($l_1 = l_2 = 10\,\mathrm{nm}$; d = 10nm)

N = 7

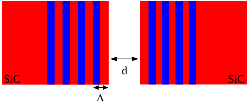


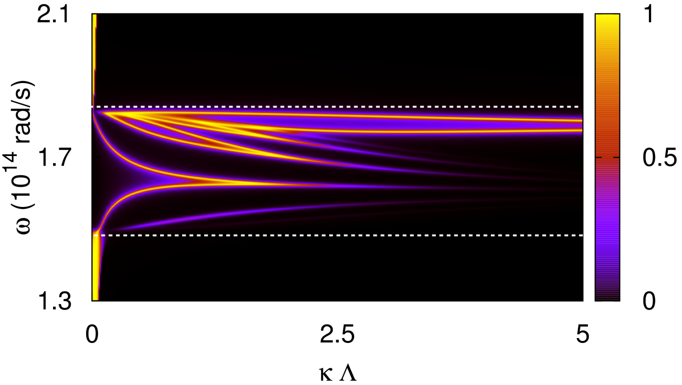


Building up hyperbolic bands

Transmission coefficient $\mathcal{T}_{\mathrm{p}}$ ($l_1 = l_2 = 10\,\mathrm{nm}$; d = 10nm)

N = 9

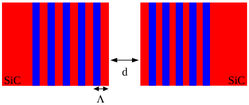


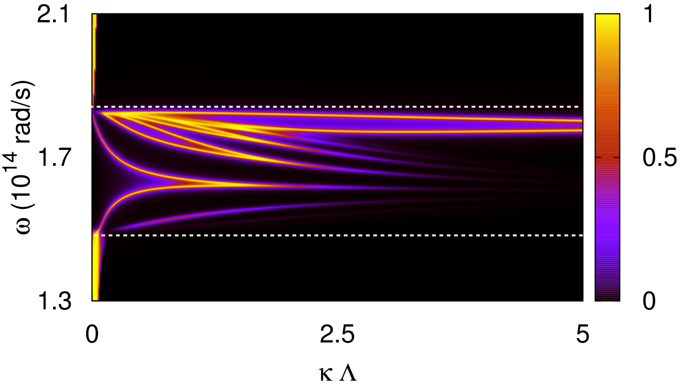


Building up hyperbolic bands

Transmission coefficient $\mathcal{T}_p$ ($l_1 = l_2 = 10\,\mathrm{nm}$; d = 10nm)

N = 11

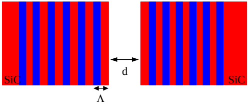


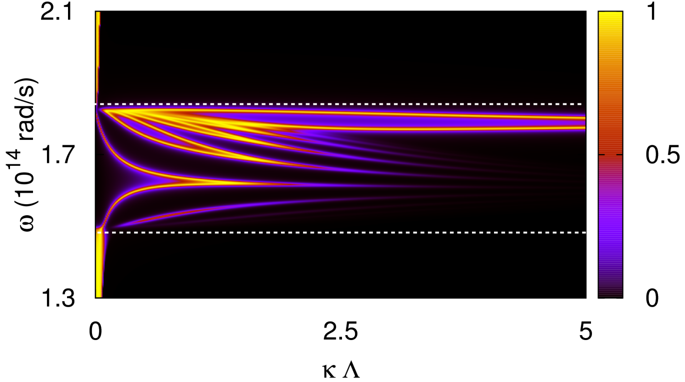


Building up hyperbolic bands

Transmission coefficient $\mathcal{T}_p$ ($l_1 = l_2 = 10\,\text{nm}$; d = 10nm)

N = 13

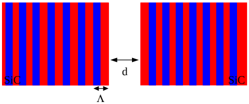


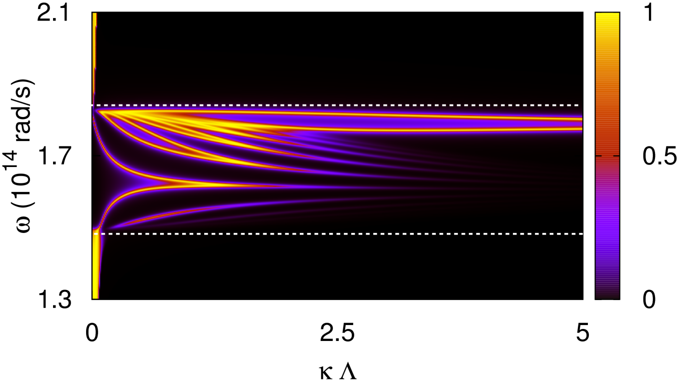


Building up hyperbolic bands

Transmission coefficient $\mathcal{T}_p$ ($l_1 = l_2 = 10\,\mathrm{nm}$; d = 10nm)

