SCATTERING APPROACH FOR MOMENTUM AND HEAT TRANSFER

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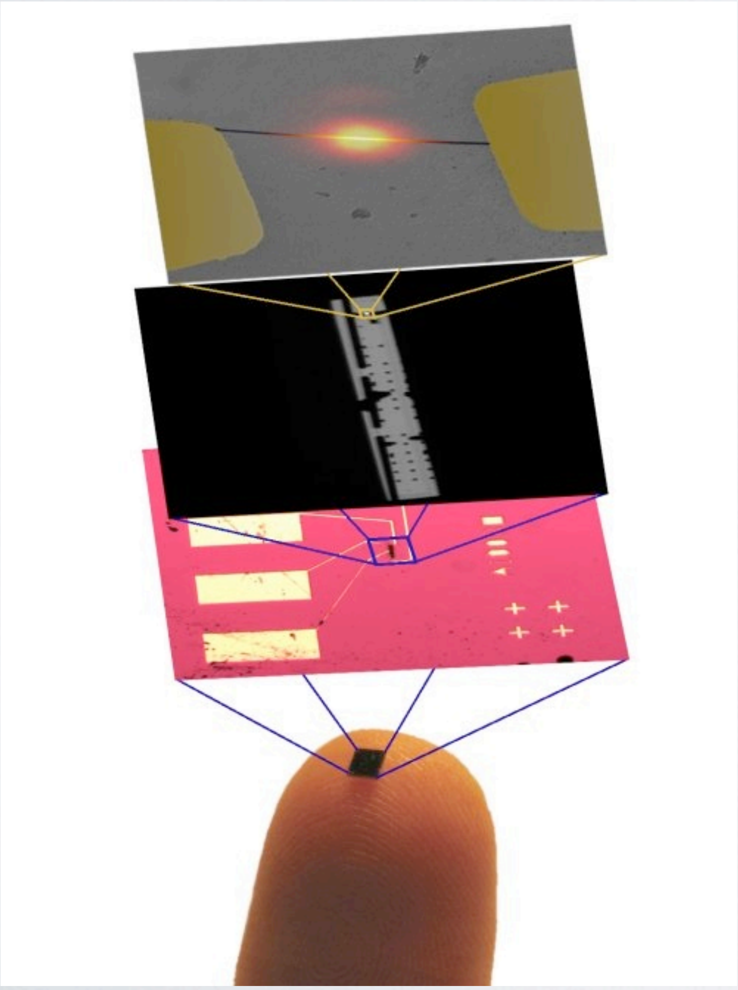
Nanoscale Radiative Heat Transfer, Les Houches, 16/05/2013

Outline

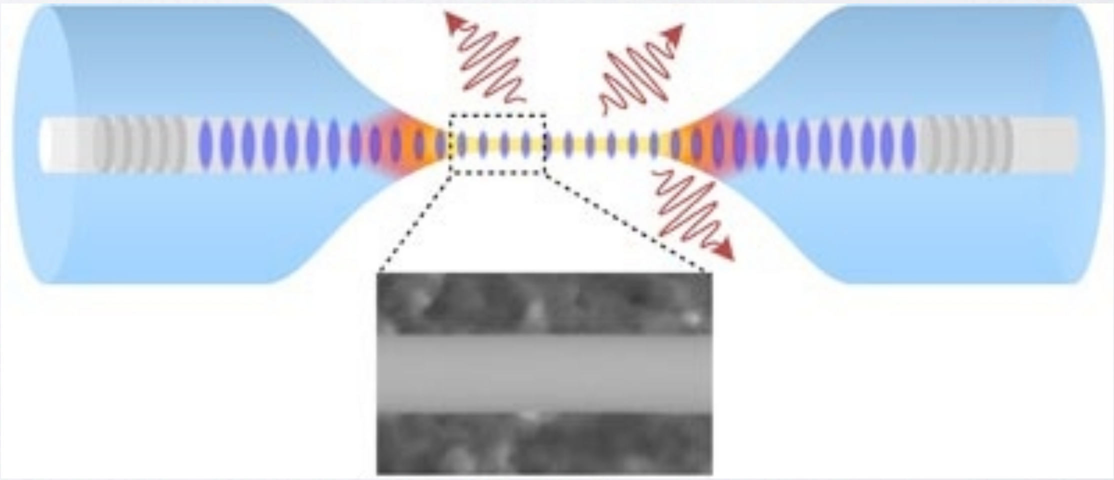
- Equilibrium fluctuations:
  - interactions between macroscopic bodies
  - influenced by shape, material properties, and temperature
  - correlated !
- Non-equilibrium fluctuations:
  - bodies at different temperatures
  - novel interaction effects
  - heat radiation and transfer
- Outlook, new directions

HEAT RADIATION AND TRANSFER

- *Breaking the law, at the nanoscale* *[MITnews, July 29, 2009]*
- Planck's law is modified for small objects and short separations
- *Probing Planck's Law with Incandescent Light Emission from a Single Carbon Nanotube* *[Y. Fan, S.B. Singer, R. Bergstrom, & B.C. Regan, Phys. Rev. Lett.102, 187402 (2009)]*

