Casimir Forces, Friction and Radiative Heat Transfer in Graphene Systems

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- Introduction
- Fluctuations produce forces
- Casimir forces between graphene sheets
- Casimir friction
- Using graphene to detect Casimir friction
- Radiative heat transfer in graphene systems
- Conclusion
Fluctuations produce forces

H. Casimir 1948
E. Lifshitz 1954

\[ \frac{F_C}{A} = \frac{\hbar c \pi}{240d^4} = \frac{1.3 \times 10^{-27}}{d^4} \text{Nm}^2, \]
Rytov’s theory

\[ \nabla \times \mathbf{E} = i \frac{\omega}{c} \mathbf{B} \]

\[ \nabla \times \mathbf{H} = -i \frac{\omega}{c} \mathbf{D} + \frac{4\pi}{c} \mathbf{j}^f \]

\[
\left\langle j^f_{i'}(\mathbf{r}) j^{f*}_{k'}(\mathbf{r}') \right\rangle_{\omega} = \frac{\hbar}{(2\pi)^2} \left( \frac{1}{2} + n(\omega) \right) \omega^2 \text{Im} \varepsilon_{ik}(\mathbf{r}, \mathbf{r}', \omega)
\]

\[ n(\omega) = \frac{1}{e^{\hbar \omega/k_B T} - 1} \]
Application of Rytov’s theory

- Polder D. and Van Hove M. Theory of the radiative heat transfer 1971
Temperature Effect

Quantum fluctuations dominate for $d < \lambda_T = \frac{c\hbar}{k_B T}$

Thermal fluctuations dominate for $d > \lambda_T = \frac{c\hbar}{k_B T}$

Sushkov A.O. et al 2011
Doppler Effect

J. Pendry 1997
A.I. Volokitin and B.N.J. Persson 2008
Casimir force between moving bodies

\[ F_z = F_{zT} + F_{z0}, \]

where the temperature dependent term \( F_{zT} \) and the zero-temperature contribution \( F_{z0} \) are given by

\[
F_{zT} = \frac{\hbar}{\pi^3} \int_0^\infty dq_y \int_0^\infty dq_x q e^{-2qd} \left\{ \int_0^\infty d\omega \left( \frac{\text{Im} R_1(\omega) \text{Re} R_2(\omega^+) n_1(\omega) + \text{Re} R_1(\omega) \text{Im} R_2(\omega^+) n_2(\omega^+)}{|1 - e^{-2qd} R_1(\omega) R_2(\omega^+)|^2} \right) + (1 \leftrightarrow 2) \right\},
\]

\[
F_{z0} = \frac{\hbar}{2\pi^3} \int_0^\infty dq_y \int_0^\infty dq_2 \left\{ \text{Re} \int_0^\infty d\omega s e^{-2sd} \left( \frac{R_1(i\omega) R_2(i\omega + q_x v)}{1 - e^{-2qd} R_1(i\omega) R_2(i\omega + q_x v)} \right) \right\},
\]

\[
\omega^\pm = \omega \pm q_x v
\]

The rise of graphene

Quantum fluctuations dominate for \( d < \zeta_T = v_F\hbar/k_B T \)
Thermal fluctuations dominate for \( d > \zeta_T = v_F\hbar/k_B T \)
Drift velocity \( v_D \approx 10^6 \text{m/s} \)
Casimir force between graphene sheets

\[ F_z, \text{quantum} \]

\[ F_z, \text{thermal} (v=2 \cdot 10^6 \text{ m/s}) \]

\[ F_z, \text{thermal} (v=0) \]

\[ F_{zT}, 10^4 \text{ N/m}^2 \]

Resonant photon tunneling enhancement

\[ v > 2v_F \approx 2 \cdot 10^6 \text{ m/s} \]