N = 15

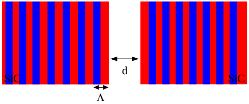


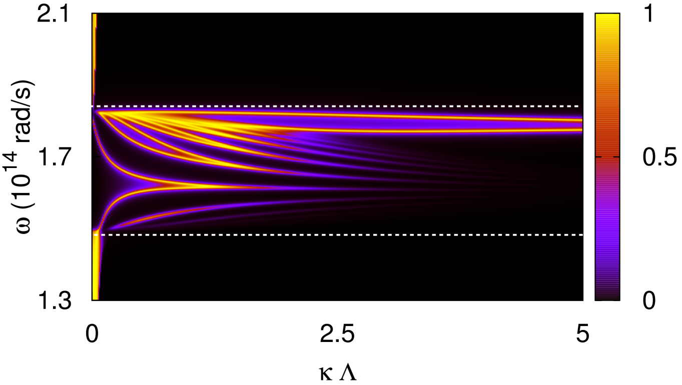


Building up hyperbolic bands

Transmission coefficient $\mathcal{T}_p$ ($l_1 = l_2 = 10\,\mathrm{nm}$; d = 10nm)

N = 17

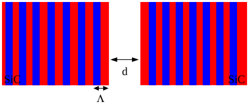


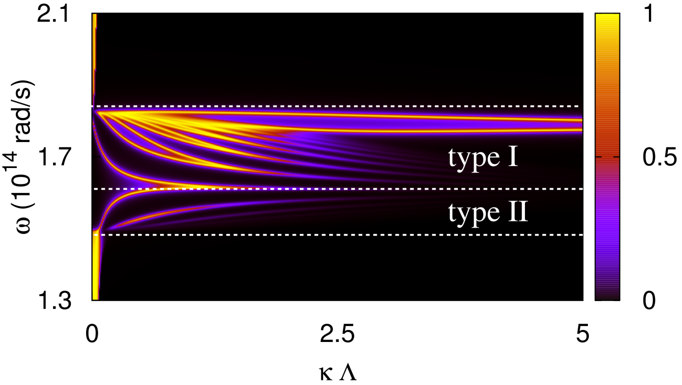


Building up hyperbolic bands

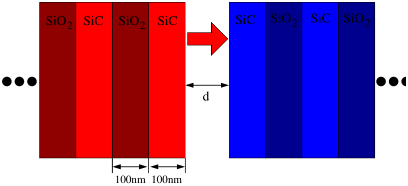
Transmission coefficient $\mathcal{T}_{\mathrm{p}}$ ($l_1 = l_2 = 10\,\mathrm{nm}$; d = 10nm)

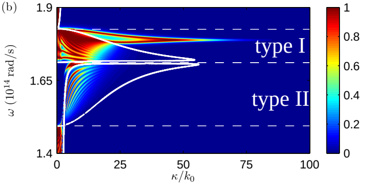
N = 19

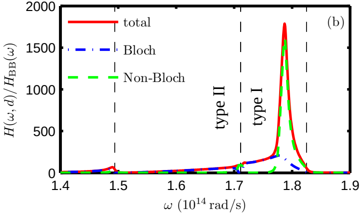


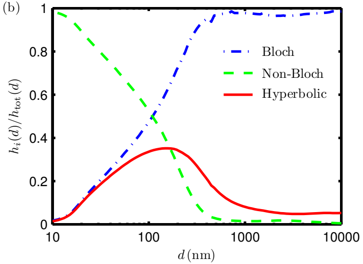


SiC/SiO$_2$ multilayer with SiC on top





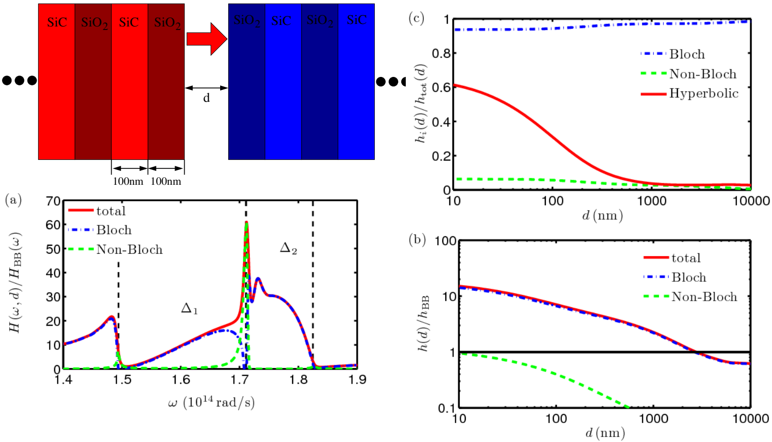




Y. Guo, C. L. Cortes, S. Molesky, and Z. Jacob, APL **101**, 131106 (2012)

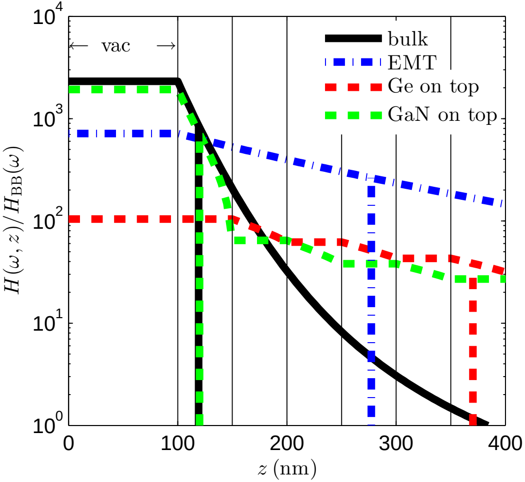
Biehs, Tschikin, Messina, und Ben-Abdallah, Appl. Phys. Lett. **102**, 131106 (2013)