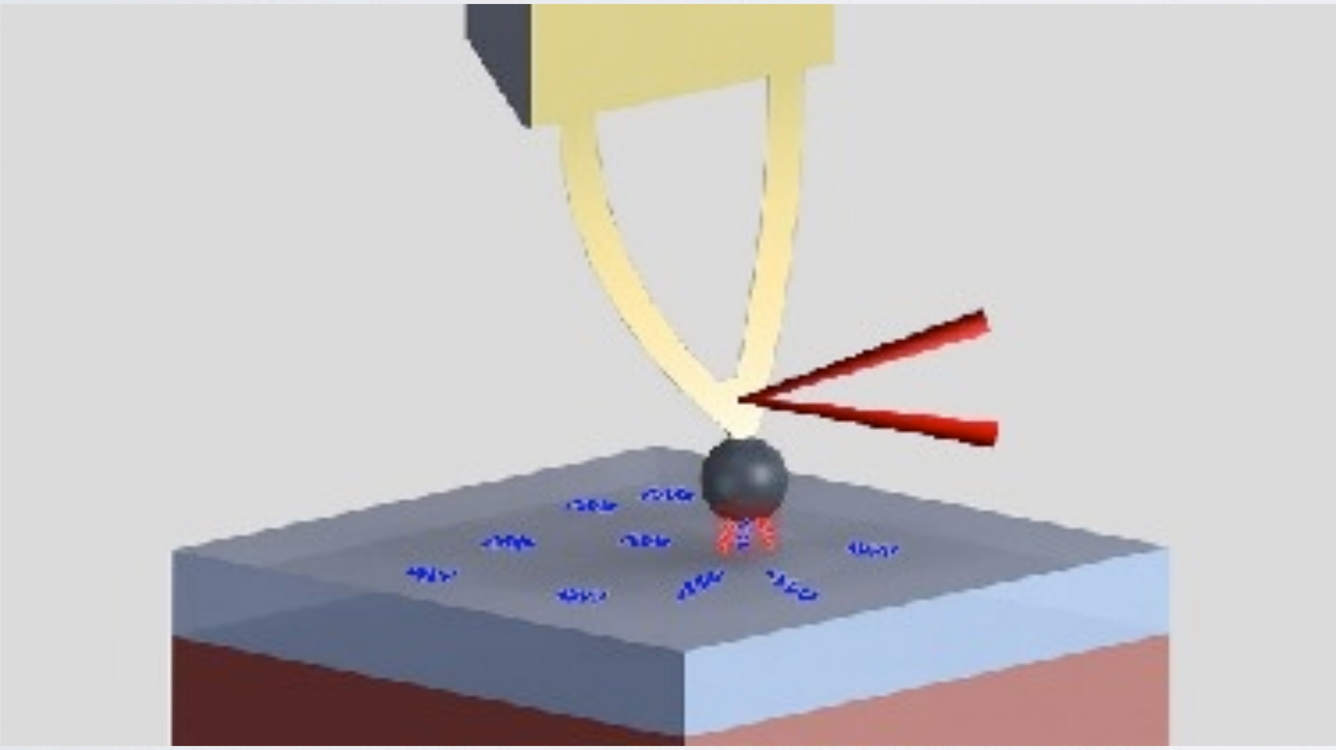


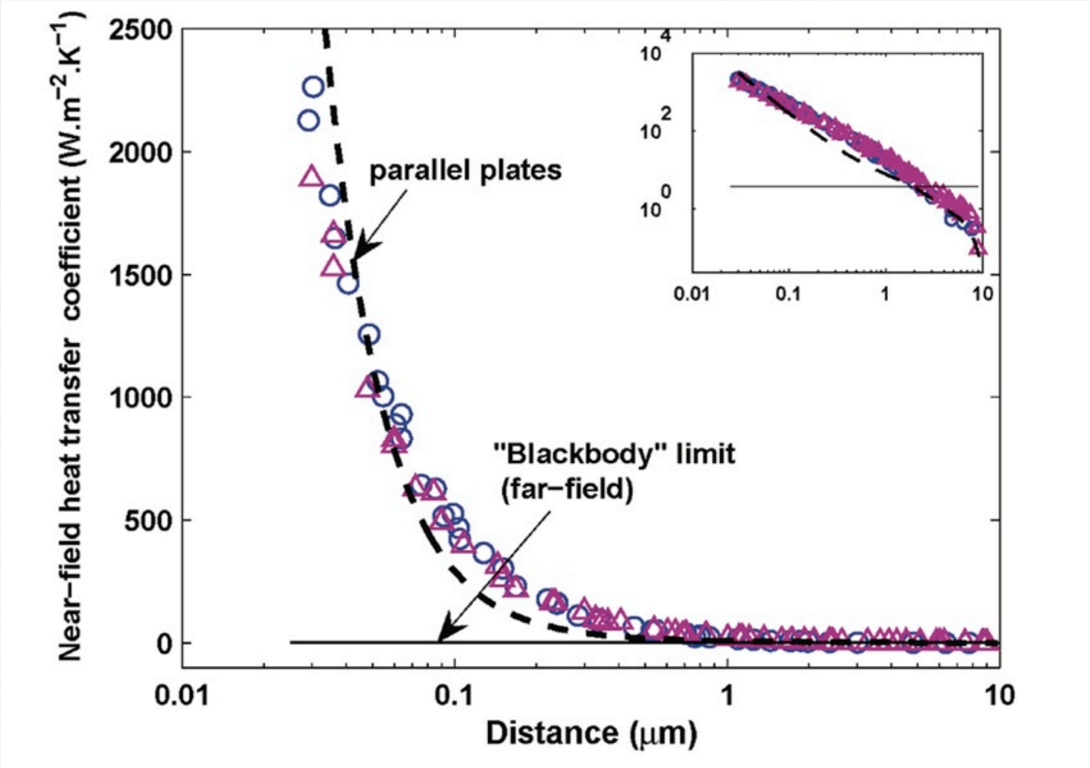
- *Probing Planck's Law for an Object Thinner than the Thermal Wavelength* *[C. Wuttke and A. Rauschenbeutel, arXiv: 1209.0536 [quant-ph]]*



SURFACE PHONON POLARITONS MEDIATED ENERGY TRANSFER BETWEEN NANOSCALE GAPS

- Beyond Stefan-Boltzmann law
- Understand heat transfer in nano-systems
- Near field effects can give huge enhancement of transfer (tunneling of evanescent waves)





S. Shen, A. Narayanaswamy, & G. Chen, Nano Lett. 9, 2909 (2009)

EQUILIBRIUM QED: INTERACTIONS

- Start from path integral for free energy of electromagnetic field at inverse temperature $1/\beta$ and imaginary frequency $\omega = ic\kappa$

$$F(\beta) = -\frac{1}{\beta}\log Z(\beta)$$

$$Z = \prod_{\kappa}\int \mathcal{D}\mathbf{A}\mathcal{D}\mathbf{A}^* \ \exp\left[-\beta\int d\mathbf{x}\,\mathbf{E}^*(\kappa,\mathbf{x})\left(\mathbb{H}_0(\kappa) + \frac{1}{\kappa^2}\mathbb{V}(\kappa,\mathbf{x})\right)\mathbf{E}(\kappa,\mathbf{x})\right]$$

- Free photons: $\mathbb{H}_0(\kappa) = \mathbb{I} + \frac{1}{\kappa^2}\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times$
- Interaction: $\mathbb{V}(\kappa,\mathbf{x}) = \mathbb{I}\,\kappa^2\left(\epsilon(ic\kappa,\mathbf{x}) - 1\right) + \boldsymbol{\nabla}\times\left(\frac{1}{\mu(ic\kappa,\mathbf{x})} - 1\right)\boldsymbol{\nabla}\times$
- In terms of current fluctuations:

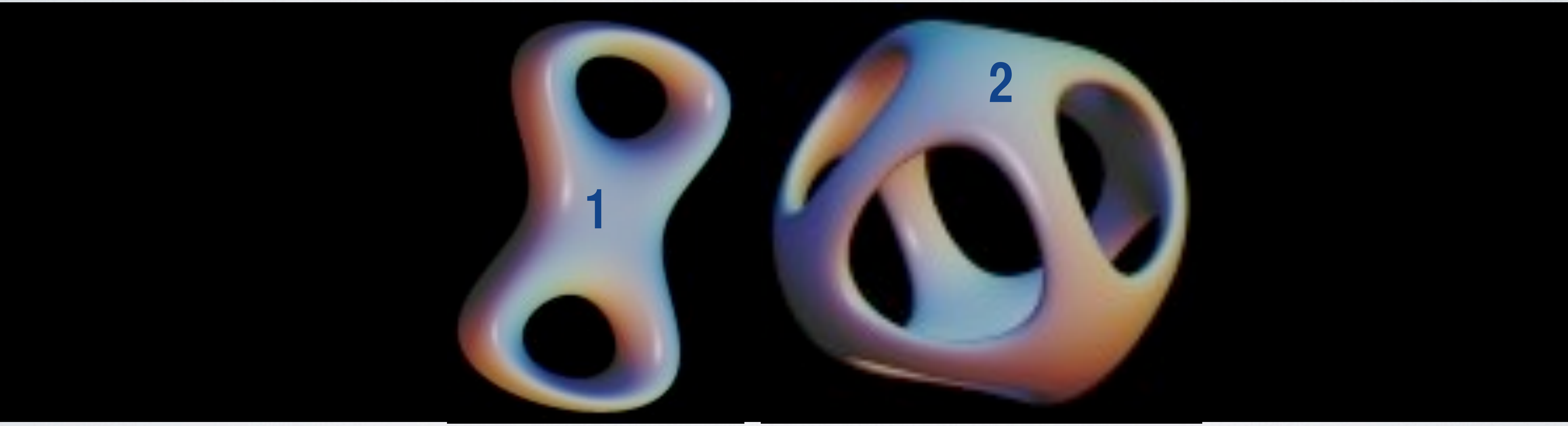
$$Z \sim \int \mathcal{D}\mathbf{J}\mathcal{D}\mathbf{J}^*|_{\mathrm{obj}}\exp\left[-\beta\int d\mathbf{x}d\mathbf{x}'\,\mathbf{J}^*(\mathbf{x})\left(\mathbb{G}_0(\kappa,\mathbf{x},\mathbf{x}') + \mathbb{V}^{-1}(\kappa,\mathbf{x})\delta^{(3)}(\mathbf{x}-\mathbf{x}')\right)\mathbf{J}(\mathbf{x}')\right]$$

The T-operator and its Decomposition

$$\mathbb{T}^{-1} = \mathbb{G}_0(\kappa, \mathbf{x}, \mathbf{x}') + \mathbb{V}^{-1}(\kappa, \mathbf{x})\delta^{(3)}(\mathbf{x} - \mathbf{x}')$$

is inverse T-operator: induced source = T-operator × applied field”

$\mathbb{U}^{\alpha\beta}$ couples induced sources on different objects



$$\mathcal{E} = \frac{\hbar c}{2\pi}\int_0^\infty d\kappa \log\det(\mathbb{M}\mathbb{M}_\infty^{-1})$$

$$\mathbb{M} = \begin{pmatrix} (\mathbb{T}^1)^{-1} & \mathbb{U}^{12} & \mathbb{U}^{13} & \cdots \\ \mathbb{U}^{21} & (\mathbb{T}^2)^{-1} & \mathbb{U}^{23} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} \qquad \mathbb{M}_\infty^{-1} = \begin{pmatrix} \mathbb{T}^1 & 0 & 0 & \cdots \\ 0 & \mathbb{T}^2 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

INTERMEZZO: OPERATOR FORMALISM

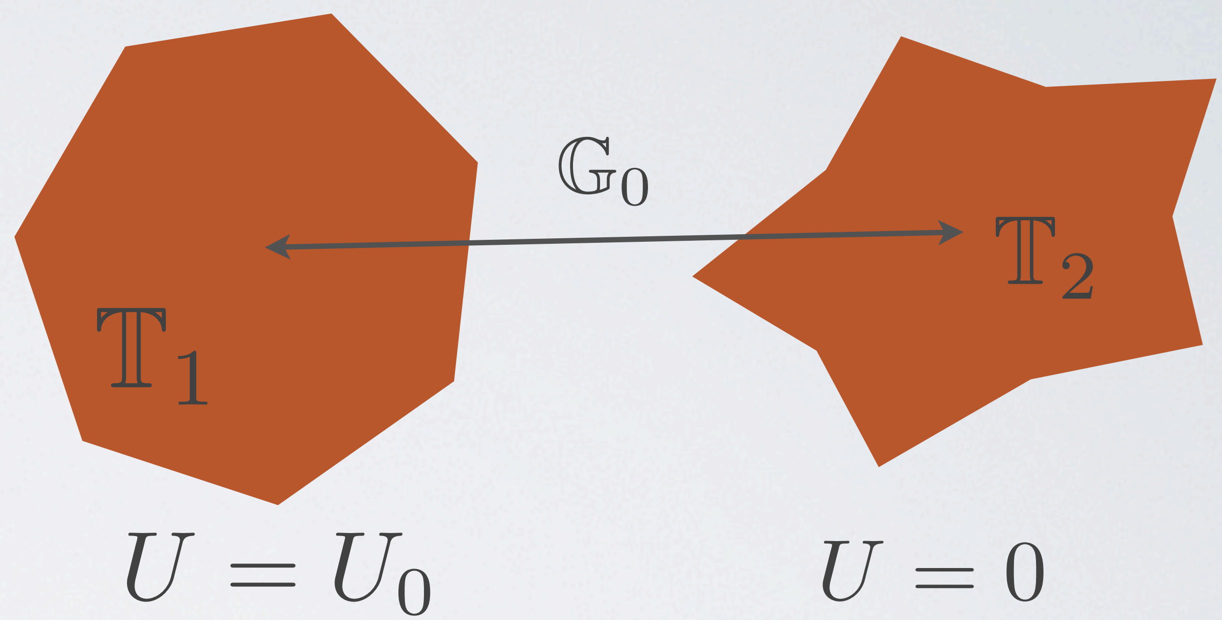
- Simple electrostatic example: Two metallic objects at fixed potentials

