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Engineering

Role of the spatial dispersion for radiative heat transfer in wire media

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Outline

1. Hyperbolic media
2. Theory of the thermal radiation transfer through the hyperbolic medium.
3. Wire media and effects of spatial dispersion
4. Arrays of carbon nanotubes as perfect hyperbolic media for the IR range.
5. Transmission through wire media with the spatial dispersion.
6. Discussion. Transmission and the total spectral radiation heat density
7. Conclusions

Hyperbolic media

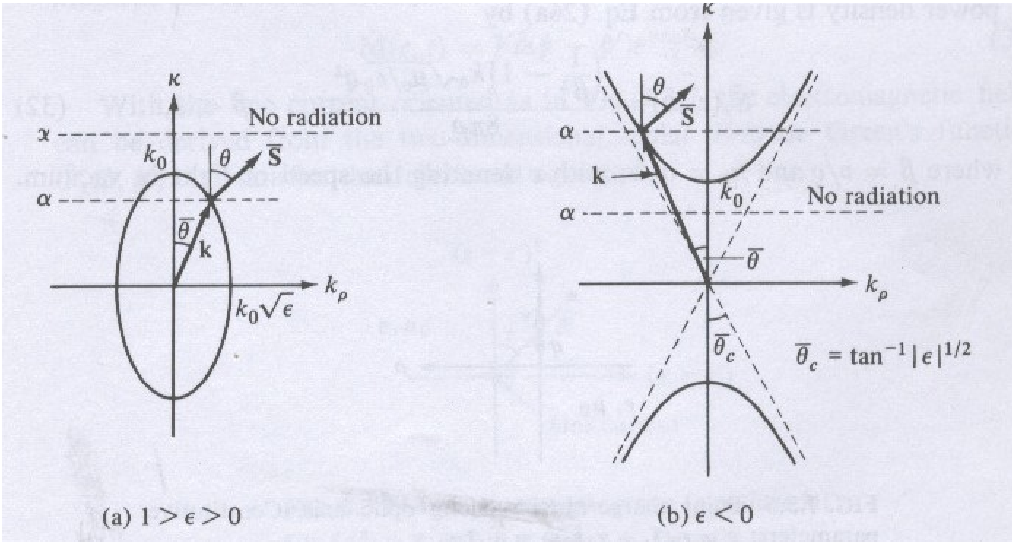


FIG. 7.3.5 Interpretation via wavenumber surfaces.

L.F. Felsen, N. Marcuvitz, *Radiation and Scattering of Waves,* 1973 (references to **E. Arbel, L.B. Felsen**, 1963)

- infinite power, radiated by a point-like source

D.R. Smith, D. Schurig, *PRL* **90** 2003
Term *indefinite* medium, negative refraction, near-field focusing

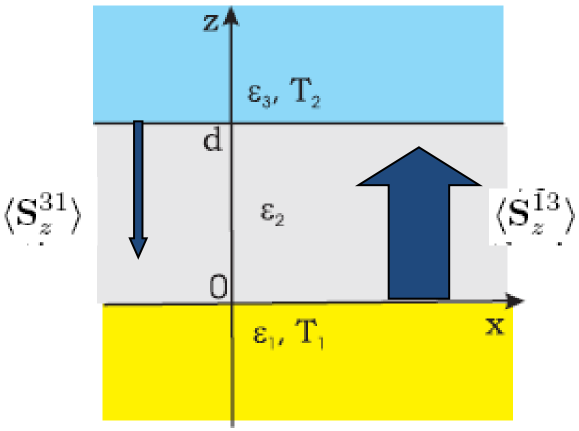
Z. Jacob, L. V. Alekseyev, E. E. Narimanov, Opt. Express 14, 8247 (2006). Optical Hyperlens: Far-field imaging beyond the diffraction limit.

Z. Jacob, et al.
App. Phys. B **100**, 215 (2010).
Engineering photonic density of states in hyperbolic metamaterials.

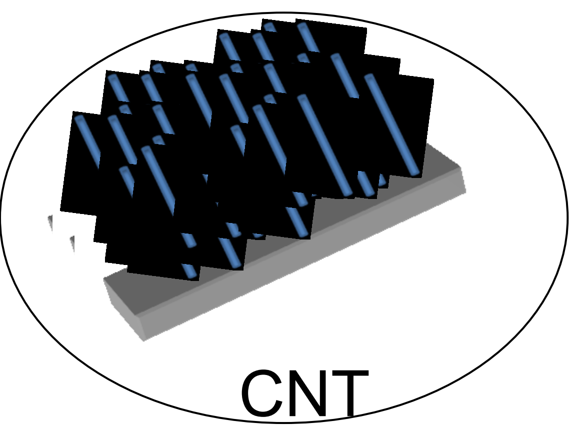
M. A. Noginov, et al.
Optics Letters 35, 1863 (2010)
Control of spontaneous emission