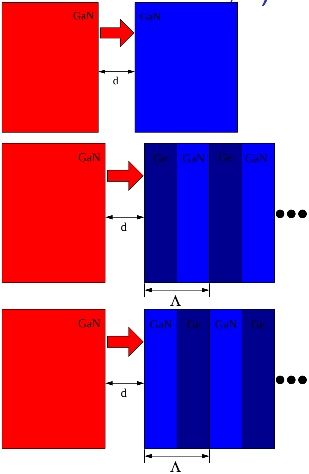
SiC/SiO_2 with SiO_2 on top



Biehs, Tschikin, Messina, und Ben-Abdallah, Appl. Phys. Lett. **102**, 131106 (2013)

Spectral heat flux GaN/Ge multilayer Systems ($d = 100\,\mathrm{nm}$, $\Lambda = 100\,\mathrm{nm}$, $\omega = 1.36 \cdot 10^{14}\mathrm{rad/s}$)

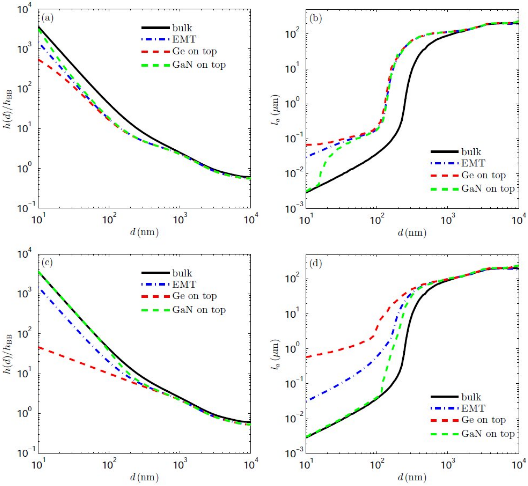




Lang et al., APL **104**, 121903 (2014)

Tschikin et al. JQSRT **158**, 17 (2015).

HTC and penetration depth in mHMM

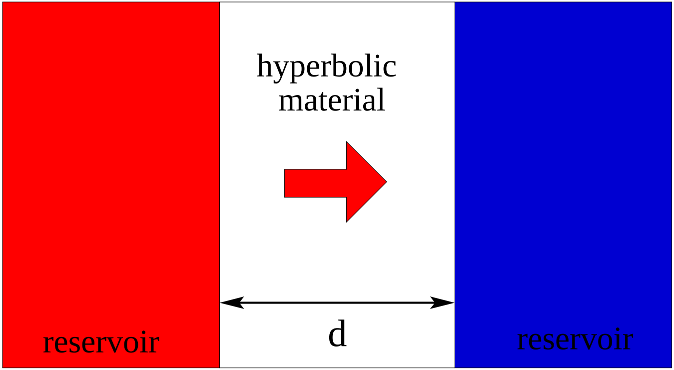


$\Lambda = 10\,\mathrm{nm}$

$\Lambda = 100\,\mathrm{nm}$

Lang et al., APL **104**, 121903 (2014); Tschikin et al. JQSRT **158**, 17 (2015).

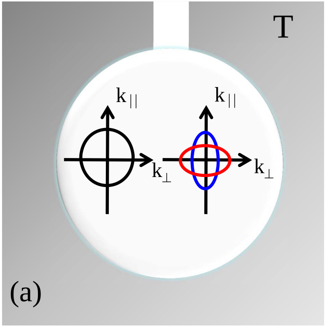
Laws of thermal radiation inside hyperbolic materials

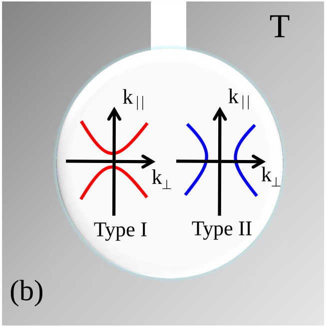


Liu and Narimanov, Phys. Rev. B **91**, 041403(R) (2015)

Biehs, Lang, Petrov, Eich, Ben-Abdallah , PRL **115**, 174301 (2015)

Equilibrium properties of thermal radiation





- Local density of states (LDOS)

$$D(\omega, \mathbf{r}) = \frac{\omega}{c^2\pi}\mathrm{ImTr}\left[\boldsymbol{\epsilon}\mathbb{G}^{\mathrm{EE}}(\mathbf{r},\mathbf{r},\omega) + \boldsymbol{\mu}\mathbb{G}^{\mathrm{HH}}(\mathbf{r},\mathbf{r},\omega)\right]$$

