Fluctuation Electrodynamics – foundations

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thanks to:

Martin Wilkens, Humboldt foundation, DFG

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www.quantum.physik.uni-potsdam.de

Fluctuation electrodynamics

what?

Basic concepts

how?

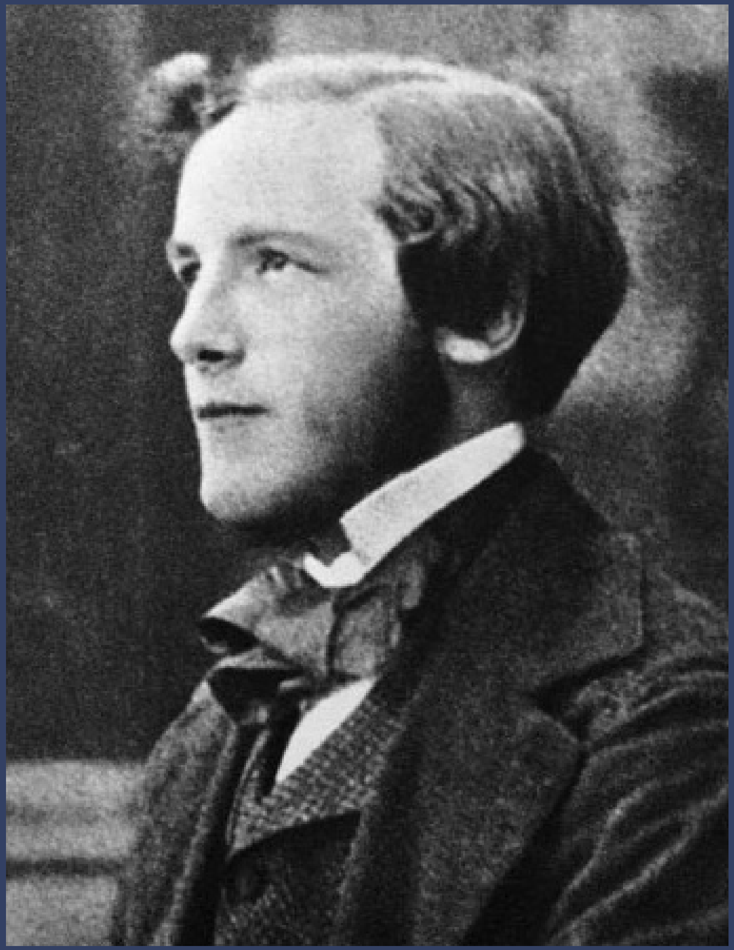
Simple examples

why not?

Beyond typical approximations

got it?

Suggested problems



en.wikipedia.org

(1831–79)

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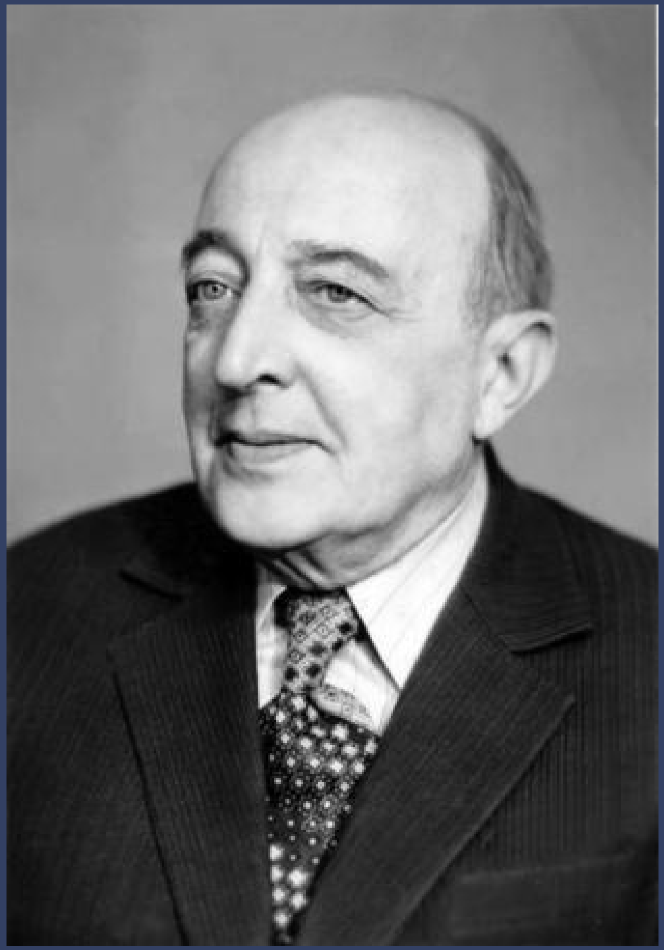
(1872–1946)



ru.wikipedia.org

Sergei Michailovich Rytov

(1908–96)

Fluctuation electrodynamics

– Maxwell equations ~ 1861

$$\mathrm{d}F = 0 \qquad \mathrm{d}^\star F = j$$

– Langevin equations $\sim 1905/08$

$$m\frac{dv}{dt} + \Gamma v = F(t)$$

Langevin force $F(t) \to \begin{cases} \text{diffusion} \\ \text{thermal equilibrium} \end{cases}$

Rytov equations $\quad \nabla \times \mathrm{H} - \partial_t \mathrm{D} = \mathrm{j}(\mathrm{x}, t)$ $\quad \leq 1953$