A.I. Volokitin and B.N.J. Persson 2013
Casimir friction

\[ F_{xT} = \frac{\hbar}{\pi^3} \int_0^\infty dq_x \int_0^\infty dq_y q_x e^{-2qd} \left\{ \int_0^\infty d\omega \left( \frac{\text{Im} R_1(\omega) \text{Im} R_2(\omega^+)}{|1 - e^{-2qd} R_1(\omega) R_2(\omega^+)|^2} \times [n_1(\omega) - n_2(\omega^+)] + (1 \leftrightarrow 2) \right) \right\} \]

\[ - \int_0^{q_{xv}} d\omega \left( \frac{\text{Im} R_{d1}(\omega) \text{Im} R_{g1}(\omega^-)}{|1 - e^{-2qd} R_1(\omega) R_2(\omega^-)|^2} n_1(\omega) + (1 \leftrightarrow 2) \right), \]

\[ F_{x0} = -\frac{\hbar}{2\pi^3} \int_0^\infty dq_y \int_0^\infty dq_x q_x e^{-2qd} \int_0^{q_{xv}} d\omega \left( \frac{\text{Im} R_1(\omega) \text{Im} R_2(\omega^-)}{|1 - e^{-2qd} R_1(\omega) R_2(\omega^-)|^2} n_1(\omega) + (1 \leftrightarrow 2) \right). \]

Resonant photon tunneling

Thermal fluctuations dominate at $v < v_T = \frac{k_B T d}{\hbar}$
Quantum fluctuations dominate at $v > v_T = \frac{k_B T d}{\hbar}$

(a) $vq_x = \omega_1 + \omega_2$
(b) $vq_x = \omega_1 - \omega_2$
Quantum Friction

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We investigate the van der Waals friction between graphene and an amorphous SiO$_2$ substrate. We find that due to this friction the electric current is saturated at a high electric field, in agreement with experiment. The saturation current depends weakly on the temperature, which we attribute to the quantum friction between the graphene carriers and the substrate optical phonons. We calculate also the frictional drag between two graphene sheets caused by van der Waals friction, and find that this drag can induce a voltage high enough to be easily measured experimentally.

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Current density-electric field dependence in graphene on SiO$_2$

\[ F_x(T_d, T_g, v)v = S_z(T_d, T_g, v) + \alpha_{phon}(T_g - T_d) \]
\[ v_{sat} \sim \omega_{ph}/k_F \sim 10^6 \text{m/s} \]
\[ J_{sat} = en_s v_{sat} \sim 1 \text{mA/µm} \]
Two ways to study Casimir friction

Left: a metallic block is sliding relative to the metallic substrate with velocity $v$.
Right: A drift motion of the free carries of charge (electrons or ions) is induced in the upper medium.
Frictional Drag in 2D-systems

Layer 1

Layer 2

Experiment - Quantum wells T. J. Gramila et.al 1991, U. Sivan et.al 1992
Experiment - Graphene Sheets S. Kim et.al 2012, R.V. Gorbachev et.al 2012
Frictional Drag between Graphene Sheets

Low Velocities

At \( v \ll v_F \) induced electric field \( E = \rho_D J = \mu^{-1}v \).

\[
\rho_D = \frac{\Gamma}{(ne)^2} = \frac{\hbar \pi \zeta(3)}{e^2} \left( \frac{k_B T}{\epsilon_F} \right)^2 \frac{1}{(k_F d)^2} \frac{1}{(k_{TF} d)^2},
\]

\[
F_{x0} = \frac{\hbar v}{d^4} \frac{15 \zeta(5)}{128 \pi^2} \left( \frac{v}{v_F} \right)^2 \frac{1}{(k_{TF} d)^2}.
\]

\[
F_{x0}/F_{xT} \approx (15/8\pi^2)(v/v_T)^2, \text{ where } v_T = \omega_T d \approx 10^5 \text{ m/s}.
\]
Frictional Drag between Graphene Sheets

High Velocities

(a) $T=600$ K, $300$ K, $0$ K

(b) $T=300$ K, $100$ K, $0$ K

$d=1$ nm

$d=10$ nm
Radiative Heat Transfer.