G. S. Agarwal, Phys. Rev. B **11**, 253 (1975)

Eckhardt, Zeitschrift für Physik B: Condensed Matter **46**, 85 (1982)

- Green's function for anisotropic media

Weiglhofer, IEE Proceedings **137**, 5 (1990)

Thermodynamic potentials - dielectrics

- internal and free energy per unit volume

$$U = \int_0^\infty \mathrm{d}\omega\, D(\omega)\mathcal{U}(\omega, T), \qquad F = \int_0^\infty \mathrm{d}\omega\, D(\omega)\mathcal{F}(\omega, T)$$

where

$$\mathcal{U}(\omega, T) = \frac{\hbar\omega}{\mathrm{e}^{\frac{\hbar\omega}{k_\mathrm{B} T}} - 1}, \qquad \mathcal{F}(\omega, T) = k_\mathrm{B} T \ln\left(1 - \mathrm{e}^{-\frac{\hbar\omega}{k_\mathrm{B} T}}\right)$$

- entropy per unit volume $S = \frac{U-F}{T}$
- dielectrics $\epsilon_\| > 0$ and $\epsilon_\perp > 0$ (W. Eckardt, Opt. Commun. **27**, 299 (1990))

$$U_\mathrm{D}^\mathrm{o} = U_\mathrm{BB}^\mathrm{s} \epsilon_\perp \sqrt{\epsilon_\perp} \quad \text{and} \quad U_\mathrm{D}^\mathrm{e} = U_\mathrm{BB}^\mathrm{p} \epsilon_\| \sqrt{\epsilon_\perp}$$

and

$$F_\mathrm{D}^{\mathrm{o/e}} = -\frac{1}{3} U_\mathrm{D}^{\mathrm{o/e}} \quad \text{and} \quad S_\mathrm{D}^{\mathrm{o/e}} = \frac{4}{3} \frac{U_\mathrm{D}^{\mathrm{o/e}}}{T}$$

Thermodynamic potentials - hyperbolic media

- DOS for $k_{\perp,\max} \gg \frac{\omega}{c}\sqrt{|\epsilon_\parallel|}$:

$$D_{\rm I}^{\rm e} \approx D_{\rm II}^{\rm e} \approx \frac{\omega}{\pi^2 c^2} \frac{\sqrt{|\epsilon_\perp \epsilon_\parallel|}}{2} k_{\perp,\max}$$

- Planck's law:

$$D_{\rm I/II}^{\rm e}(\omega)\mathcal{U}(\omega, T)$$

- internal energy ($l_c = \hbar/k_{\rm B} T$)

$$U_{\rm I/II}^{\rm e} \propto k_{\perp,\max} T^3 \quad \text{and} \quad \frac{U_{\rm I/II}^{\rm e}}{U_{\rm BB}^{\rm p}} \propto \frac{l_c}{\Lambda}$$

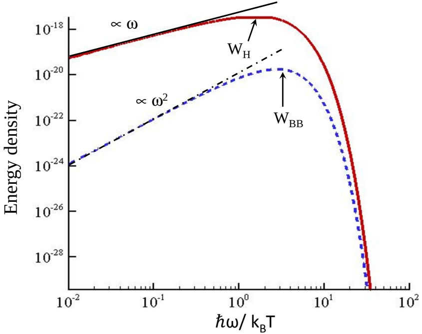
- including dispersion see

  Smolyanov and Narimanov, PRL **105**, 067402 (2010)

Biehs, Lang, Petrov, Eich, Ben-Abdallah, PRL **115**, 174301 (2015)

Comparison with normal blackbody

$|\epsilon_\perp \epsilon_{||}| = 1$, $k_{\perp,\max} = \frac{\pi}{\Lambda}$, $\Lambda = 100\,\text{nm}$, $T = 300\,\text{K}$

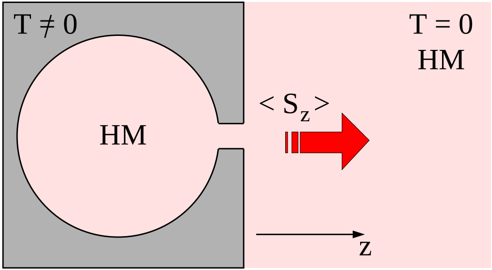


- Wien's law

$$\text{dielectric: } \frac{\hbar\omega_{\max}}{k_B T} = 2.82 \quad \text{hyperbolic: } \frac{\hbar\omega_{\max}}{k_B T} = 1.59$$

Biehs, Lang, Petrov, Eich, Ben-Abdallah, PRL **115**, 174301 (2015)