Rytov-Langevin current $\mathrm{j}(\mathrm{x}, t) \to$ thermal equilibrium

- local equilibrium $T(\mathrm{x})$

Nano-scale radiative heat transfer

fluctuat'n e'dyn

atoms $\ll$ nano $\ll \lambda$

OK

thermodynamics, T, entropy S,

$300\,$K : $\lambda \sim 10 \ldots 50\,\mu$m

heat, (ir)reversible processes

matter $\sim$ macroscopic continuum

non-equilibrium:

driven by external forces ∇T, E_L, $\mathrm{v}_2 - \mathrm{v}_1$

transfer of momentum $=$ force, torque

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driven by external forces ∇T, E_L, $\text{v}_2 - \text{v}_1$

transfer of momentum = force, torque

This lecture

- splash into Maxwell–Langevin equations
- examples: damped oscillator
  hot nano-particle
  heat transfer
- "quantum" remarks

$\longrightarrow$ *suggested problems*

Maxwell & Langevin

macroscopic electrodynamics

$$\partial_t \mathrm{B} = -\nabla \times \mathrm{E} \qquad \nabla \cdot \mathrm{B} = 0$$

$$\partial_t \mathrm{D} = \nabla \times \mathrm{H} - \mathrm{j} \qquad \nabla \cdot \mathrm{D} = \rho$$

linear response of (bulk) material

. . . always matter that provides nonlinearity

$$\mathrm{D} = \varepsilon_0 \varepsilon(\mathrm{x}, \omega) \mathrm{E}$$

$$\mathrm{H} = \mu_0^{-1} \mu^{-1}(\mathrm{x}, \omega) \mathrm{B}$$

$\varepsilon(\omega)$, $\mu(\omega)$ must be complex (!)

$\varepsilon(\mathrm{x})$, $\mu(\mathrm{x})$ cannot be local (!)

Rytov & Langevin: losses come with fluctuating sources ('forces')

$$\mathrm{j} = \mathrm{j}_{\mathrm{free}} + \partial_t \mathrm{P}(\mathrm{x}, t) + \nabla \times \mathrm{M}(\mathrm{x}, t)$$

$$\rho = \rho_{\mathrm{free}} - \nabla \cdot \mathrm{P}(\mathrm{x}, t)$$

cf. $$m \frac{dv}{dt} = -\nabla U - \Gamma v + F(t)$$

Maxwell & Langevin

macroscopic electrodynamics

$$-\mathrm{i}\frac{\omega}{c^2}\varepsilon(\mathrm{x},\omega)\mathrm{E} \quad = \quad \nabla\times\mu^{-1}(\mathrm{x},\omega)\mathrm{B} - \mu_0 \mathrm{j}(\mathrm{x},\omega) \qquad\qquad -\mathrm{i}\omega\mathrm{B} = \nabla\times\mathrm{E}$$

Rytov & Langevin: losses come with fluctuating sources (‘forces’)

$$\mathrm{j} \quad = \quad \mathrm{j}_{\mathrm{free}} - \mathrm{i}\omega\mathrm{P}(\mathrm{x},\omega) + \nabla\times\mathrm{M}(\mathrm{x},\omega)$$

$$\rho \quad = \quad \rho_{\mathrm{free}} - \nabla\cdot\mathrm{P}(\mathrm{x},\omega)$$

Maxwell-Langevin equation: ‘stochastic differential equation’

= physicists’ code word for averages

$$0 \quad = \quad \langle \mathrm{P}(\mathrm{x},t)\rangle$$

$$0 \quad \neq \quad \langle \mathrm{P}(\mathrm{x},t)\mathrm{P}(\mathrm{x}',t')\rangle = \int\frac{\mathrm{d}\omega}{2\pi}S_P(\mathrm{x},\mathrm{x}',\omega)\,\mathrm{e}^{\mathrm{i}\omega(t-t')}$$

spectral density

Maxwell & Langevin

macroscopic electrodynamics

$$-\mathrm{i}\frac{\omega}{c^2}\varepsilon(\mathrm{x},\omega)\mathrm{E} = \nabla\times\mu^{-1}(\mathrm{x},\omega)\mathrm{B} - \mu_0 \mathrm{j}(\mathrm{x},\omega) \qquad -\mathrm{i}\omega\mathrm{B} = \nabla\times\mathrm{E}$$

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spectral density

Rytov: $$\boxed{S_{P,ij}(\mathrm{x},\mathrm{x}',\omega) \approx 2\hbar\bar{N}(\omega)\varepsilon_0\,\mathrm{Im}\,\varepsilon_{ij}(\mathrm{x},\omega)\delta(\mathrm{x}-\mathrm{x}')}$$

. . . fluctuation-dissipation relation

Bose-Einstein distribution $\bar{N}(\omega) = \dfrac{1}{\exp(\hbar\omega/k_B T) - 1} \approx \dfrac{k_B T}{\hbar\omega}$

Maxwell & Langevin

Suggested problems

(1) Energy balance in lossy medium

water IR absorption $\alpha \sim 10^3\,\text{cm}^{-1}$

field $E = 1\,\text{mV}/\mu\text{m}$: energy density $\sim 0.03\,\text{meV}/\mu\text{m}^3$

absorbed power density $\sim 5\,\text{meV}/\text{ps}/\mu\text{m}^3$

time scale for losses $c\alpha \sim \omega/2\pi$ frequency

(2) Temperature: Bjerrum length $\dfrac{e^2}{\varepsilon_0 k_B T} = 0.7\,\mu\text{m}$