- Find potential U and surface charges σ_α from T-operators

$$\mathbb{T}_\alpha = \frac{1}{2}(\mathbb{S}_\alpha - 1)$$

- Conditions: $\mathbb{G}_\alpha = \mathbb{G}_0 - \mathbb{G}_0 \mathbb{T}_\alpha \mathbb{G}_0$

$$U = \mathbb{G}_0(\sigma_1 + \sigma_2)$$
$$U = \mathbb{G}_2 \sigma_1$$
$$U = U_0 + \mathbb{G}_1 \sigma_2$$



- Surface charges:

$$\sigma_1 = \mathbb{G}_0^{-1}(1 - \mathbb{G}_0 \mathbb{T}_1 \mathbb{G}_0 \mathbb{T}_2)^{-1} U_0 = \mathbb{G}_0^{-1} \sum_{n=1}^{\infty} (\mathbb{G}_0 \mathbb{T}_1 \mathbb{G}_0 \mathbb{T}_2)^n U_0$$
$$\sigma_2 = -\mathbb{T}_2 \sum_{n=0}^{\infty} (\mathbb{G}_0 \mathbb{T}_1 \mathbb{G}_0 \mathbb{T}_2)^n U_0$$

- Potential:

$$U = (1 - \mathbb{G}_0 \mathbb{T}_2) \sum_{n=0}^{\infty} (\mathbb{G}_0 \mathbb{T}_1 \mathbb{G}_0 \mathbb{T}_2)^n U_0$$

multiple "scattering" & multipole expansion

CASIMIR POTENTIAL

- The Casimir energy can be expressed as

$$\mathcal{E} = \frac{\hbar c}{2\pi}\int_0^\infty d\kappa \log\det(\mathbb{M}\mathbb{M}_\infty^{-1})$$

with

$$\mathbb{M} = \begin{pmatrix} (\mathbb{T}^1)^{-1} & \mathbb{U}^{12} & \mathbb{U}^{13} & \cdots \\ \mathbb{U}^{21} & (\mathbb{T}^2)^{-1} & \mathbb{U}^{23} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} \qquad \mathbb{M}_\infty^{-1} = \begin{pmatrix} \mathbb{T}^1 & 0 & 0 & \cdots \\ 0 & \mathbb{T}^2 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

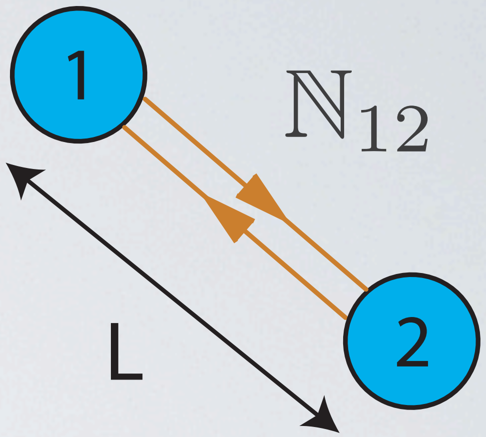
- Diagonal: Scattering amplitudes (T-matrix elements) of individual objects. They describe shape and material properties.

- Off-diagonal: Translation matrices. They are universal (depend only on dimension of space and type of field) and describe relative position of objects, i.e., geometry.

- For two objects: $\mathcal{E}_2 = \frac{\hbar c}{2\pi}\int_0^\infty d\kappa \ln\det(1 - \mathbb{N}_{12})$ $\quad \mathbb{N}_{12} = \mathbb{T}^1\mathbb{U}^{12}\mathbb{T}^2\mathbb{U}^{21}$

INTERPRETATION FOR COMPACT OBJECTS

- 2 Objects: Expansion in number (2p) of scatterings



$$\mathcal{E}_2 = \frac{\hbar c}{2\pi}\int_0^\infty d\kappa \ln\det(1-\mathbb{N}_{12}) = \frac{\hbar c}{2\pi}\int_0^\infty d\kappa \,\mathrm{Tr}\,\log(1-\mathbb{N}_{12})$$

$$= -\frac{\hbar c}{2\pi}\int_0^\infty d\kappa \,\mathrm{Tr}\left(\mathbb{N}_{12} + \frac{1}{2}\mathbb{N}_{12}^2 + \frac{1}{3}\mathbb{N}_{12}^3 + \ldots + \frac{1}{p}\mathbb{N}_{12}^p + \ldots\right)$$

with $\quad \mathbb{N}_{12} = \mathbb{T}^1\mathbb{U}^{12}\mathbb{T}^2\mathbb{U}^{21} \sim e^{-2\kappa L}$

- For each scattering consider l partial waves

- Expand scattering amplitude in $R\kappa \sim R/L$

- For $1/L$ - Series of Casimir energy up to order

$$\sim (R/L)^\eta$$

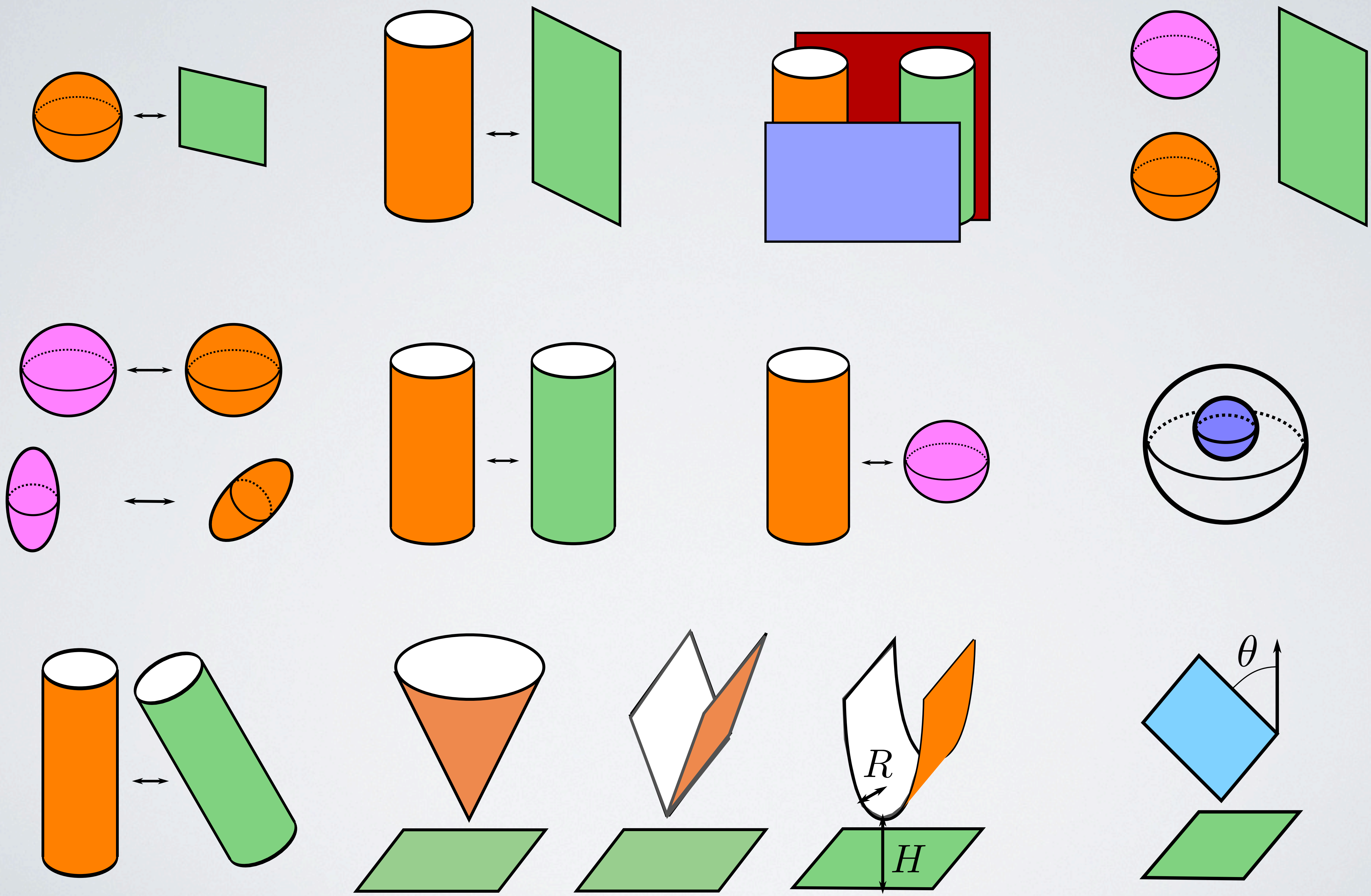
one needs only finite p, l

η	p	l	$\dim(\mathbb{N})$
7,8	1	1	6
9,10	1	2	16
11,12	1	3	30
13,14	2	4	48
15,16	2	5	70

- Scale-free objects (cone, wedge, …): low order expansion accurate

SOME EXAMPLES

EQUILIBRIUM INTERACTIONS BETWEEN...