I.S. Nefedov, C.R. Simovski, PRB **84,** 195459 (2011), Giant radiation heat transfer through micron gaps

I.S. Nefedov, C.R. Simovski, Giant radiation heat transfer through the micron gaps, PRB **84,** 195459 (2011).



Hot medium



ensemble-averaging

$$q''_\omega = \int_0^\infty \left[\langle \mathbf{S}_z^{13}(k_x, \omega, T_1) \rangle - \langle \mathbf{S}_z^{31}(k_x, \omega, T_2) \rangle \right] k_x dk_x . \qquad (1)$$

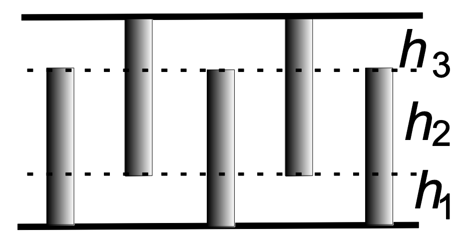
Our own algorithm: 1st we calculate the incident Poynting vector

$$\langle \mathbf{S}_z^1(\mathbf{r}, \omega) \rangle = \int_0^\infty \frac{1}{2} \langle \mathrm{Re} E_x(\mathbf{r}, \omega) H_y^*(\mathbf{r}, \omega') \rangle d\omega' \qquad (2)$$

produced by a half-space filled with fluctuating currents $\mathbf{j}$

which obey the fluctuation-dissipation theorem

$$\langle j_m(\mathbf{r}, \omega) j_n^*(\mathbf{r}', \omega') \rangle = \frac{4}{\pi} \omega \epsilon_0 \epsilon''(\omega) \delta_{mn} \delta(\mathbf{r} - \mathbf{r}') \delta(\omega - \omega') \Theta(\omega, T).$$



energy of a Planck oscillator:

$$\Theta(\omega, T) = \frac{\hbar \omega}{\exp(\hbar \omega / k_B T) - 1}$$

Without loss of generality we can assume that $\mathbf{j}$ are fluctuating current sheets

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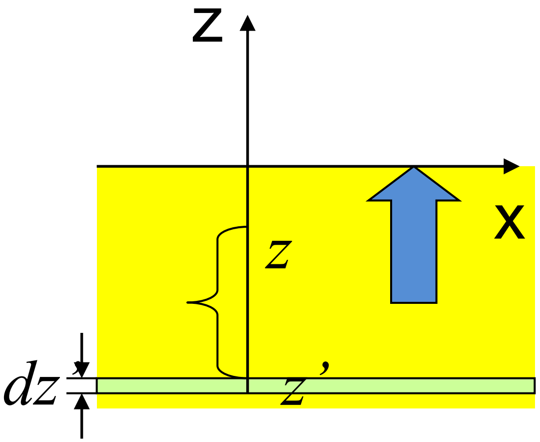
$$\mathbf{j}(z) = \mathbf{j}_0(z')\delta(z - z')e^{j(\omega t - k_x x - k_y y)}. \quad \Rightarrow \quad \begin{aligned} dE_x &= \frac{\eta dz'}{2k\epsilon_1}(j_{0x}k_{1z} - j_{0z}k_x)e^{j[\omega t - k_x x - k_{1z}(z-z')]}, \\ dH_y &= \frac{dz'}{2k_{1z}}(-j_{0x}k_{1z} + j_{0z}k_x)e^{j[\omega t - k_x x - k_{1z}(z-z')]}, \end{aligned}$$

$$k_{1z} = \sqrt{k^2\epsilon_1 - k_x^2}, \ \eta = \sqrt{\varepsilon_0/\mu_0}.$$



Then at z=0 we have

$$\langle E_x H_y^* \rangle = \frac{\eta e^{-jk_{1z}z}}{4kk_{1z}\epsilon_1}\int_{-\infty}^{0} e^{j(k_{1z}-k_{1z}^*)z'}e^{j(\omega-\omega')t}\langle [j_{0x}(\omega, z') k_{1z} - j_{0z}(\omega, z')k_x][-j_{0x}(\omega', z')k_{1z} + j_{0z}(\omega', z')k_x]\rangle dz'. \qquad (3)$$