(3) Fluctuating dipole moment in volume ΔV: $\mathrm{d}(t) = \int_{\Delta V} \mathrm{d}x\, \mathrm{P}(\mathrm{x}, t)$

estimate polarization energy $\dfrac{\Delta V}{\varepsilon_0} \dfrac{\langle d^2 \rangle_\omega \Delta\omega}{(\Delta V)^2}$

(4) Count degrees of freedom in volume $1\,\mu\text{m}^3$:

photon modes & matter (phonon) modes (bandwidth $\Delta\omega$)

Maxwell & Langevin

Remarks

— equilibrium: fluctuation–dissipation (FD) relation for (quantum) fields   Callen & Welton 1951

— FD relations (theorem)
Einstein (1905), Langevin (1908): diffusion $D \leftrightarrow \Gamma$ friction
detailed balance: up/down rate $\sim \mathrm{e}^{-\Delta E/k_B T}$ Boltzmann
Kubo-Martin-Schwinger (KMS) relation $\langle A(t)B(t')\rangle_T$ vs $\langle B(t')A(t)\rangle_T$

Maxwell & Langevin

Remarks

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— FD relations (theorem)
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Kubo-Martin-Schwinger (KMS) relation $S_{AB}(\omega) = \mathrm{e}^{-\hbar\omega/k_B T} S_{BA}(\omega)$

— Rytov in full power: non-equilibrium
local temperature $S_P(\mathrm{x}, \omega) \sim \bar{N}(\omega, T(\mathrm{x}))\,\mathrm{Im}\,\varepsilon(\mathrm{x}, \omega)$ Lifshitz & Pitaevskii 1980s

— local equilibrium: on sub-λ scale, matter thermalizes ‘fast enough’
‘photons are a poor thermostat’ Planck 1900 / PLANCK 2013

— local approximation $\varepsilon(\mathrm{x}, \omega)$ for dielectric response
non-local media (plasma/screening, effective medium/unit cell)
genuine surface response $P(x, y, \omega)\delta(z) \sim \chi(\omega) E(x, y, z = 0, \omega)$

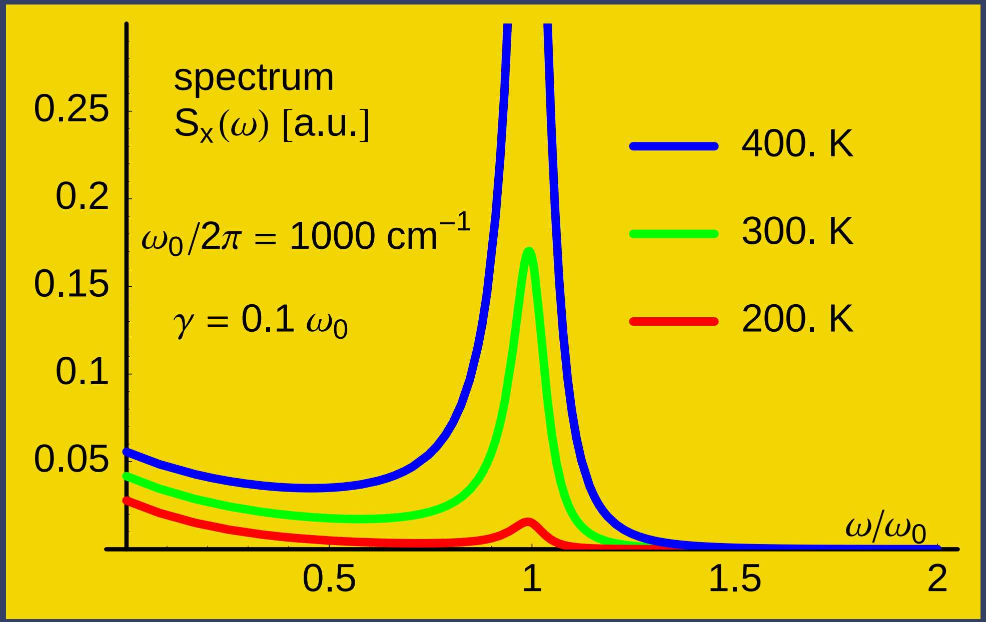
Examples

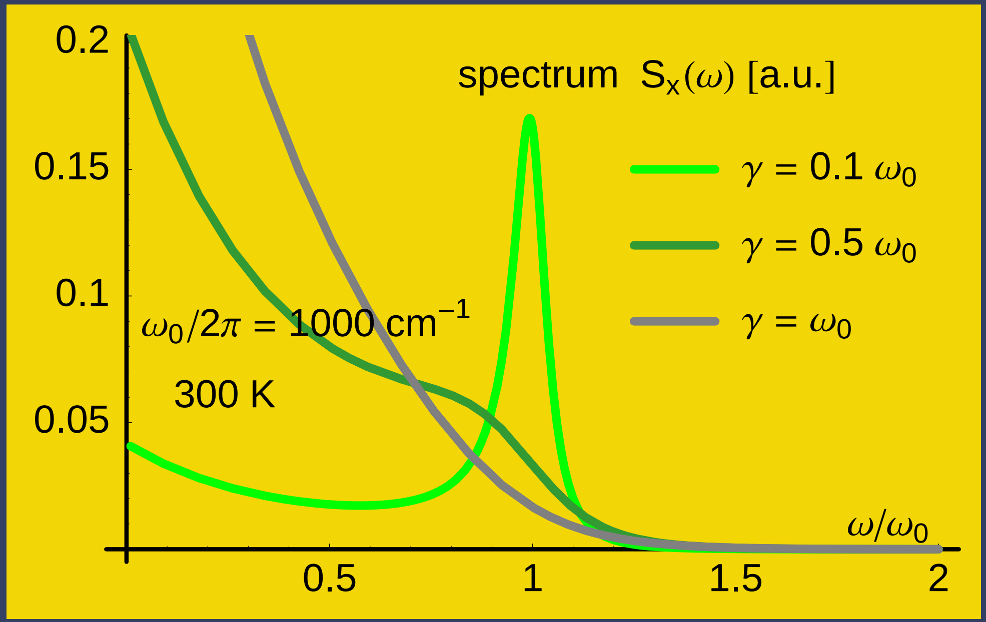
Damped harmonic oscillator $m\ddot{x} + \Gamma\dot{x} + Kx = F(t)$