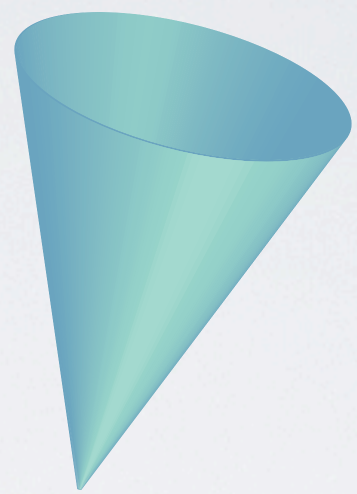
Sharp shaped metals

T independent of κ

$$T^{cone}_{M\,\lambda m} = -\frac{\partial_{\theta_0} P^{-m}_{i\lambda-1/2}(\cos\theta_0)}{\partial_{\theta_0} P^{m}_{i\lambda-1/2}(-\cos\theta_0)}$$

$$T^{cone}_{E\,\lambda m} = -\frac{P^{-m}_{i\lambda-1/2}(\cos\theta_0)}{P^{m}_{i\lambda-1/2}(-\cos\theta_0)}$$

$$T^{cone}_{Gh\,m} = \frac{P^{-|m|}_{0}(\cos\theta_0)}{P^{-|m|}_{0}(-\cos\theta_0)}$$



$$T^{\text{wedge}}_{M\pm\mu k_z} = \frac{e^{\mu\theta_0} \mp e^{-\mu\theta_0}}{e^{\mu(\pi-\theta_0)} \mp e^{-\mu(\pi-\theta_0)}}$$

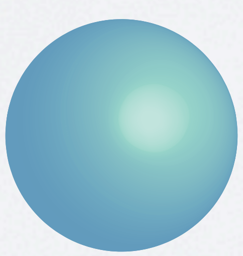
$$T^{\text{wedge}}_{E\pm\mu k_z} = -T^{\text{wedge}}_{M\mp\mu k_z}$$

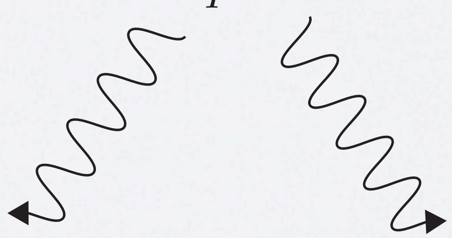
$$T^{\text{plate}}_{M\mathbf{k}_\parallel} = -1 \quad T^{\text{plate}}_{E\mathbf{k}_\parallel} = +1$$

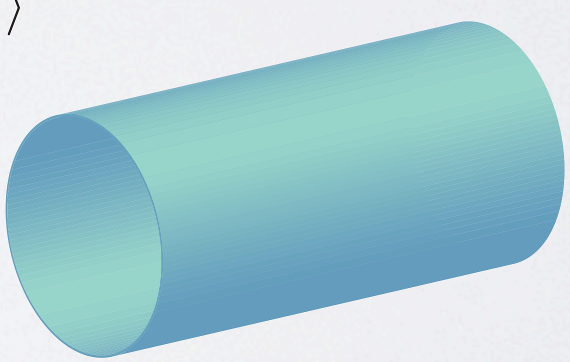
$$|\mathbf{E}_P\rangle = \sum_{P'} \mathcal{U}_{PP'} |\mathbf{E}_{P'}\rangle$$

T^{sphere}_{Mlm}

T^{sphere}_{Elm}







$T^{\text{cylinder}}_{Mmk_z}$

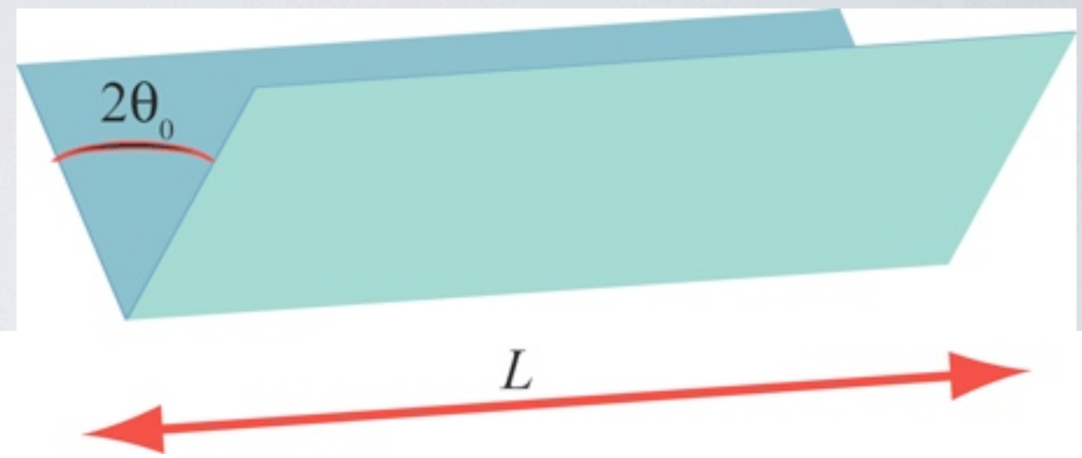
$T^{\text{cylinder}}_{Emk_z}$

T depends on κR

M. F. Maghrebi, S. J.Rahi, T. Emig, N. Graham, R. L. Jaffe, M. Kardar, PNAS 108, 6867 (2011).

PLATE - WEDGE

Multiple scattering expansion is highly accurate: analytical results at all distances