Rewrite FDT for amplitudes of current sheets

$$\langle j_{0m}(z', \omega) j_{0n}^*(z', \omega')\rangle = \frac{4}{\pi}\omega\epsilon_0\epsilon''(\omega)\delta_{mn}\delta(\omega - \omega')\Theta(\omega, T).$$

Then after integration over ω' in (2) and z' in (3) we have:

$$\langle \mathbf{S}_z^1(k_x, \omega)\rangle = \frac{\epsilon_1''(\omega)/2\pi}{\epsilon_1 k_{1z}\mathrm{Im}(k_{1z})}(k_x^2 + k_{1z}k_{1z}^*)\Theta(\omega, T) + \mathrm{c.c.}$$

Suitable also for ε-negative medium

We found the incident Poynting vector

The Poynting vector transmitted from medium 1 to medium 3:

$$\langle \mathbf{S}_z^{13}(k_x,\omega)\rangle = \frac{1}{2}\langle \mathbf{S}_z^{1}(k_x,\omega)\rangle |\tau|^2 \frac{Z_1^*}{Z_3^*} + \text{c.c.}$$

where

$$Z_i = -E_{ix}/H_{iy} = \eta k_{iz}/k \qquad (i=1,2,3)$$

transmission coefficient:

$$\tau = \frac{2}{M_{11} + M_{12}/Z_3 + M_{21}Z_1 + M_{22}Z_1/Z_3}$$

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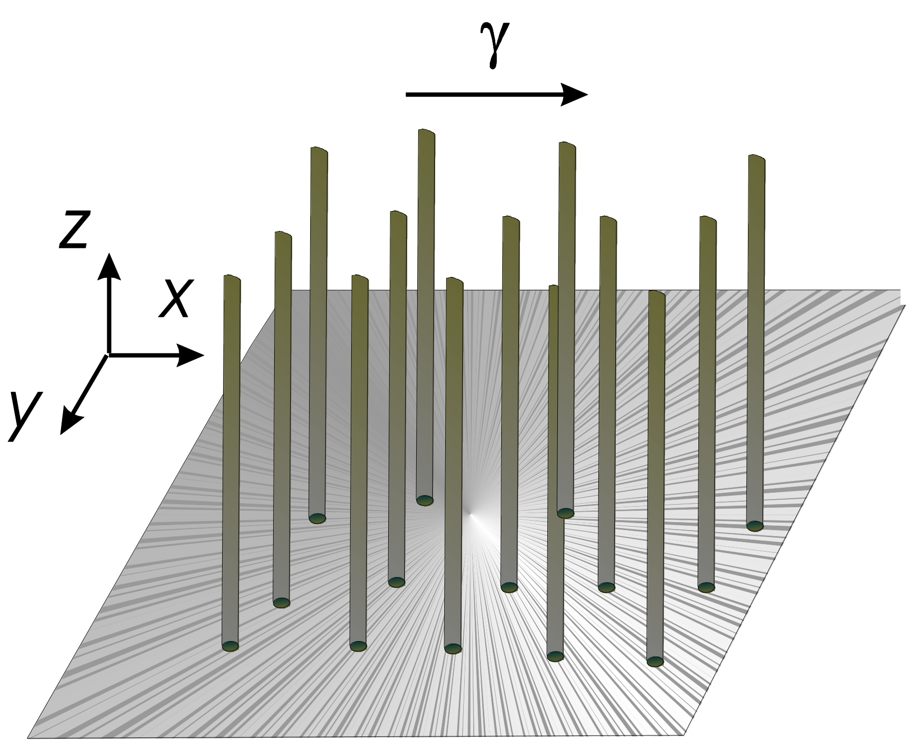
Wire Media as hyperbolic metamaterials



Is a wire medium an ε-negative material?

$$\bar{\bar{\epsilon}} = \epsilon_{zz}\mathbf{z}_0\mathbf{z}_0 + \epsilon_0(\mathbf{x}_0\mathbf{x}_0 + \mathbf{y}_0\mathbf{y}_0)$$

$$\varepsilon_{zz} = \varepsilon_h\left(1 - \frac{k_p^2}{k^2\varepsilon_h}\right)$$

$$\kappa_p = \frac{}{a^2 \ln(a/r)}$$

J.B. Pendry, A.J. Holden, W.J. Stenwart, I. Youngs, Phys. Rev. Lett. 76, 4773 (1996).