Langevin force spectrum $S_F(\omega) = 2\hbar\bar{N}(\omega)\omega\,\mathrm{Re}\,\Gamma(\omega)$ white vs colored (discussion)

Oscillator spectrum (equilibrium, long-time limit)

$$S_x(\omega) = \frac{S_F(\omega)}{|-m\omega^2 - \mathrm{i}\omega\Gamma(\omega) + K|^2} = 2\hbar\bar{N}(\omega)\,\mathrm{Im}\Big(\frac{1}{-m\omega^2 - \mathrm{i}\omega\Gamma(\omega) + K}\Big)$$





non-equilibrium: relaxation dynamics, two-temperature driving . . .

quantum questions: zero-point energy, negative frequencies, overdamped limit . . .

Examples

Hot nano-sphere

spectrum of dipole moment

$$S_d(\omega) \approx 2\hbar \bar{N}(\omega) 4\pi a^3 \varepsilon_0 \, \mathrm{Im} \, \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 2}$$

polarizability $\mathrm{Im}\,\alpha(\omega)$

- emitted field $$\mathrm{E}(\mathrm{r}, \omega) = \frac{\omega^2 \, \mathrm{e}^{\mathrm{i}\omega r/c}}{4\pi\varepsilon_0 c^2 r} \left\{ (\mathrm{d} - 3\hat{\mathrm{r}}(\hat{\mathrm{r}} \cdot \mathrm{d})) \left(\frac{-1}{(\omega r/c)^2} + \frac{\mathrm{i}}{\omega r/c} \right) + \mathrm{d} - \hat{\mathrm{r}}(\hat{\mathrm{r}} \cdot \mathrm{d}) \right\}$$

$\langle \mathrm{E}(\mathrm{r}, \omega) \rangle = 0$ $$= \mathrm{G}(\mathrm{r}, 0, \omega) \cdot \mathrm{d}(\omega)$$

Green function/tensor/dyadic

$\longrightarrow$ this week

Examples

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Green function/tensor/dyadic
$\longrightarrow$ this week

Suggested problems

average intensity "$\langle |\mathrm{E}(\mathrm{r}, \omega)|^2 \rangle$" vs distance r

average Poynting vector "$\mathrm{Re}\,\langle \mathrm{E}^*(\mathrm{r}, \omega) \times \mathrm{H}(\mathrm{r}, \omega) \rangle$"

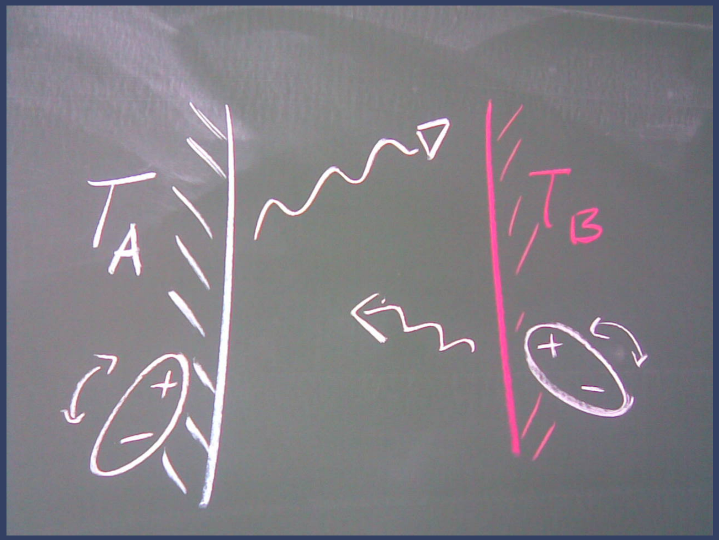
total emitted power & cooling rate/time

array of nano-particles
$\longrightarrow$ P Ben-Abdallah

- core technique for non-equilibrium Rytov theory
  = 'incoherent summation' over elementary source volumes

Examples

Radiative heat transfer



Poynting vector $\mathrm{Re}\,\langle \mathrm{E}^*(\mathrm{r},\omega) \times \mathrm{H}(\mathrm{r},\omega)\rangle_z$ T_{z0} current

$= ($left sources$) + ($right sources$)$

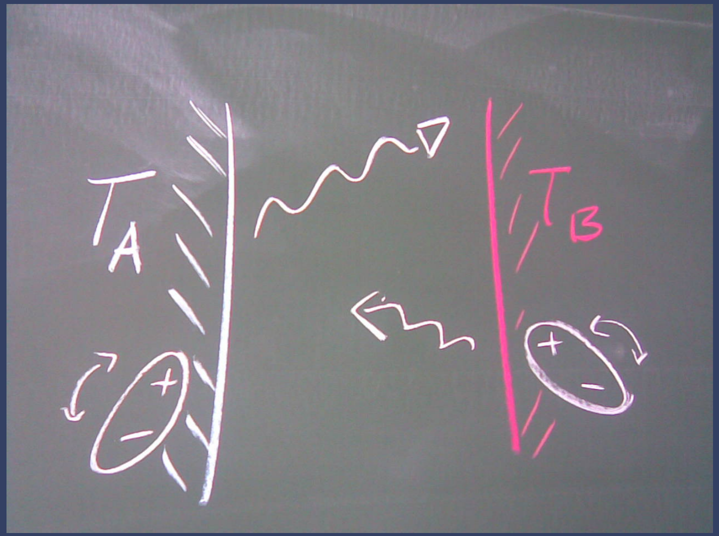
$\approx (T_B - T_A) \times h(\text{distance}, \bar{T})$