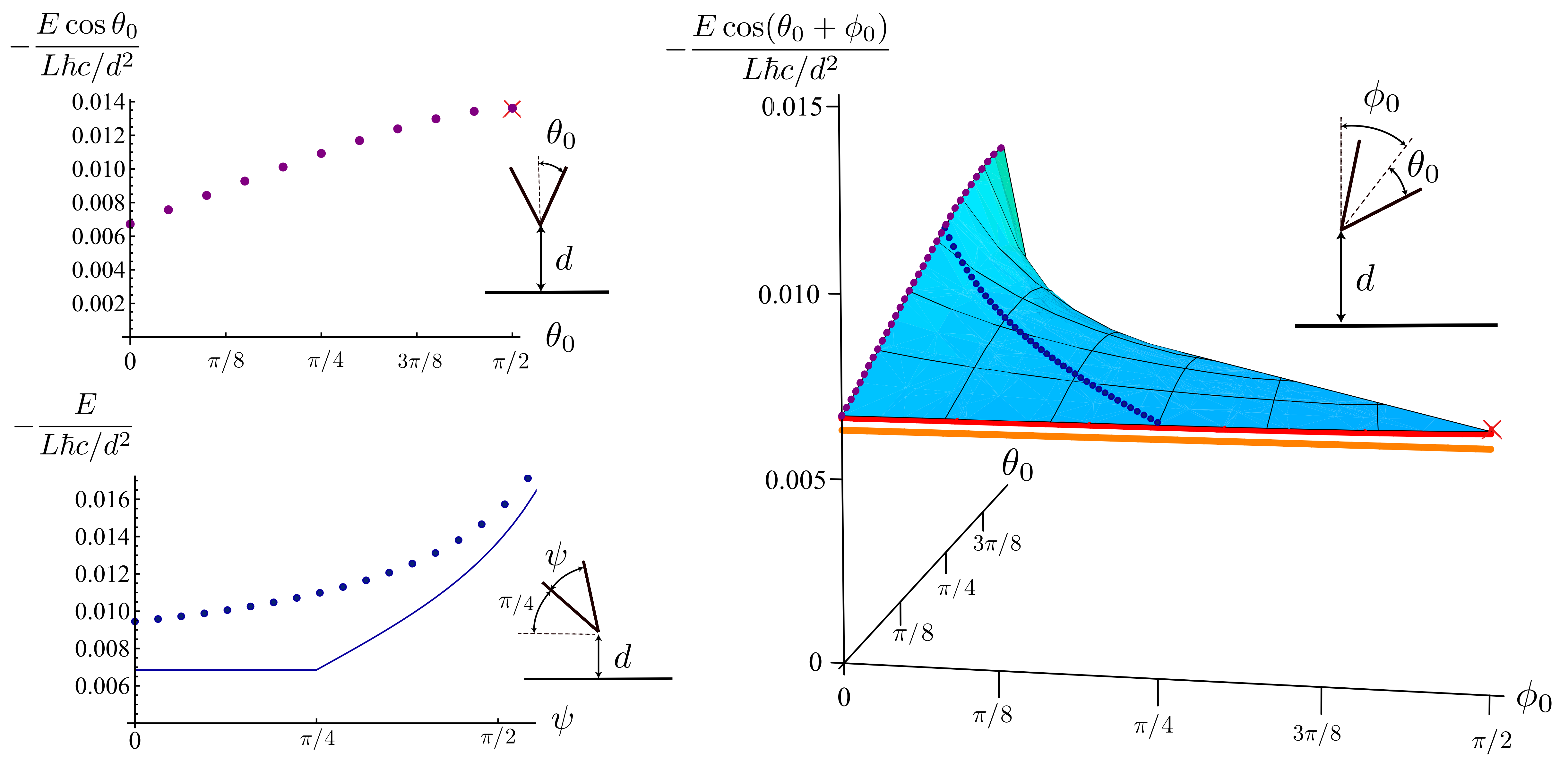
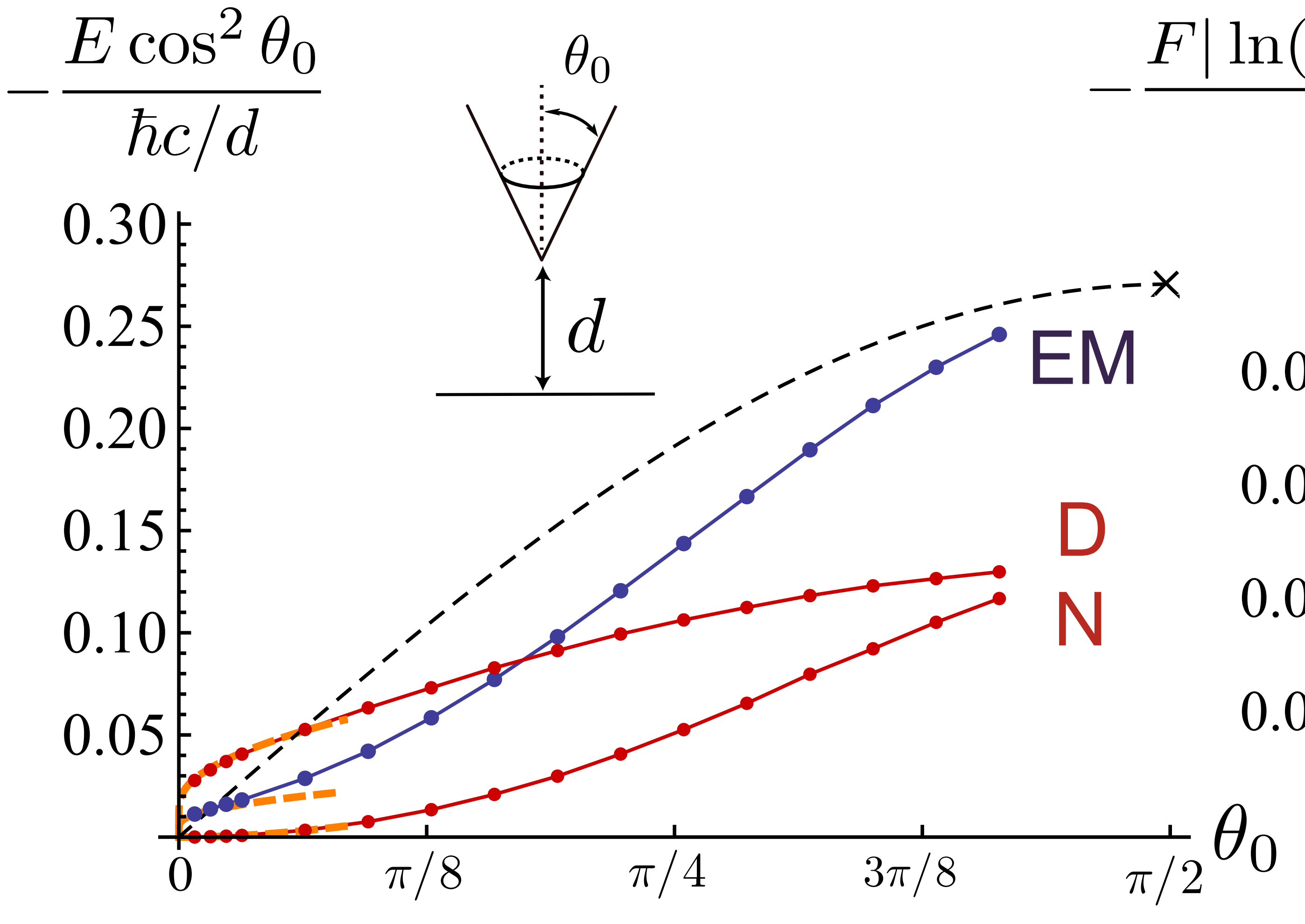
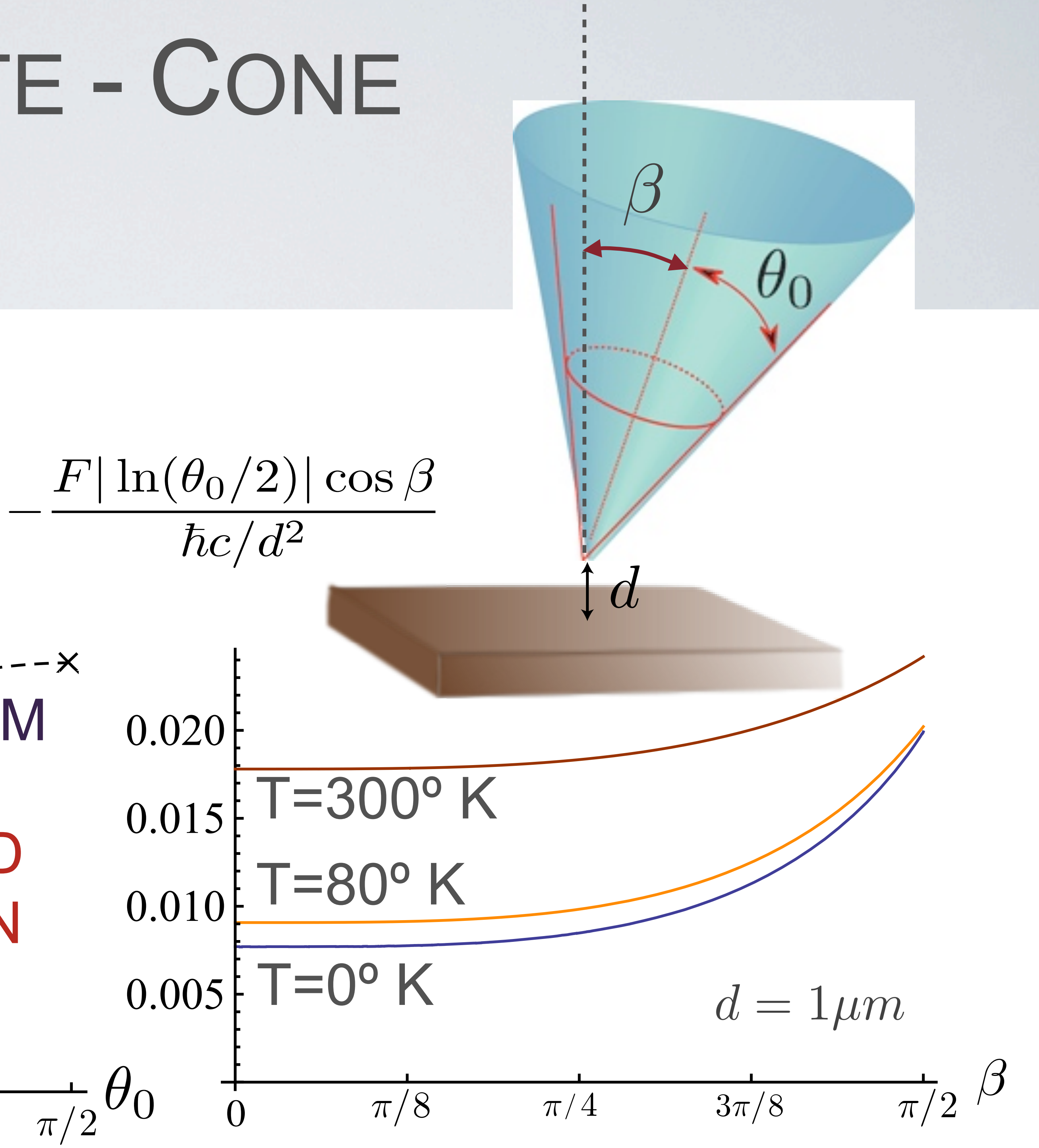