$$k_z = \sqrt{k^2 - k_\perp^2\frac{\varepsilon_h}{\varepsilon_{zz}}},\ \varepsilon_{zz} < 0,\ k < k_p.$$

Waves are propagating ones for any transverse wave vector component

Effect of spatial dispersion at low frequencies

P.A. Belov, R. Marques, S.I. Maslovski, I.S. Nefedov, M. Silveirinha, C.R. Simovski, S.A. Tretyakov, Phys. Rev. B 67, 113103 (2003).

$$\varepsilon_{zz} = \varepsilon_h \left(1 - \frac{k_p^2}{k^2\varepsilon_h - k_z^2}\right)$$

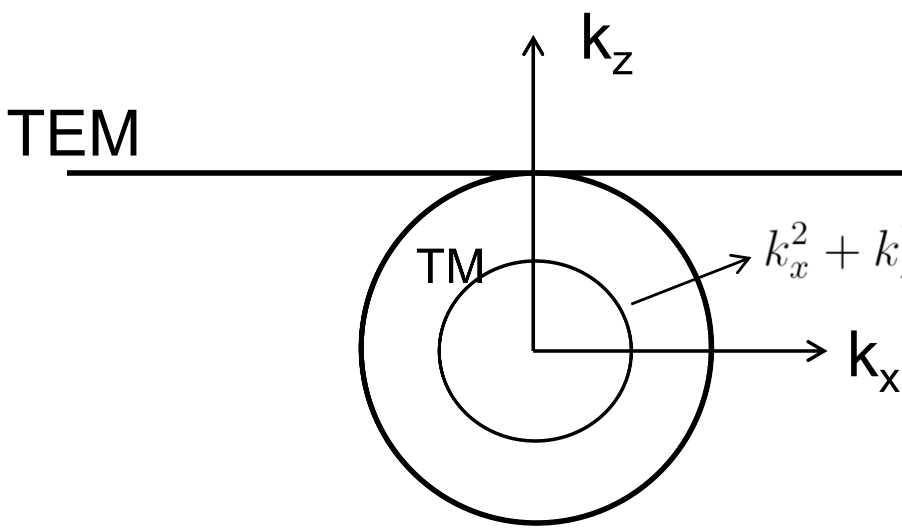
spatial dispersion term

TEM mode: $k_{z1} = k\sqrt{\varepsilon_h}$

TM mode: $k_{z2} = \sqrt{k^2\varepsilon_h - k_\perp^2 - k_p^2}$ $\quad 0 < k_\perp < \infty$

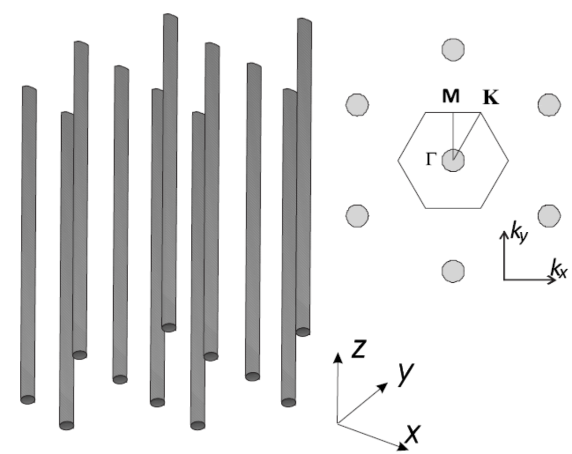
No propagation below k_p for the TM wave

Isofrequencies of TM and TEM waves in perfectly conducting wire media

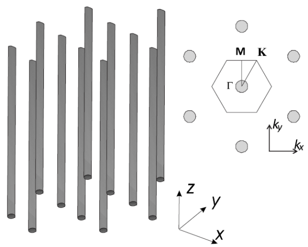


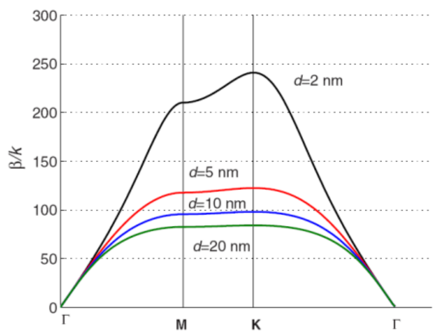
$$k_x^2 + k_z^2 = k^2 - k_p^2, \qquad k > k_p$$