$\rightarrow$ this week's programme

- Casimir interaction = momentum transfer, stress tensor $\langle T_{ij} \rangle$

Examples

Radiative heat transfer



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$= (\text{left sources}) + (\text{right sources})$

$\approx (T_B - T_A) \times h(\text{distance}, \bar{T})$

$\rightarrow$ this week's programme

Typical / tacit assumptions

— homogeneous temperature profile *suggested problem* convection faster than radiation

— Langevin sources not correlated radiative coupling slower than local relaxation

— no heat extracted from vacuum fluctuations but: unstable vacua

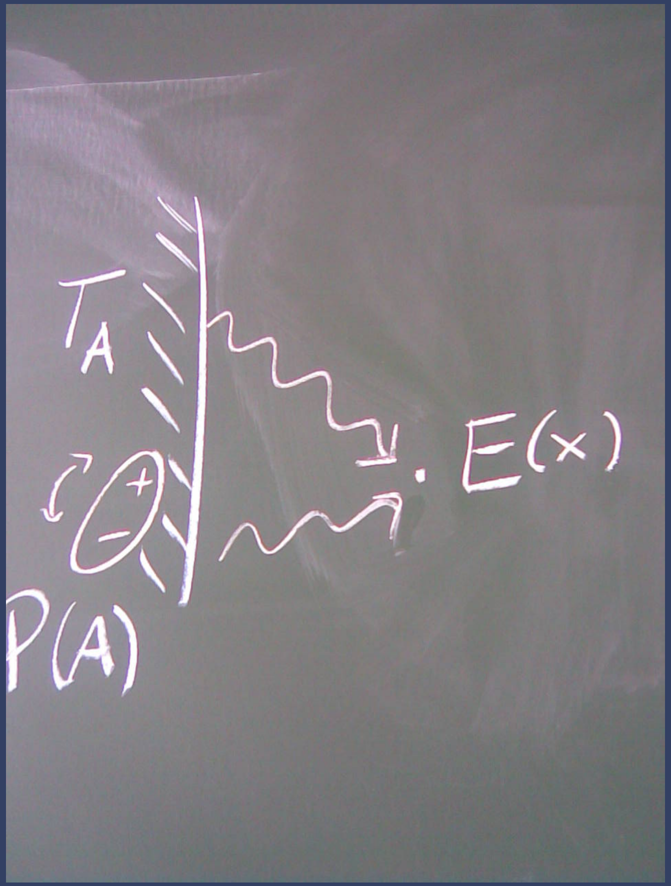
- Casimir interaction = momentum transfer, stress tensor $\langle T_{ij} \rangle$

Examples

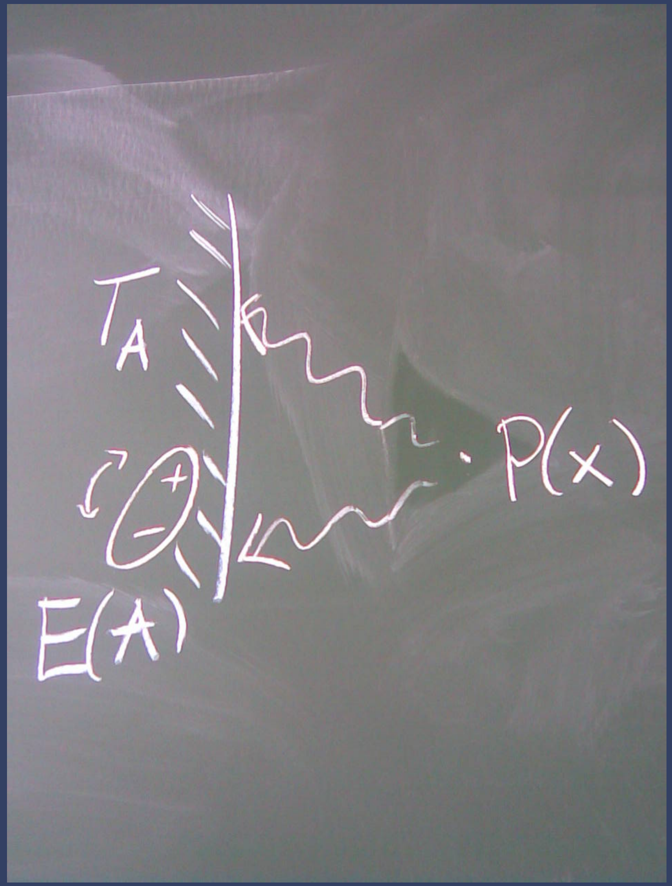
Radiative heat transfer

- convenient shortcut: generalized Kirchhoff law

Dorofeyev & Vinogradov (*Phys Rep* 2011)