PLATE - CONE



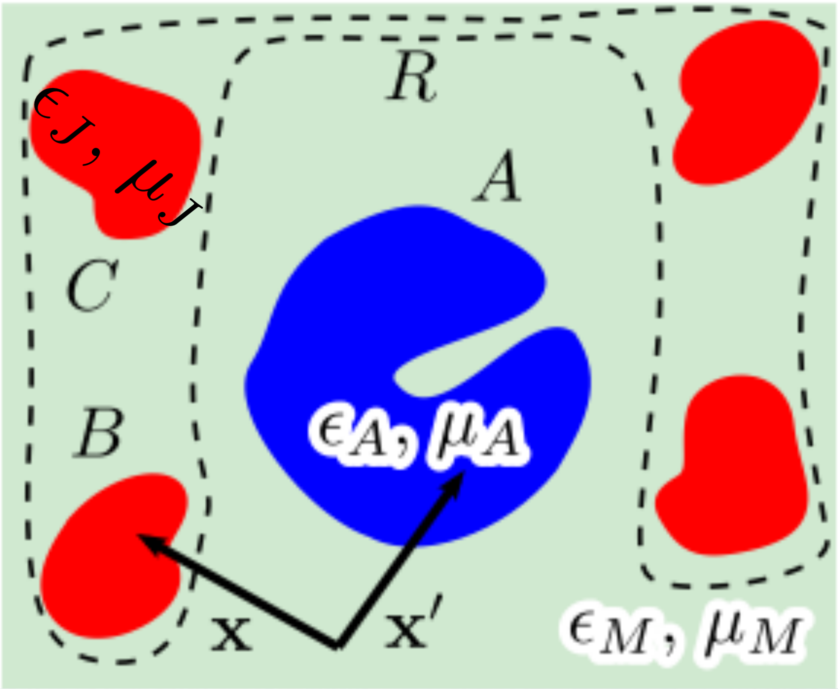


$$\mathcal{E} = -\frac{\ln 4 - 1}{16\pi}\frac{\hbar c}{d}\frac{1}{|\ln\theta_0/2|} + \mathcal{O}(\theta_0^2)$$

$$F \sim \frac{-\hbar c}{16\pi|\ln\frac{\theta_0}{2}|}\left[\frac{\ln 4 - 1}{d^2} - \frac{2}{3\lambda_T^2}\ln\frac{2d}{\lambda_T} + \frac{0.810}{\lambda_T^2} + \cdots\right]$$

Stable Equilibrium?

- Earnshaw's theorem: A charged body cannot be held in stable equilibrium by electrostatic forces from other charged bodies.



- Extension to fluctuation-induced forces?

- Start from scattering formulation (T-operators):

$$\mathcal{E} = \frac{\hbar c}{2\pi}\int_0^\infty d\kappa \; \mathrm{tr}\log \mathbb{T}^{-1}\mathbb{T}_\infty = \frac{\hbar c}{2\pi}\int_0^\infty d\kappa \; \mathrm{tr}\log(\mathbb{I} - \mathbb{T}_A \mathbb{G} \mathbb{T}_R \mathbb{G})$$

- Move object A by **d** with the "rest" of objects (R) fixed.

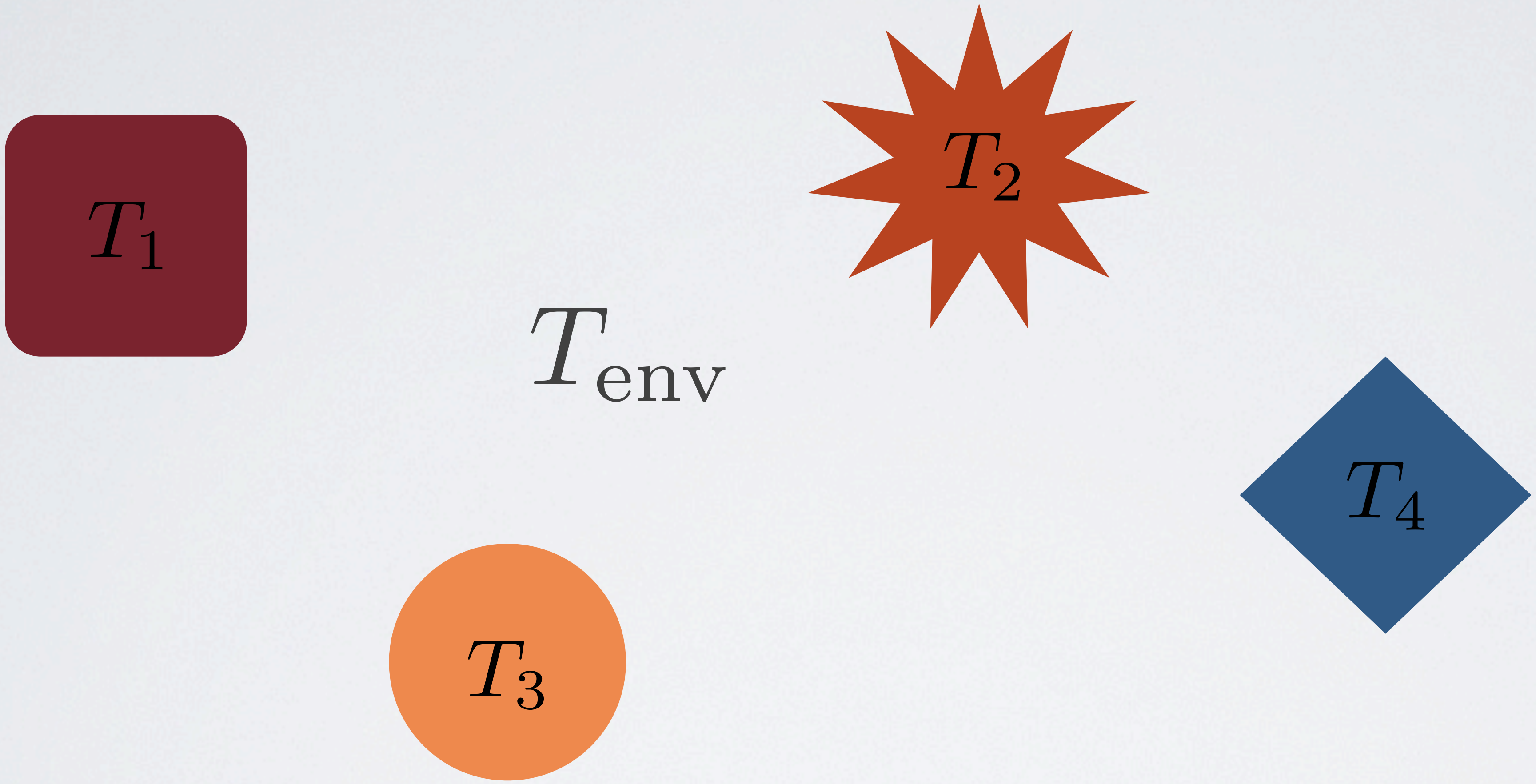
- Object A is unstable ($\nabla^2_{\mathbf{d}}\,\mathcal{E}\big|_{\mathbf{d}=0} \leq 0$) if $\mathrm{sign}(\mathbb{T}_A)\mathrm{sign}(\mathbb{T}_R) \geq 0$

- Stability not possible for (i) $\epsilon_J/\epsilon_M > 1,\; \mu_J/\mu_M \leq 1\,(\text{positive}\,\mathbb{T}_J)$
  (ii) $\epsilon_J/\epsilon_M < 1,\; \mu_J/\mu_M \geq 1\,(\text{negative}\,\mathbb{T}_J)$
  on the imaginary frequency axis (where always $\epsilon_J > 1$)

S. J. Rahi, M. Kardar, T. Emig, Phys. Rev. Lett. 105, 070404 (2010).

NON-EQUILIBRIUM QED

- Objects at different temperatures T_α (local equilibrium)
- Environment can have different temperature T_{env}



- Modification of equilibrium force ?
  Parallel plates: M. Antezza, L.P. Pitaevskii, S. Stringari, V.B. Svetovoy, Phys. Rev. A 77, 022901 (2008).
  General shapes: M. Krüger, T. Emig, G. Bimonte and M. Kardar, EPL 95 21002 (2011), M. Antezza et al. (2011).
- Radiation and transfer of heat ?
  M. Krüger, T. Emig and M. Kardar, PRL 106, 210404 (2011).