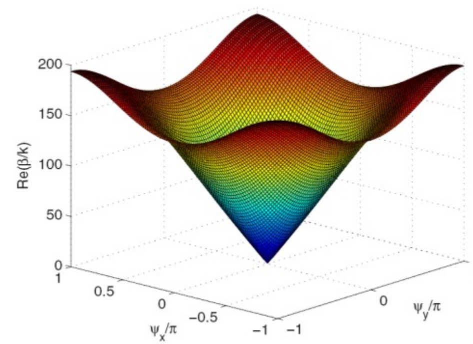
$$k_x^2 + k_z^2 = k^2$$



Arrays of carbon nanotubes as perfect hyperbolic media (HM) for the IR range







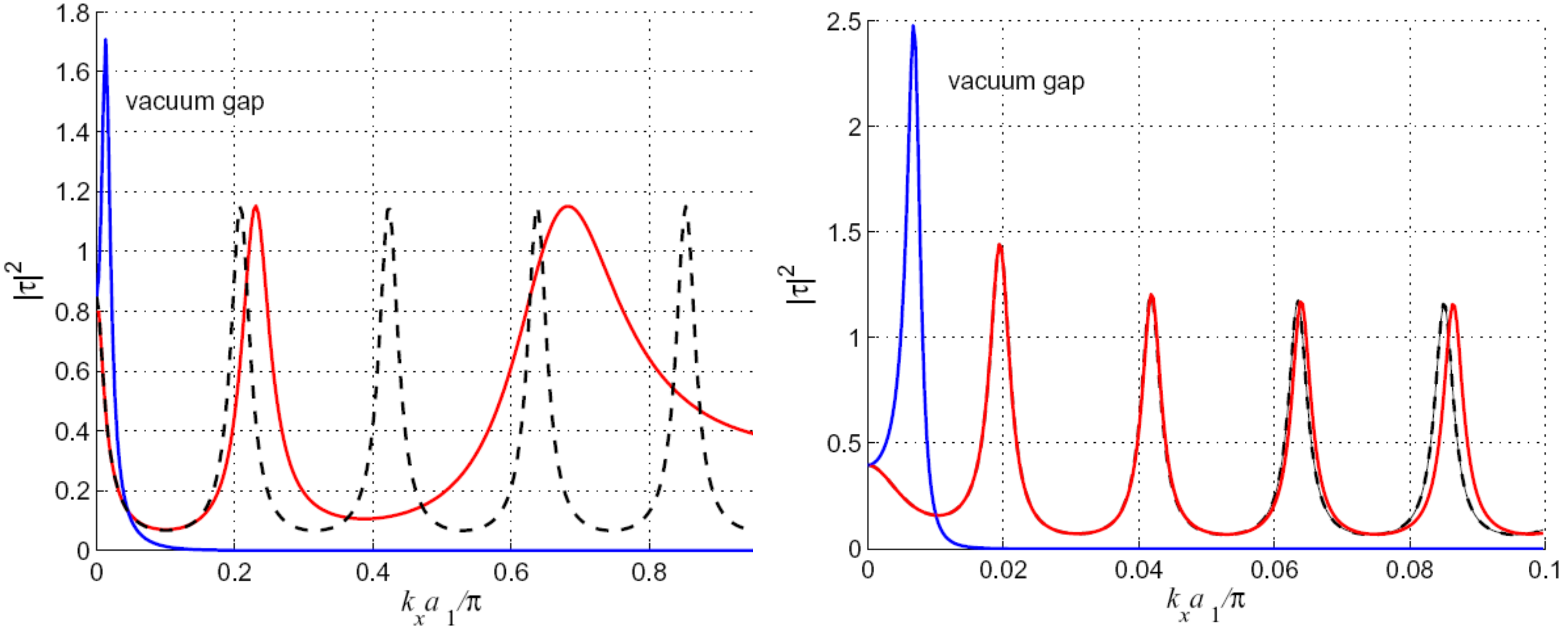
I.S. Nefedov, *PRB* **82,** 155423 (2010)

In opposite to wire media CNT arrays behave as ε-negative crystals. Spatial dispersion is totally suppressed due to a very high kinetic inductance of carbon nanptubes !!

$$k_z^2 = \frac{k^2\left(k^2 - k_\perp^2 - k_p^2\right)}{k^2 - k_p^2}$$

I.S. Nefedov, S.A. Tretyakov, *PRB* **84,** 113410 (2011).