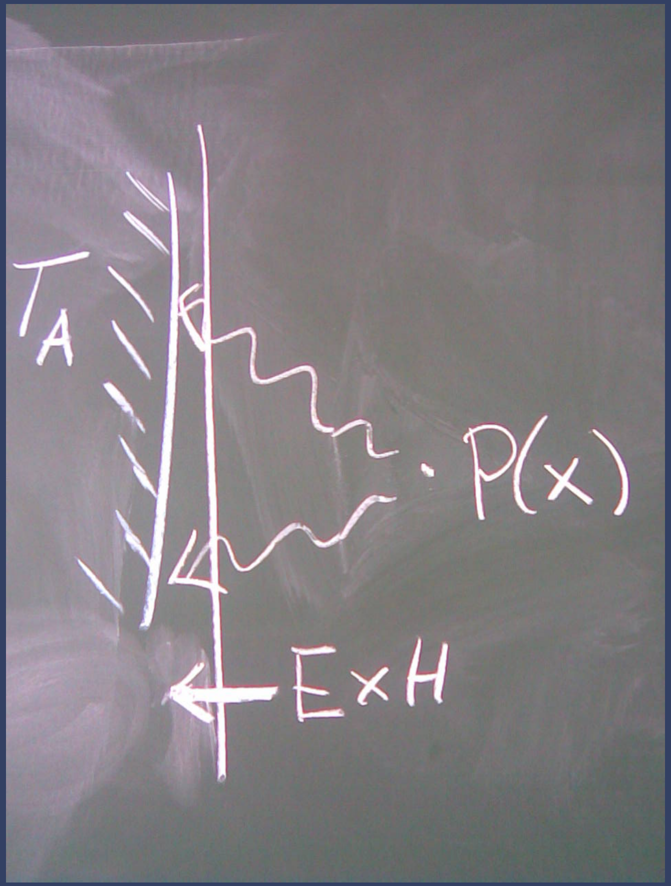


emission by sources



reciprocity

$$G(\mathrm{x}, \mathrm{x}') = G^{\mathsf{T}}(\mathrm{x}', \mathrm{x})$$



Poynting vector

Examples

“Quantum” remarks

(1) ‘first quantization’	QM	electrodynamics
Heisenberg	$\Delta p \Delta x \sim \hbar$	(Fourier)
Planck & Einstein	$\hbar\omega$	$\varepsilon \mathrm{E}^2 + \mu \mathrm{H}^2$
de Broglie	$\hbar \mathrm{k}$	$\varepsilon \mathrm{E} \times \mathrm{B}$
(2) ‘second quantization’		
Pauli & Jordan		$\Delta E \Delta A \sim \hbar$

$$\mathrm{i}\,[A_i(\mathrm{x}), E_j(\mathrm{x}')] = \frac{\hbar}{\varepsilon_0} \delta^{\perp}_{ij}(\mathrm{x} - \mathrm{x}')$$

$$\mathrm{i}\,[B_i(\mathrm{x}), E_j(\mathrm{x}')] = -\frac{\hbar}{\varepsilon_0} \epsilon_{ijk} \partial_k \delta(\mathrm{x} - \mathrm{x}')$$

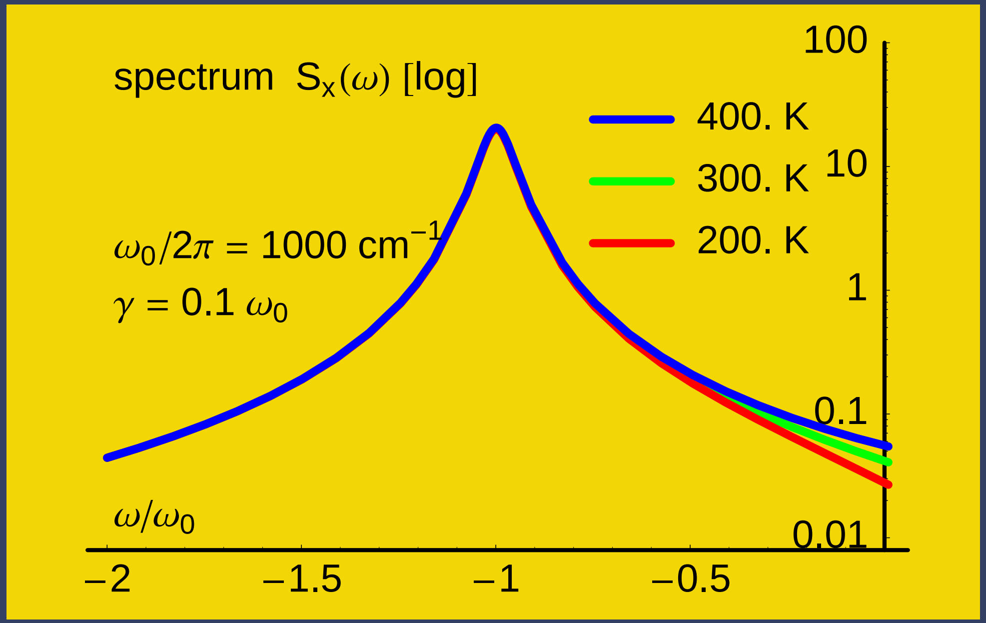
Ordering of operators

Feynman	$\langle T\{E(t)E(t')\}\rangle$	$D_E(\omega)$ even	
Green	$\mathrm{i}\langle[E(t),\, E(t')]\rangle\Theta(t - t')$	$\operatorname{Im} G(\omega)$ odd	
Hadamard	$\langle\{E(t), E(t')\}\rangle$	$H_E(\omega)$ real and even	$\sim \bar{N}(\omega) + \frac{1}{2}$
Glauber (?)	$\langle E(t)E(t')\rangle$	$S_E(\omega)$ positive	$\sim \bar{N}(\omega), \quad \omega > 0$
		$\mathrm{e}^{\mathrm{i}\omega(t-t')}$	