Towards Stefan-Boltzmann's law for HM



- Poynting vector (Rytov's theory; Levi-Civita tensor $\zeta_{\alpha\beta\gamma}$)

$$\langle S_\gamma \rangle = \epsilon_{\alpha\beta\gamma} 2\mathrm{Re} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \frac{2\omega^3 \mu_0}{c^2} \mathcal{U}(\omega, T) \times \int_V \mathrm{d}\mathbf{r}'' \left(\mathbb{G}^{\mathrm{EE}}(\mathbf{r}, \mathbf{r}'') \mathrm{Im}(\epsilon) \mathbb{G}^{\mathrm{HE}^\dagger}(\mathbf{r}, \mathbf{r}'') \right)_{\alpha\beta}$$

- Green's function for anisotropic media

Weiglhofer, IEE Proceedings **137**, 5 (1990)

Stefan-Boltzmann's law for HM

- lossless limit

$$\Phi^{o/e} \equiv \langle S_z \rangle = \int_0^\infty \frac{d\omega}{2\pi} \mathcal{U}(\omega, T) \int_0^\infty \frac{dk_\perp}{2\pi} k_\perp \frac{\mathrm{Re}(\gamma_{o/e})^2}{\gamma_{o/e}^2}$$

with

$$\gamma_o^2 \equiv \frac{\omega^2}{c^2}\epsilon_\perp - k_\perp^2 \quad \text{and} \quad \gamma_e^2 \equiv \frac{\omega^2}{c^2}\epsilon_\perp - k_\perp^2 \frac{\epsilon_\perp}{\epsilon_\parallel}$$

- Stefan-Boltzmann's law in dielectrics

$$\Phi_D^{o/e} = \Phi_{BB}^{s/p} \begin{Bmatrix} \epsilon_\perp \\ \epsilon_\parallel \end{Bmatrix}$$

- Stefan-Boltzmann's law for HM ($k_{\perp,\max} \gg \frac{\omega}{c}\sqrt{|\epsilon_\parallel|}$)

$$\Phi_{I/II}^{e} \propto k_{\perp,\max}^2 T^2 \quad \text{and} \quad \frac{\Phi_{I/II}^{e}}{\Phi_{BB}^{p}} \approx \left(\frac{l_c}{\Lambda}\right)^2 \frac{5}{2\pi^2}$$

Biehs, Lang, Petrov, Eich, Ben-Abdallah, PRL **115**, 174301 (2015)

Summary

- phonon-polaritonic near-field emitters
  - narrow-band thermal radiation
  - ultra-small penetration depth
- hyperbolic near-field emitters
  - broad-band thermal radiation
  - large penetration depth
- blackbody laws of hyperbolic materials
  - Planck's law
  - Wien's law
  - Stefan-Boltzmann law

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  - M. Tschikin
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  - S. Lang, A. Petrov, M. Eich
    (TU Hamburg)
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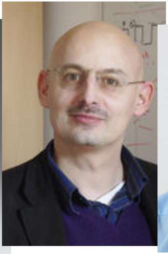
    (University of Montpellier)
  - A. Kittel
    (U Oldenburg)
  - G. S. Agarwal
    (Oklahoma State University, Stillwater)

Maria Tschikin



Philippe Ben-Abdallah

Slawa Lang

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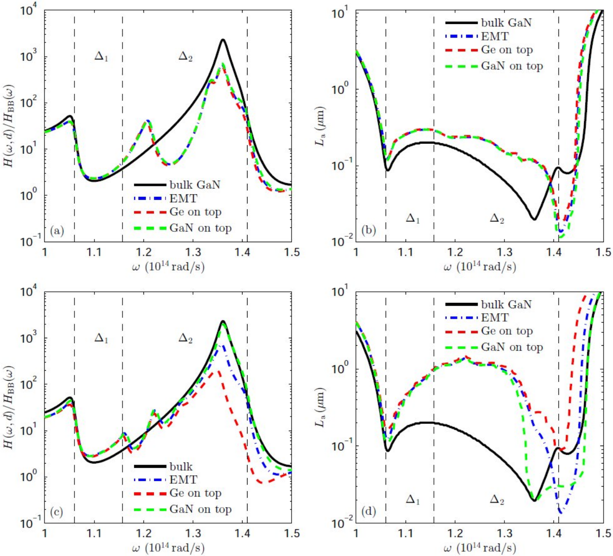
Deutscher Akademischer Austauschdienst
German Academic Exchange Service

Thank you very much for your attention!!!