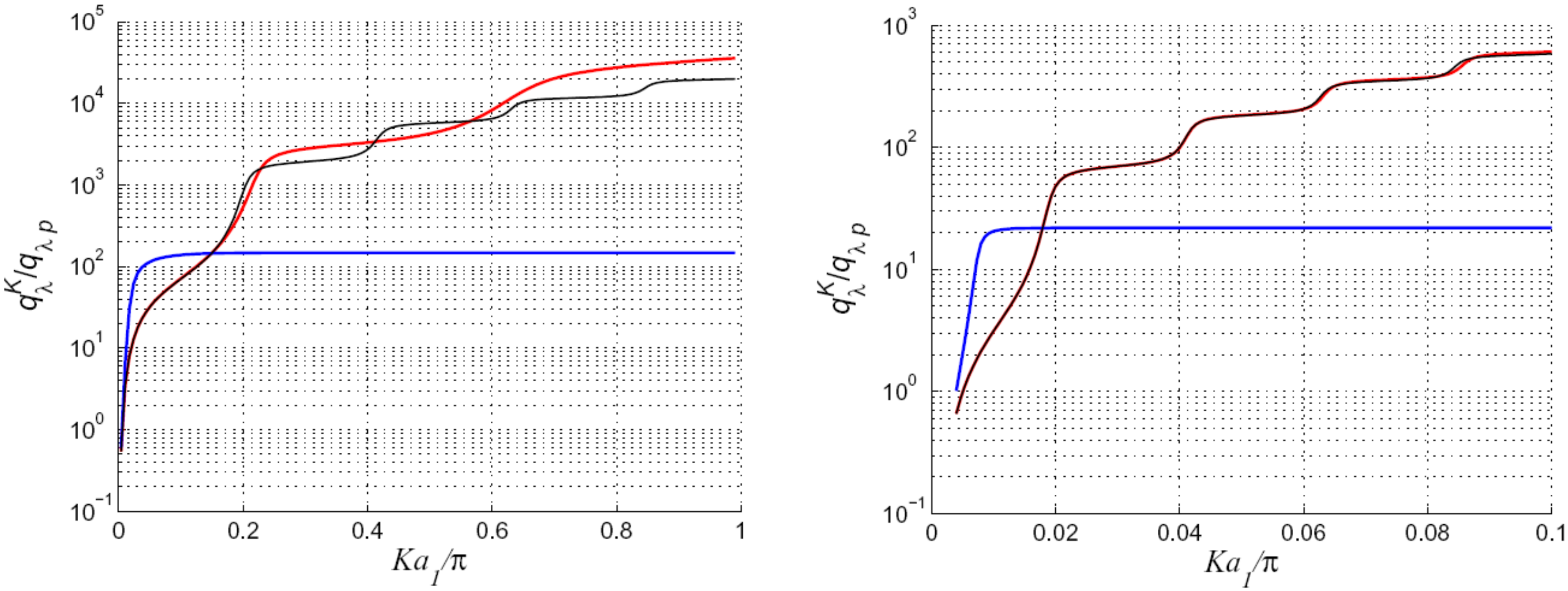
Transmission



Transmission coefficient for the vacuum gap (blue), where it is supported by evanescent waves, and that for waves with same spatial frequencies, propagating through the CNT array (red), calculated for *d* = 100 nm (left) and *d* = 1000 nm (right). Dashed curves correspond to the effective medium theory.

Total spectral radiation heat density

$$q_\lambda^K = \int_0^K \left[\langle \mathbf{S}_z^{13}(k_x,\lambda,T_1)\rangle - \langle \mathbf{S}_z^{31}(k_x,\lambda,T_2)\rangle\right] k_x dk_x$$



Normalized total spectral density of the transferred heat calculated for the vacuum gap (blue) and the gap filled with CNTs (red), *d* = 100 nm (left) and *d* = 1000 nm right) (logarithmic scale).