\[ d \gg \lambda_T = \frac{c\hbar}{k_B T} \]

\[ S = \frac{\pi^2 k_B^4}{60\hbar^3 c^2} \left( T_1^4 - T_2^4 \right), \]

Theory. D. Polder and M. Van Hove 1971

Experiment. Rousseau E. et al 2009; Shen S. et al 2009
Radiative heat transfer between moving bodies

\[
S_z = \frac{\hbar}{\pi^3} \int_0^\infty dq_y \int_0^\infty dq_x e^{-2qd} \left\{ \int_0^\infty d\omega \left( -\frac{\omega \text{Im} R_1(\omega) \text{Im} R_2(\omega^+)}{|1 - e^{-2qd} R_1(\omega) R_2(\omega^+)|^2} \times \right) \right. \\
\left. [n_1(\omega) - n_2(\omega^+)] + \frac{\omega^+ \text{Im} R_d(\omega^+) \text{Im} R_g(\omega)}{|1 - e^{-2qd} R_1(\omega^+) R_2(\omega)|^2} [n_2(\omega) - n_1(\omega^+)] \right) + \\
\left\{ \int_0^{q_x \nu} d\omega \frac{\omega \text{Im} R_1(\omega) \text{Im} R_2(\omega^-)}{|1 - e^{-2qd} R_1(\omega) R_2(\omega^-)|^2} [n_2(\omega^-) - n_1(\omega)] \right\},
\]

\[
F_x(T_d, T_g) = S_z(T_d, T_g) + \alpha_{ph}(T_g - T_d)
\]

Friction generates Heat Transfer

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Near-field radiative heat transfer between closely spaced graphene and amorphous SiO$_2$

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We study the near-field radiative energy transfer between graphene and an amorphous SiO$_2$ substrate. In comparison with the existing theories of near-field radiative heat transfer our theory takes into account that the free carriers in graphene are moving relative to the substrate with a drift velocity $v$. In this case the heat flux is determined by both thermal and quantum fluctuations. We find that quantum fluctuations give an important contribution to the radiative energy transfer for low temperatures and high electric field (large drift velocities). For nonsuspended graphene the near-field radiative energy transfer gives a significant contribution to the heat transfer in addition to the contribution from phononic coupling. For suspended graphene (large separation) the corresponding radiative energy transfer coefficient at a nanoscale gap is $\sim$3 orders of magnitude larger than radiative heat transfer coefficient of the blackbody radiation limit.

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$$F_x(T_d, T_g, v)v = S_z(T_d, T_g, v) + \alpha_{phon}(T_g - T_d)$$
 Phononic and Radiative Heat Transfer

\[ n = 10^{16} \text{ } m^{-2}, \quad d = 0.35 \text{ } nm, \quad \alpha_{ph} = 1.0 \times 10^8 \text{ } W \text{ } m^{-2} \text{ } K^{-1} \]

\[ \alpha = \frac{S_z(T_d, T_g) + \alpha_{ph}\Delta T}{\Delta T} \approx \frac{(\alpha_{ph} + S'_{z0})F_{t0}v - S_{z0}F'_{t0}v}{F'_{t0}v - S_{z0}} \]
$n = 10^{16} \text{ } m^{-2}, \quad d = 0.35 \text{ nm}$
Radiative Energy Transfer

\[
\frac{\alpha}{\alpha_0} = \frac{F_{fr}(T, \nu)v}{F_{fr}(T, V)v - S_z(T, \nu)}
\]
Dependence of Heat Flux on Electric Field

\[ d = 1 \, nm \]
Conclusion

- Electric current in graphene sheet can produce measurable change of Casimir force between graphene sheets.
- The thermal Casimir force as well as the Casimir friction are strongly enhanced in the case of resonant photon tunneling.
- Casimir friction and its limiting case - quantum friction can be studied in frictional drag experiment between graphene sheets and by measuring electric current-electric field dependence for graphene field-effect transistor.
- Quantum fluctuations can generate radiative energy transfer comparable with radiative heat transfer due to thermal fluctuations.
Thank you for your attention!