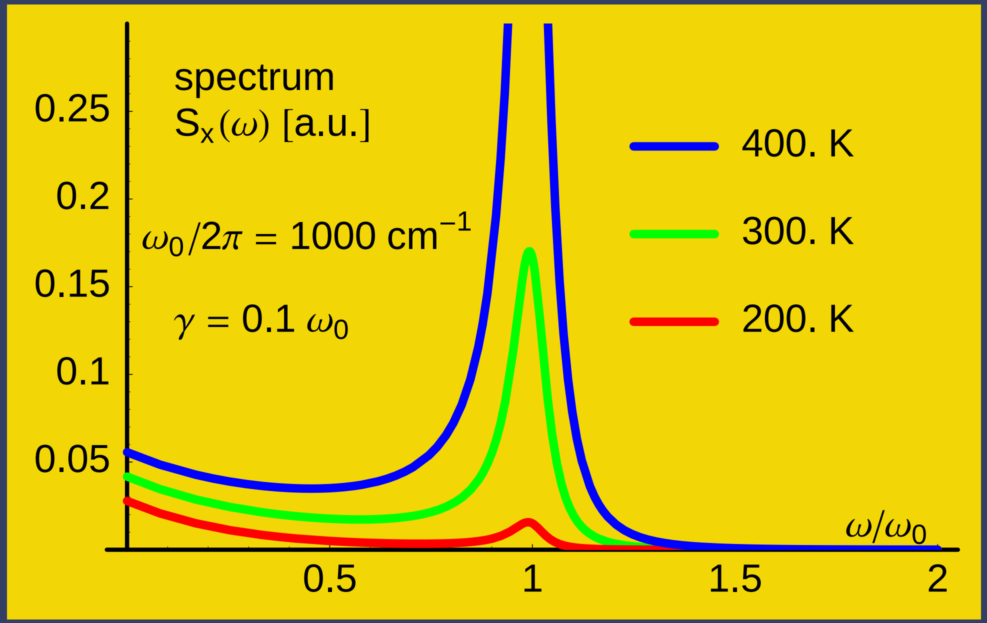
“Quantum” remarks

Damped oscillator in equilibrium

Glauber $S_x(\omega)$ at negative . . .



. . . and positive frequencies

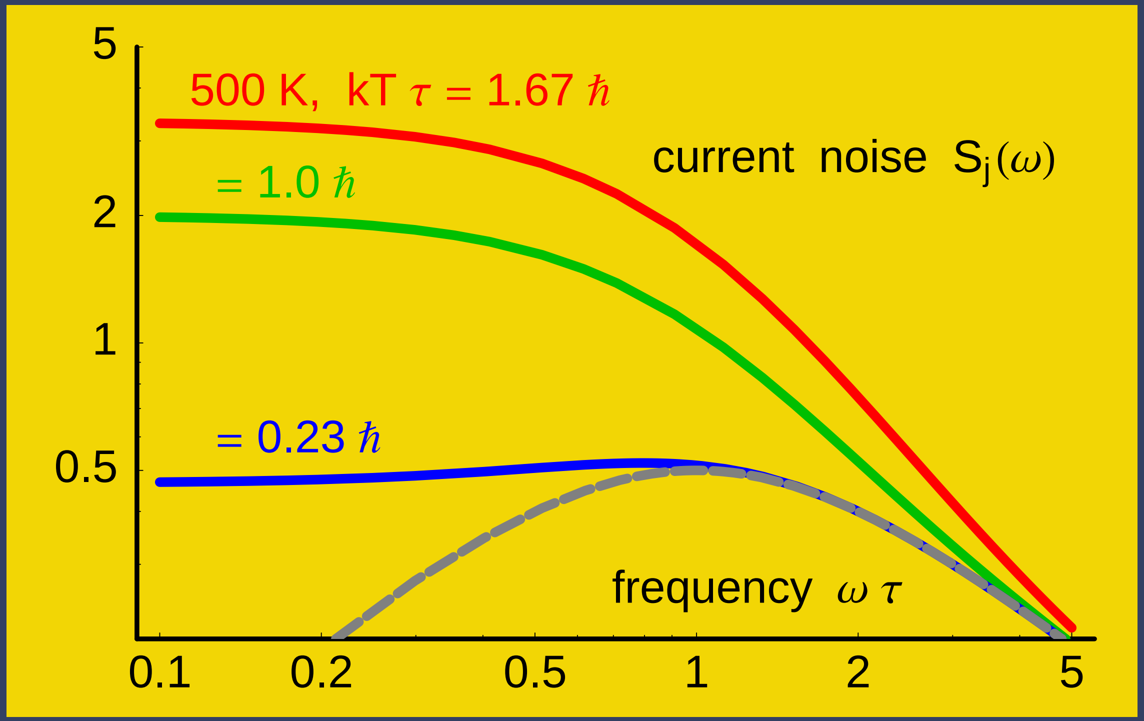


quantum noise / zero-point fluctuations

"Quantum" remarks

Electric current noise in metal

symmetrized $H_j(\omega) \sim (\bar{N}(\omega) + \frac{1}{2})\,\omega\,\mathrm{Re}\,\sigma(\omega)$



quantum noise of overdamped/diffusive field

- Casimir interaction: see Intravaia & CH (*Phys Rev Lett* 2009)