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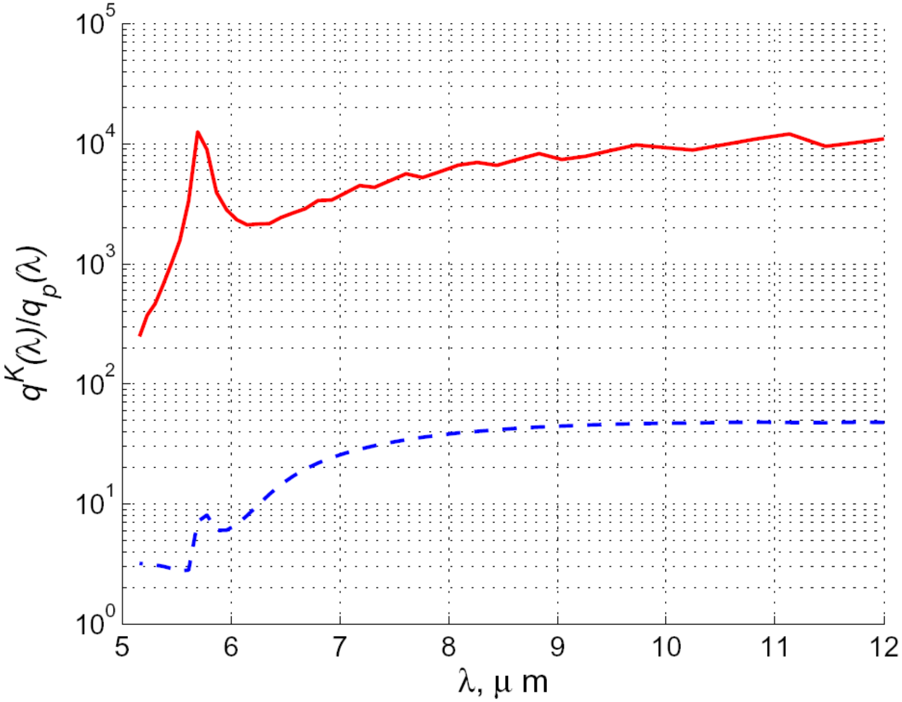
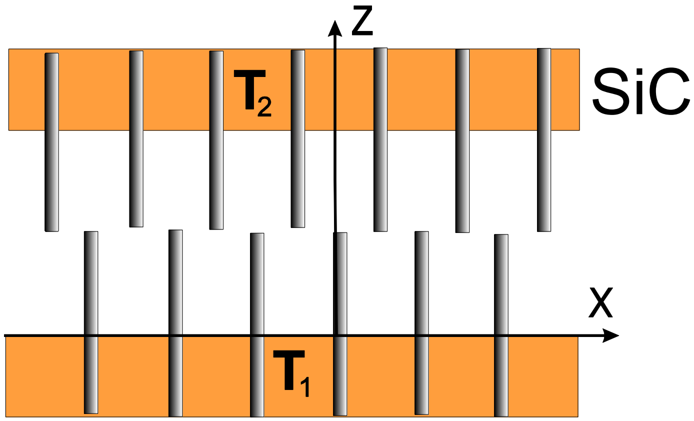
Total spectral density



$q^K(\lambda)/q_p(\lambda),\ K = 0.5\pi/a_1,$ calculated for the vacuum gap (dashed blue) and the gap filled with CNTs (solid red), *d* = 1 μm (logarithmic scale).

Thermal radiative heat transfer through metal wires



$$\frac{\varepsilon_{\parallel}}{\varepsilon_h} = 1 + \frac{1}{\frac{\varepsilon_h}{p(\varepsilon_m - \varepsilon_h)} - \frac{k_h^2 - k_z^2}{k_p^2}}$$

$$k_h = k_0\sqrt{\varepsilon_h}$$

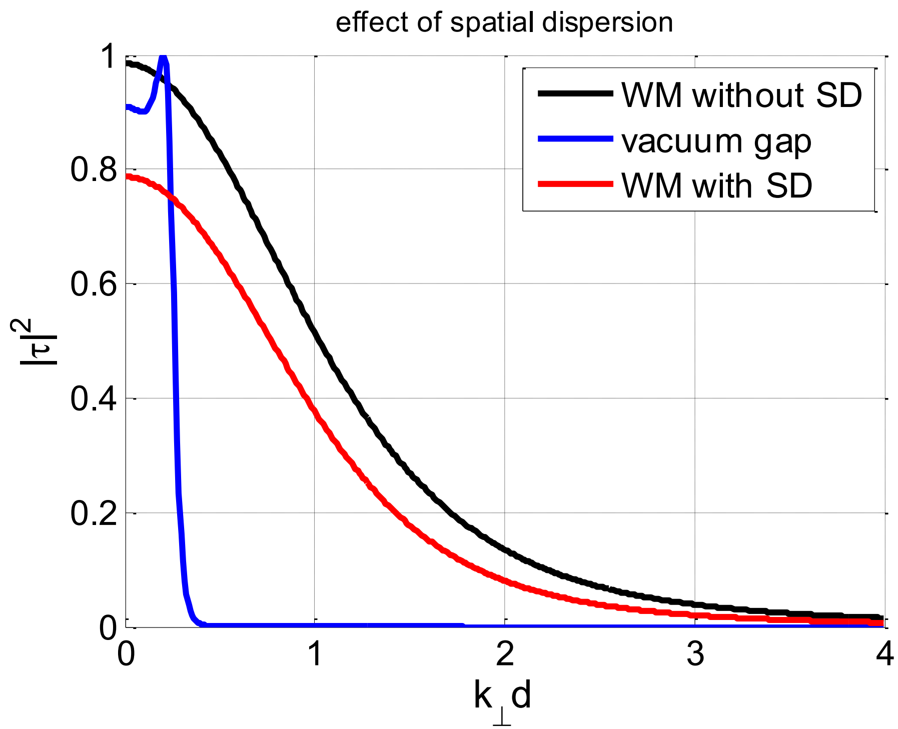
ABC: $E_{n1}\varepsilon_{z1} = E_{n2}\varepsilon_{h2}$