Fluctuation-Dissipation Theorem

- Equilibrium field correlations: $a_T(\omega) \equiv \frac{\omega^4 \hbar (4\pi)^2}{c^4} (\exp[\hbar\omega / k_B T] - 1)^{-1}$ $\quad a_0(\omega) \equiv \frac{\omega^4 \hbar (4\pi)^2}{2c^4}$

$$C^{eq}(T) = \langle \mathbf{E}(\omega; \mathbf{r}) \mathbf{E}^*(\omega; \mathbf{r}') \rangle^{eq} = [a_T(\omega) + a_0(\omega)] \frac{c^2}{\omega^2} \mathrm{Im}\mathbb{G}(\omega; \mathbf{r}, \mathbf{r}') = C_0 + \sum_\alpha C_\alpha^{sc}(T) + C^{env}(T)$$

- Three contributions:
  - zero point fluctuations: $C_0 = a_0(\omega) \frac{c^2}{\omega^2} \mathrm{Im}\mathbb{G}$
  - thermal currents in object α : $C_\alpha^{sc}(T) = a_T(\omega) \mathbb{G} \mathrm{Im}\varepsilon_\alpha \mathbb{G}^*$
  - environment fluctuations $C^{env}(T) = -a_T(\omega) \frac{c^2}{\omega^2} \mathbb{G} \mathrm{Im}\mathbb{G}_0^{-1} \mathbb{G}^*$
- Non-equilibrium correlations: change temperatures

$$C^{neq}(T_{env}, \{T_\alpha\}) = C_0 + \sum_\alpha C_\alpha^{sc}(T_\alpha) + C^{env}(T_{env}) = C^{eq}(T_{env}) + \sum_\alpha [C_\alpha^{sc}(T_\alpha) - C_\alpha^{sc}(T_{env})]$$

- Scattering theory: radiation of object α : $C_\alpha(T_\alpha) \equiv a_{T_\alpha}(\omega) \mathbb{G}_\alpha \mathrm{Im}\varepsilon_\alpha \mathbb{G}_\alpha^*$
  scattered at all other objects:

$$C_\alpha^{sc}(T_\alpha) = \mathbb{O}_{\alpha,\beta} C_\alpha(T_\alpha) \mathbb{O}_{\alpha,\beta}^\dagger, \quad \text{with}$$

$$\mathbb{O}_{\alpha,\beta} = (1 - \mathbb{G}_0 \mathbb{T}_\beta) \frac{1}{1 - \mathbb{G}_0 \mathbb{T}_\alpha \mathbb{G}_0 \mathbb{T}_\beta}$$

HEAT RADIATION OF SINGLE OBJECT

- Poynting vector: $$\mathbf{S}(\mathbf{r}) = \frac{c}{4\pi}\int \frac{d\omega}{2\pi}\langle \mathbf{E}(\mathbf{r})\times\mathbf{B}^*(\mathbf{r})\rangle$$
- Heat emitted by object α :

$$H_\alpha = \mathrm{Re}\oint_{\Sigma_\alpha} \mathbf{S}\cdot\mathbf{n}_\alpha = -\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{V_\alpha} d^3\mathbf{r}\,\langle \mathbf{E}(\mathbf{r})\cdot\mathbf{J}^*(\mathbf{r})\rangle$$

- Use $\mathbf{E} = 4\pi i\frac{\omega}{c^2}\mathbb{G}_0\,\mathbf{J}$ to get general result

- Since $\mathrm{Im}[\mathbb{G}_0]$ involves only propagating waves, in matrix notation:

$$H_\alpha = \frac{2\hbar}{\pi}\int_0^\infty d\omega\frac{\omega}{\exp(\hbar\omega/k_BT)-1}\mathrm{Tr}\left\{\mathrm{Im}[\mathbb{G}_0]\,\mathrm{Im}[\mathbb{T}] - \mathrm{Im}[\mathbb{G}_0]\,\mathbb{T}\,\mathrm{Im}[\mathbb{G}_0]\,\mathbb{T}^*\right\}$$

$$H_\alpha = \frac{\hbar}{2\pi}\int d\omega\frac{\omega}{e^{\frac{\hbar\omega}{k_BT}}-1}\mathrm{Tr}_{pr}\left[\mathcal{I}-\mathcal{S}\mathcal{S}^\dagger\right]\geq 0$$

HEAT TRANSFER BETWEEN TWO BODIES

- Two bodies, at T_1 and at T_2 , in cold environment. Total heat transferred from 1 to 2:

$$H_{\rm tot} = H_1^{(2)}(T_1) - H_2^{(1)}(T_2)$$

where $H_1^{(2)}(T_1)$ is radiation of 1, partly absorbed by 2.

- We get for transfer rate

$$H_1^{(2)}(T_1) = \frac{2\hbar}{\pi}\int_0^\infty d\omega \frac{\omega}{e^{\frac{\hbar\omega}{k_B T_1}} - 1} J(\mathbb{T}_1, \mathbb{T}_2)$$

with

$$J(\mathbb{T}_1, \mathbb{T}_2) = \mathrm{Tr}\left\{[\mathrm{Im}[\mathbb{T}_2] - \mathbb{T}_2^*\mathrm{Im}[\mathbb{G}_0]\mathbb{T}_2]\frac{1}{1 - \mathbb{G}_0\mathbb{T}_1\mathbb{G}_0\mathbb{T}_2}\mathbb{G}_0\left[\mathrm{Im}[\mathbb{T}_1] - \mathbb{T}_1\mathrm{Im}[\mathbb{G}_0]\mathbb{T}_1^*\right]\mathbb{G}_0^*\frac{1}{1 - \mathbb{T}_2^*\mathbb{G}_0^*\mathbb{T}_1^*\mathbb{G}_0^*}\right\} \geq 0$$

- Since J is symmetric (trace is cyclic), one gets

$$H_{\rm tot} = \frac{2\hbar}{\pi}\int_0^\infty d\omega\omega\left(\frac{1}{e^{\frac{\hbar\omega}{k_B T_1}} - 1} - \frac{1}{e^{\frac{\hbar\omega}{k_B T_2}} - 1}\right) J(\mathbb{T}_1, \mathbb{T}_2)$$

TOTAL ABSORBED HEAT

- Total heat absorbed by one object (1) in the presence of a second object (2) and the environment:

$$H^{(1)}(T_1, T_2, T_{\text{env}}) = H_2^{(1)}(T_1) + H_1^{(1)}(T_2) + H_{\text{env}}^{(1)}(T_{\text{env}})$$
$$= \sum_{\alpha=1,2} H_\alpha^{(1)}(T_\alpha) - H_\alpha^{(1)}(T_{\text{env}})$$

- Here we have included the heat emitted by object 1 (in the presence of object 2) which is negative,

$$H_1^{(1)} = -\frac{2\hbar}{\pi}\int_0^\infty d\omega \frac{\omega}{e^{\frac{\hbar\omega}{k_B T_1}} - 1} \text{ImTr}\left\{(1 + \mathbb{G}_0\mathbb{T}_2)\frac{1}{1 - \mathbb{G}_0\mathbb{T}_1\mathbb{G}_0\mathbb{T}_2}\mathbb{G}_0\left[\text{Im}[\mathbb{T}_1] - \mathbb{T}_1\text{Im}[\mathbb{G}_0]\mathbb{T}_1^*\right]\frac{1}{1 - \mathbb{G}_0^*\mathbb{T}_2^*\mathbb{G}_0^*\mathbb{T}_1^*}\right\}$$

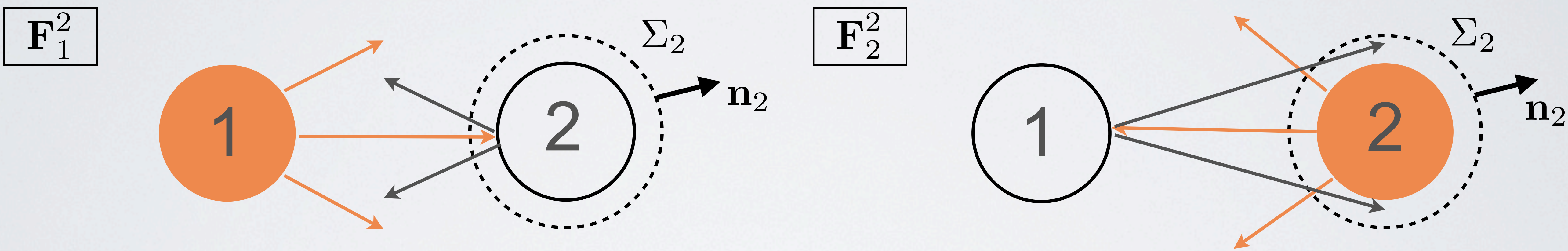
and the radiation absorbed from the environment.

- Important for heating or cooling rate.