“Quantum” remarks

Suggested problems

Glauber spectrum: $-\bar{N}(-\omega) = \bar{N}(\omega) + 1$ asymmetric, still positive

Magnetic flux quantization? usually need electron charge for flux quantum, so it's beyond field quantization? (cf. magnetic monopole)

Why do annihilation operators evolve with positive frequencies?

$$a(t) = a(0)\,\exp\left(-\mathrm{i}\omega t\right)$$

Can all observers agree on the sign of a frequency? (not always!)

anomalous Doppler effect (Ginzburg):

$\omega' = \gamma(\omega + \mathrm{v} \cdot \mathrm{k}) < 0$

quantum friction, pair creation, Cherenkov radiation

Summary

stochastic electrodynamics Rytov = Maxwell + Langevin

route towards thermal equilibrium
QED: preserve commutators

separation of time scales
field absorption vs heat transport

Examples

stochastic field correlations: linear in sources
natural non-equilibrium technique (local vs global eq)
'macroscopic' response functions sufficient, even for quantum noise
(ε, μ, surface response . . .)

Glimpses . . . of the land beyond Rytov

Suggested problem

wikipedia entries about S. M. Rytov
(english, français, deutsch . . .)

Thanks to the group!

Francesco Intravaia (→ Nottingham)	Casimir physics	*Phys Rev A* 2010
Jürgen Schiefele (→ Madrid)	atom chip & BEC	*Phys Lett A* 2011
Harald Haakh (→ Erlangen)	superconducting atom chips	*Eur Phys J B* 2012
Geesche Boedecker (PhD)	non-Markovian QED	*Ann Phys (Berlin)* 201
Giuseppe Cammarata (post doc)	quantization of coupled oscillators	
Alexander Kegeles (PhD)	entanglement production	
Gregor Pieplow (Dipl)	quantum friction	*New J Phys* 2013
Ralf Saplata (Dipl)	failures of adiabaticity	
Abdoulaye Diallo (Dipl)	border of BEC	

Timo Felbinger	Illarion Dorofeyev (Nizhny Novgorod)
Martin Wilkens	Vanik E. Mkrtchian (Ashtarak)

Appendix – forgotten references

von Laue: Thermal radiation in absorbing bodies (*Ann. Phys. (Leipzig)* 1910)

Bakker & Heller: 'Quantum' Brownian motion in electric resistances (*Physica* 1939)

Jauch & Watson: Phenomenological Quantum-Electrodynamics (*Phys Rev* 1948)

Callen & Welton: Irreversibility and generalized noise (*Phys Rev* 1951)

De Groot & al: Series on relativistic thermodynamics (*Physica* $\geq$ 1953)

V. L. Ginzburg: Electrical fluctuations (*Fortschr Phys* 1953)

Linder: Thermal Van der Waals interactions (*J Chem Phys* $\geq$ 1960)

van Kampen: FD relation in non-linear systems? (*Physica* 1960) vs Polevoi & Rytov (*Theor Math Phys* 1975)

Morris & Fürth: Spatial field correlations near conducting surfaces (*Physica* 1960)
see also Fuchs (*Radiophys Quant Electr* 1965)

Jaynes & Cummings: Quantum vs semiclassical radiation theories (*Proc. IEEE* 1963)

Agarwal: FD theorems and series on field quantization (*Z Phys* 1972; *Phys Rev A* 1975)

Boyer: Connection between Rytov and quantum electrodynamics (*Phys Rev D* 1975)

Eckhart: Problems with FD relations for heat transfer (*Opt Commun* $\geq$ 1982)

Henry & Kazarinov: Quantum noise in photonics (*Rev Mod Phys* 1996)

Appendix — Fluctuation-dissipation relation

Cross correlation spectrum:

$$\mathcal{E}_{ij}(\mathrm{x},\mathrm{x}';\omega) = \int_{-\infty}^{+\infty} \mathrm{d}\tau \, \mathrm{e}^{-\mathrm{i}\omega\tau} \langle E_i(\mathrm{x}, t+\tau) E_j(\mathrm{x}', t) \rangle$$

Linear response:

$$\mathcal{G}_{ij}(\mathrm{x},\mathrm{x}';\omega) = \frac{\mathrm{i}}{\hbar} \int_{0}^{+\infty} \mathrm{d}\tau \, \mathrm{e}^{\mathrm{i}\omega\tau} \langle [E_i(\mathrm{x},\tau),\, E_j(\mathrm{x}',0)] \rangle$$

Canonical ensemble:
(KMS relation)

$$\int_{-\infty}^{+\infty} \mathrm{d}\tau \, \mathrm{e}^{\mathrm{i}\omega\tau} \langle A(\tau) B(0) \rangle = \mathrm{e}^{\hbar\omega/k_B T} \int_{-\infty}^{+\infty} \mathrm{d}\tau \, \mathrm{e}^{\mathrm{i}\omega\tau} \langle B(0) A(\tau) \rangle$$

$\longrightarrow$ Fluctuation–dissipation relation:

$$\mathcal{E}_{ij}(\mathrm{x}',\mathrm{x};\omega) = 2\hbar \bar{N}(\omega) \frac{\mathcal{G}_{ji}(\mathrm{x}',\mathrm{x};\omega) - [\mathcal{G}_{ij}(\mathrm{x},\mathrm{x}';\omega)]^*}{2\mathrm{i}}$$

Reciprocity: $\mathcal{G}_{ji}(\mathrm{x}',\mathrm{x};\omega) = \mathcal{G}_{ij}(\mathrm{x},\mathrm{x}';\omega)$