Spectral HTC and penetration depth in mHMM



$d = 10\,\text{nm}$
$\Lambda = 10\,\text{nm}$

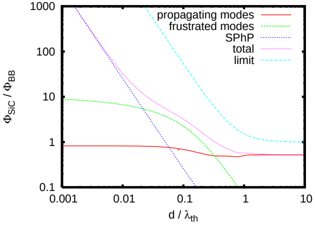
$d = 10\,\text{nm}$
$\Lambda = 100\,\text{nm}$

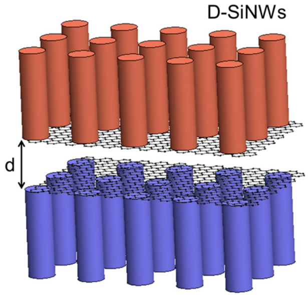
Maximizing near-field radiation

- Limit of htc

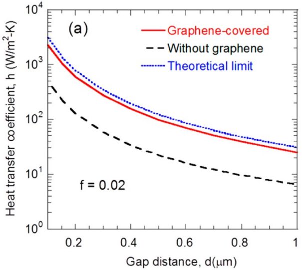
$$h_{\mathrm{max,p}} = \frac{1}{\pi d^2}\left(\frac{\pi^2 k_B^2 T}{3h}\right)\frac{\ln(2)}{2}$$

Biehs,Tschikin, Ben-Abdallah, PRL **109**, 104301 (2012)

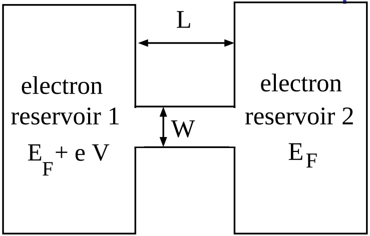


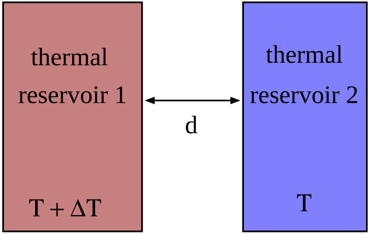


X. Liu et al., ACS Photonics **1**, 785 (2014)

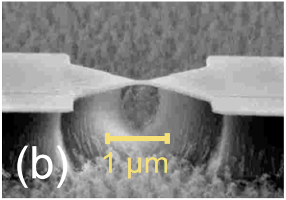


Landauer-like expression for the heat flux





$$I = \Gamma V = \frac{2e^2}{h}\left[\sum_n T_n\right] V \qquad \Phi = \frac{\pi^2 k_B^2 T}{3h}\left[\sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \overline{\mathcal{T}}_i\right] \Delta T$$



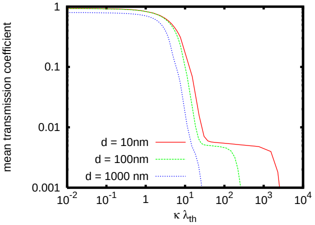
Wu et al., PRB **78**, 235421 (2008)

$$\overline{\mathcal{T}}_i = \frac{\int \mathrm{d}u\, u^2 f(u) \mathcal{T}_i(u,d)}{\int \mathrm{d}u\, u^2 f(u)}$$

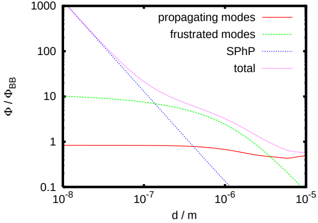
$$f(u) = \frac{u^2 \mathrm{e}^u}{(\mathrm{e}^u - 1)^2}$$

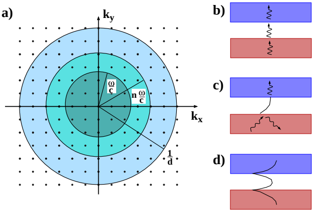
Biehs, Rousseau, Greffet, PRL **105**, 234301 (2010)

Mean transmission coefficient



$$\Phi = \frac{\pi^2 k_B^2 T}{3h} \sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \overline{\mathcal{T}}_j \Delta T$$

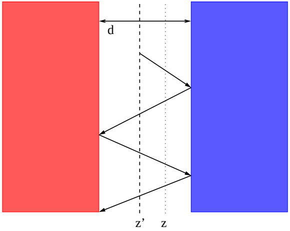




Biehs, Rousseau, Greffet, PRL **105**, 234301 (2010)

fundamental limits:Ben-Abdallah and Joulain, PRB **82**, 121419 (R)(2010)

Green's function in the gap region



- Summing up multiple reflections:

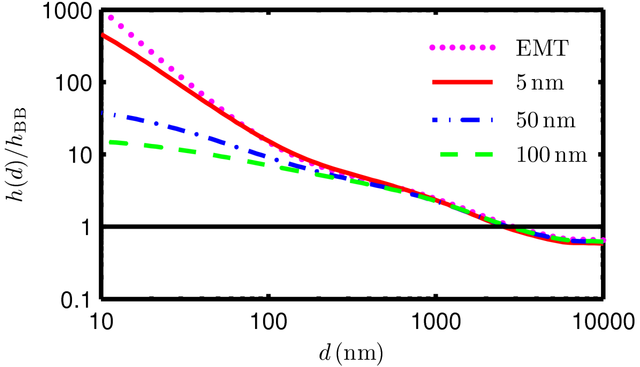
$$\mathbb{G}_A(\boldsymbol{\kappa};z,z') = \frac{\mathrm{i}}{2\gamma_\mathrm{r}}\Big[\mathbb{1}\mathrm{e}^{\mathrm{i}\gamma_\mathrm{r}(z-z')} + \mathrm{e}^{2\mathrm{i}\gamma_\mathrm{r}d}\mathrm{e}^{-\mathrm{i}\gamma_\mathrm{r}(z+z')}\mathbb{R}_2 + \mathrm{e}^{2\mathrm{i}\gamma_\mathrm{r}d}\mathrm{e}^{\mathrm{i}\gamma_\mathrm{r}(z-z')}\mathbb{R}_1\mathbb{R}_2 + \mathrm{e}^{4\mathrm{i}\gamma_\mathrm{r}d}\mathrm{e}^{-\mathrm{i}\gamma_\mathrm{r}(z+z')}\mathbb{R}_2\mathbb{R}_1\mathbb{R}_2 + \dots\Big]$$