$$\frac{\varepsilon_{\perp}}{\varepsilon_h} = 1 + \frac{2}{\frac{\varepsilon_m + \varepsilon_h}{p(\varepsilon_m - \varepsilon_h)} - 1}$$

M. Silveirinha, Phys. Rev. E 73, 046612 (2006), the model for wires, made of ε-negative materials.

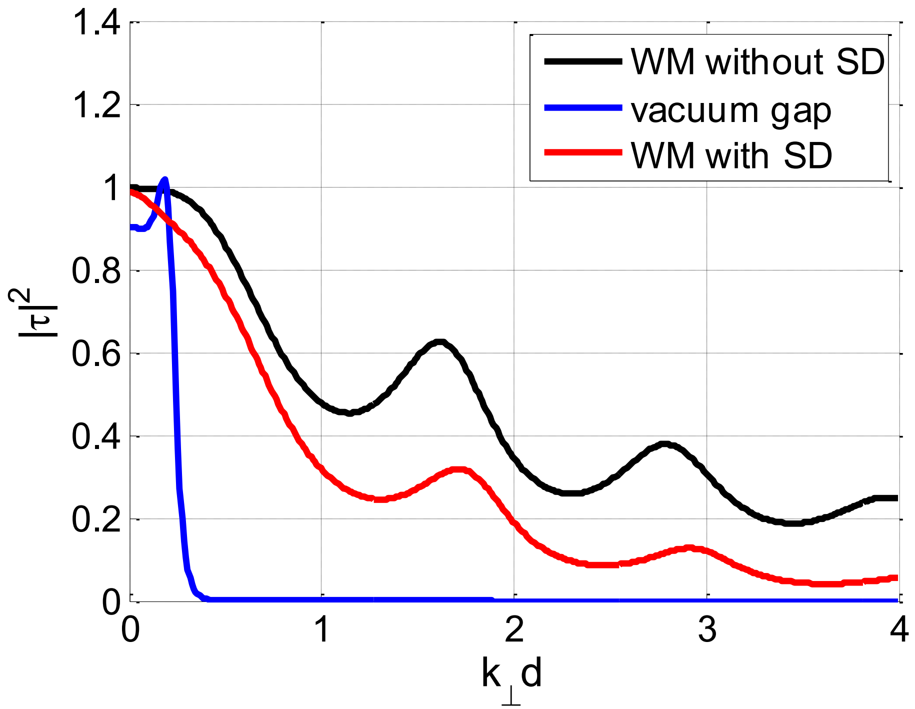
Tungsten wire media, effect of spatial dispersion

λ=2.5 μm



Radius of wires r=20 nm
Period of WM lattice d=100 nm
Width of the gap 1000 nm. Note that the transferred heat power is proportional to $|\tau|^2$.

Gold wire media



The same as in previous slide for the gold wire medium. One can see that the gold introduces quite less attenuation than tungsten. Fabry-Perot resonances are explicitly visible. Spatial dispersion considerably reduces expected radiative heat transfer, especially for large wavenumbers.

Conclusions

- **Hyperbolic media, placed inside a gap between a hot and cold bodies, provides giant enhancement of the thermal radiation heat transfer between bodies**
- **Arrays of metallic carbon nanotubes behave as low-loss hyperbolic media at terahertz and infrared frequencies and it can be used in TPV systems**
- **Metal wires can be used as HM, however, the spatial dispersion, appeared as excitation two modes, the quasi-TEM and TM modes, reduces this effect.**