- complete intracavity Green's function

$$\mathbb{G}_\mathrm{intra} = \int\frac{\mathrm{d}^2\kappa}{(2\pi)^2}\,\mathrm{e}^{\mathrm{i}\boldsymbol{\kappa}\cdot(\mathbf{x}-\mathbf{x}')}\frac{\mathrm{i}}{2\gamma_\mathrm{r}}\Big[\mathbb{D}^{12}\Big(\mathbb{1}\mathrm{e}^{\mathrm{i}\gamma_\mathrm{r}(z-z')} + \mathbb{R}_1\mathrm{e}^{\mathrm{i}\gamma_\mathrm{r}(z+z')}\Big) + \mathbb{D}^{21}\Big(\mathbb{R}_2\mathbb{R}_1\mathrm{e}^{\mathrm{i}\gamma_\mathrm{r}(z'-z)}\mathrm{e}^{2\mathrm{i}\gamma_\mathrm{r}d} + \mathbb{R}_2\mathrm{e}^{2\mathrm{i}\gamma_\mathrm{r}d}\mathrm{e}^{-\mathrm{i}\gamma_\mathrm{r}(z+z')}\Big)\Big]$$

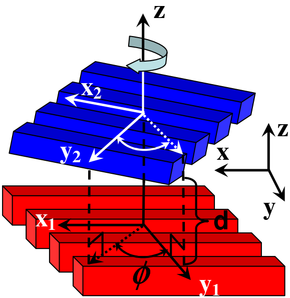
Biehs, Rosa, Ben-Abdallah, Joulain, Greffet, Opt. Expr. **19**, A1088-A1103 (2011)

SiC/SiO_2 multilayer with SiO_2 on top



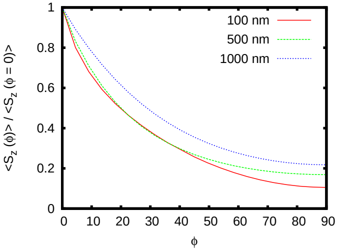
Biehs, Tschikin, Messina, und Ben-Abdallah, Appl. Phys. Lett. **102**, 131106 (2013)

heat flux modulation



Au grating

($T_1 = 300\,\mathrm{K}$, $T_2 = 0\,\mathrm{K}$, f = 0.3)



Biehs, Rosa, Ben-Abdallah, APL **98**, 243102 (2011)

- effective description:

$$\epsilon_{x,z} = \epsilon_{\mathrm{h}_i}(1 - f_i) + f_i$$

$$\epsilon_y = \frac{\epsilon_{\mathrm{h}_i}}{(1 - f_i) + f_i \epsilon_{\mathrm{h}_i}}$$

- beyond EMT:

Rodriguez et al., Phys. Rev. Lett. 107, 114302 (2011)

R. Guérout et al., Phys. Rev. B 85, 180301(R) (2012)

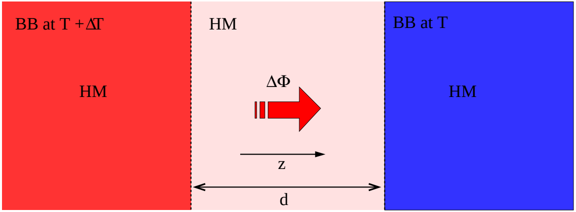
J. Lussange et al., Phys. Rev. B 86, 085432 (2012)

J. Dai et al., Phys. Rev. B 92, 035419 (2015)

J. Dai et al., Phys. Rev. B 93, 155403 (2016)

Stefan-Boltzmann's law for HM

- max heat flux for GaN/SiO_2-HMM ($\Lambda = 10$nm)



- conductivity in GaN/SiO_2: $\approx 0.79 \frac{\mathrm{W}}{\mathrm{mK}}$

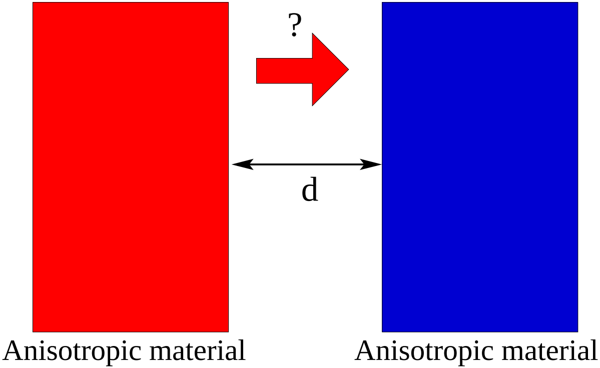
$$d = 200\,\mathrm{nm} \Rightarrow \text{htc} \approx 3.7 \times 10^6 \frac{\mathrm{W}}{\mathrm{m^2K}}$$

$$d = 400\,\mathrm{nm} \Rightarrow \text{htc} \approx 1.85 \times 10^6 \frac{\mathrm{W}}{\mathrm{m^2K}}$$

- Stefan-Boltzmann for HM (photons):

$$\text{htc} \approx 2.3 \times 10^6 \frac{\mathrm{W}}{\mathrm{m^2K}}$$

heat flux between anisotropic materials?

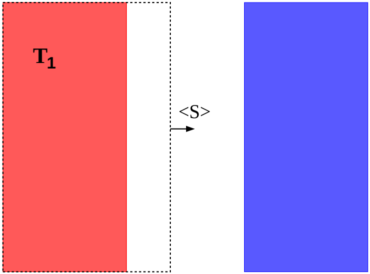


Mean Poynting vector

$$\langle S_\gamma(\mathbf{r},t)\rangle = \epsilon_{\alpha\beta\gamma}\langle E_\alpha(\mathbf{r},t)H_\beta(\mathbf{r},t)\rangle = \epsilon_{\alpha\beta\gamma}\int_0^\infty \frac{\mathrm{d}\omega}{2\pi} 2\frac{\omega^3}{c^2}\mu_0\Theta(T)\frac{\mathrm{i}}{2k_0^2}\mathbb{I}^{\mathrm{S}}_{\alpha\beta} + \text{c.c.}$$

with

$$\mathbb{I}^{\mathrm{S}} := \int_{\partial V}\mathrm{d}S'\Big[\big(\nabla'\times\mathbb{G}^{\mathrm{EE}^t}(\mathbf{r},\mathbf{r}')\big)^t\cdot\big(\mathbf{n}\times\mathbb{G}^{\mathrm{HE}^\dagger}(\mathbf{r},\mathbf{r}')\big) + \mathbb{G}^{\mathrm{EE}}(\mathbf{r},\mathbf{r}')\cdot\big(\mathbf{n}\times\nabla'\times\mathbb{G}^{\mathrm{HE}^\dagger}(\mathbf{r},\mathbf{r}')\big)\Big].$$



$\mathbb{G}(\mathbf{r},\mathbf{r}')$ electric Green's function with $\mathbf{r}$ and $\mathbf{r}'$ inside the gap

Volokitin and Persson, Rev. Mod. Phys. **79**, 1291 (2007)

A. Narayanaswamy and Y. Zheng, JQSRT **132**,12 , (2014)

Heat flux for anisotropic materials

- heat flux ($T_1 = T$ und $T_2 = 0$)

$$\Phi = \langle S_z \rangle = \int \frac{\mathrm{d}\,\omega}{2\pi} \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/(k_\mathrm{B} T)} - 1} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \mathcal{T}(\omega, \boldsymbol{\kappa}; d)$$

- Transmission coefficient

$$\mathcal{T}(\omega, \boldsymbol{\kappa}; d) = \begin{cases} \mathrm{Tr}\left[(1 - \mathbb{R}_2^\dagger \mathbb{R}_2)\mathbb{D}^{12}(1 - \mathbb{R}_1^\dagger \mathbb{R}_1)\mathbb{D}^{12^\dagger}\right], & \kappa < \frac{\omega}{c} \\ \mathrm{Tr}\left[(\mathbb{R}_2^\dagger - \mathbb{R}_2)\mathbb{D}^{12}(\mathbb{R}_1 - \mathbb{R}_1^\dagger)\mathbb{D}^{12^\dagger}\right] e^{-2|k_z|d}, & \kappa > \frac{\omega}{c} \end{cases}$$

- Reflection matrix ($i = 1, 2$)

$$\mathbb{R}_i = \begin{bmatrix} r_i^{\mathrm{s,s}}(\omega, \kappa) & r_i^{\mathrm{s,p}}(\omega, \kappa) \\ r_i^{\mathrm{p,s}}(\omega, \kappa) & r_i^{\mathrm{p,p}}(\omega, \kappa) \end{bmatrix},$$

- 'Fabry-Pérot denominator'

$$\mathbb{D}^{12} = [\mathbb{1} - \mathbb{R}_1 \mathbb{R}_2 \exp(2\mathrm{i}k_z d)]^{-1}$$

Biehs, Rosa, Ben-Abdallah, Joulain, Greffet, Opt. Expr. **19**, A1088-A1103 (2011)

Bimonte and Santamato, Phys. Rev. A **76**, 013810 (2007)

Biehs and Ben-Abdallah, Phys. Rev. B **93**, 165405 (2016)