NON-EQUILIBRIUM FORCE

- Maxwell stress tensor: $\sigma_{ab}(\mathbf{r}) \;=\; \int \frac{d\omega}{16\pi^3}\left\langle E_a E_b^* + B_a B_b^* - \frac{1}{2}\left(|E|^2 + |B|^2\right)\delta_{ab}\right\rangle$
- Total force on object 2 due to other objects and environment:

$$\mathbf{F}^2 = \mathrm{Re}\oint_{\Sigma_2}\sigma\cdot\mathbf{n}_2 = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\frac{1}{\omega}\int_{V_2} d^3\mathbf{r}\,\mathrm{Im}\langle\nabla\mathbf{E}(\mathbf{r})\cdot\mathbf{J}^*\rangle = \mathbf{F}^{2,eq}(T_{env}) + \sum_\beta\left[\mathbf{F}_\beta^2(T_\beta) - \mathbf{F}_\beta^2(T_{env})\right]$$



$$\mathbf{F}_1^{(2)} = \frac{2\hbar}{\pi}\int_0^\infty d\omega\frac{1}{e^{\frac{\hbar\omega}{k_B T_1}} - 1}\Re\mathrm{Tr}\left\{\boldsymbol{\nabla}(1 + \mathbb{G}_0\mathbb{T}_2)\frac{1}{1 - \mathbb{G}_0\mathbb{T}_1\mathbb{G}_0\mathbb{T}_2}\mathbb{G}_0\left[\Im[\mathbb{T}_1] - \mathbb{T}_1\Im[\mathbb{G}_0]\mathbb{T}_1^*\right]\mathbb{G}_0^*\frac{1}{1 - \mathbb{T}_2^*\mathbb{G}_0^*\mathbb{T}_1^*\mathbb{G}_0^*}\mathbb{T}_2^*\right\}$$

$$\mathbf{F}_2^{(2)} = \frac{2\hbar}{\pi}\int_0^\infty d\omega\frac{1}{e^{\frac{\hbar\omega}{k_B T_2}} - 1}\Re\mathrm{Tr}\left\{\boldsymbol{\nabla}(1 + \mathbb{G}_0\mathbb{T}_1)\frac{1}{1 - \mathbb{G}_0\mathbb{T}_2\mathbb{G}_0\mathbb{T}_1}\mathbb{G}_0\left[\Im[\mathbb{T}_2] - \mathbb{T}_2\Im[\mathbb{G}_0]\mathbb{T}_2^*\right]\frac{1}{1 - \mathbb{G}_0^*\mathbb{T}_1^*\mathbb{G}_0^*\mathbb{T}_2^*}\right\}$$

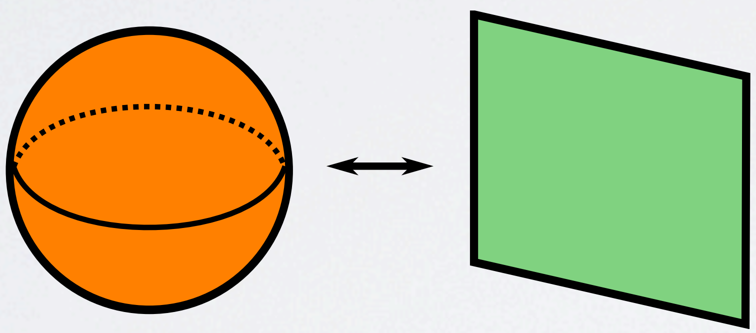
EQUILIBRIUM VS. NON-EQUILIBRIUM

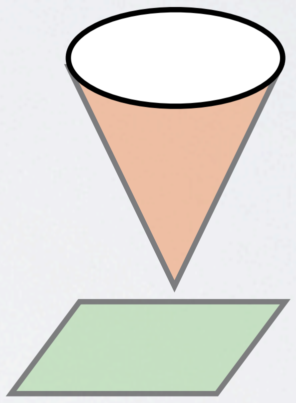
- All quantities expressed as traces over product of free Green's function and T-operators of individual bodies.
- Equilibrium:
  - Computations on imaginary frequency axis
- Non-equilibrium:
  - Traces are non-analytic function of frequency, computations on real frequency axis
  - Quantities sensitive to details of dielectric function: resonances
  - Much richer phenomenology

NON-EQUILIBRIUM EFFECTS FOR...

- Heat radiation:

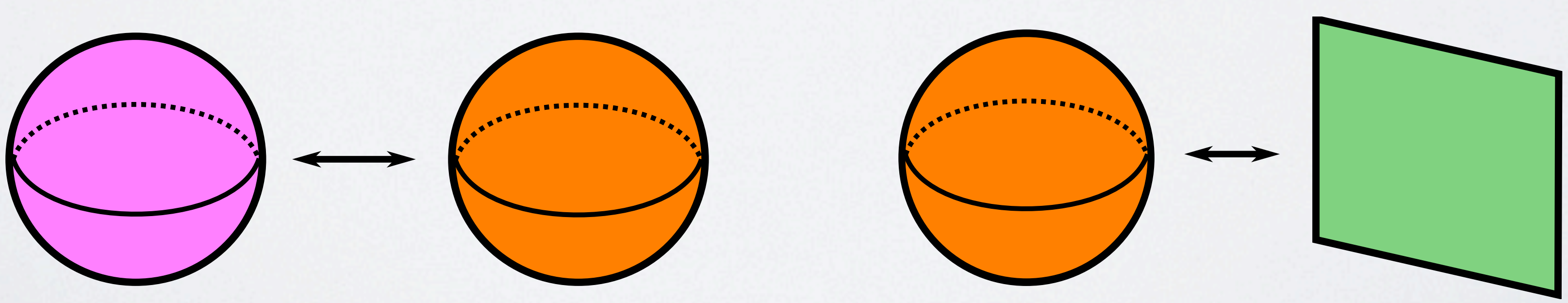
- Heat transfer: M. Krüger, T. Emig and M. Kardar, PRL 106, 210404 (2011)





A. P. McCauley, M. T. H. Reid, M. Krüger and S. G. Johnson, Phys. Rev. B 85, 165104 (2012).

- Forces M. Krüger, T. Emig, G. Bimonte and M. Kardar, EPL 95 21002 (2011)

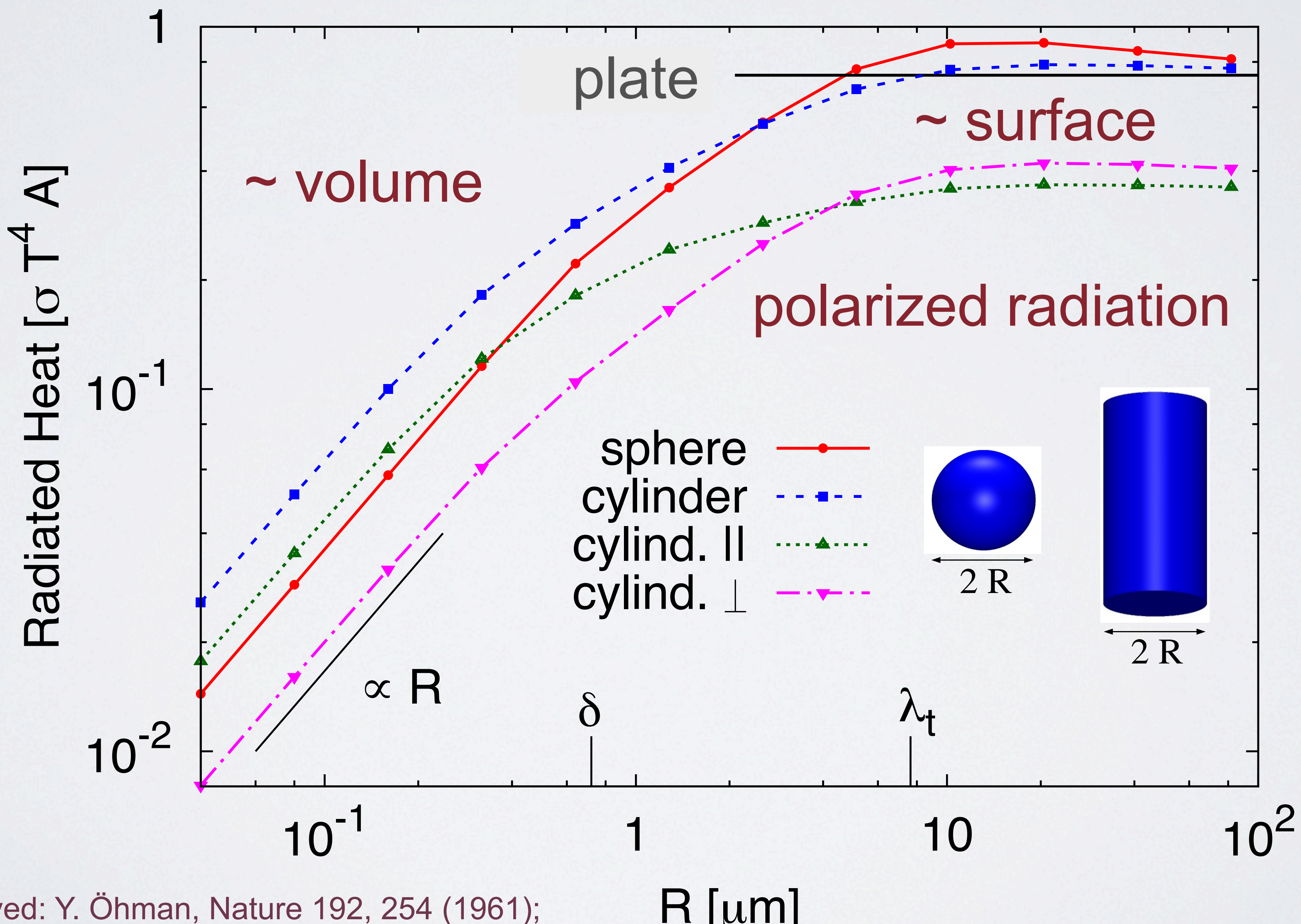


HEAT RADIATION

- Stefan-Boltzmann law for an ideal black body with surface area A:

$$H = \sigma T^4 A \qquad\qquad \sigma = \frac{\pi^2 k_B^4}{60\hbar^3 c^2}$$

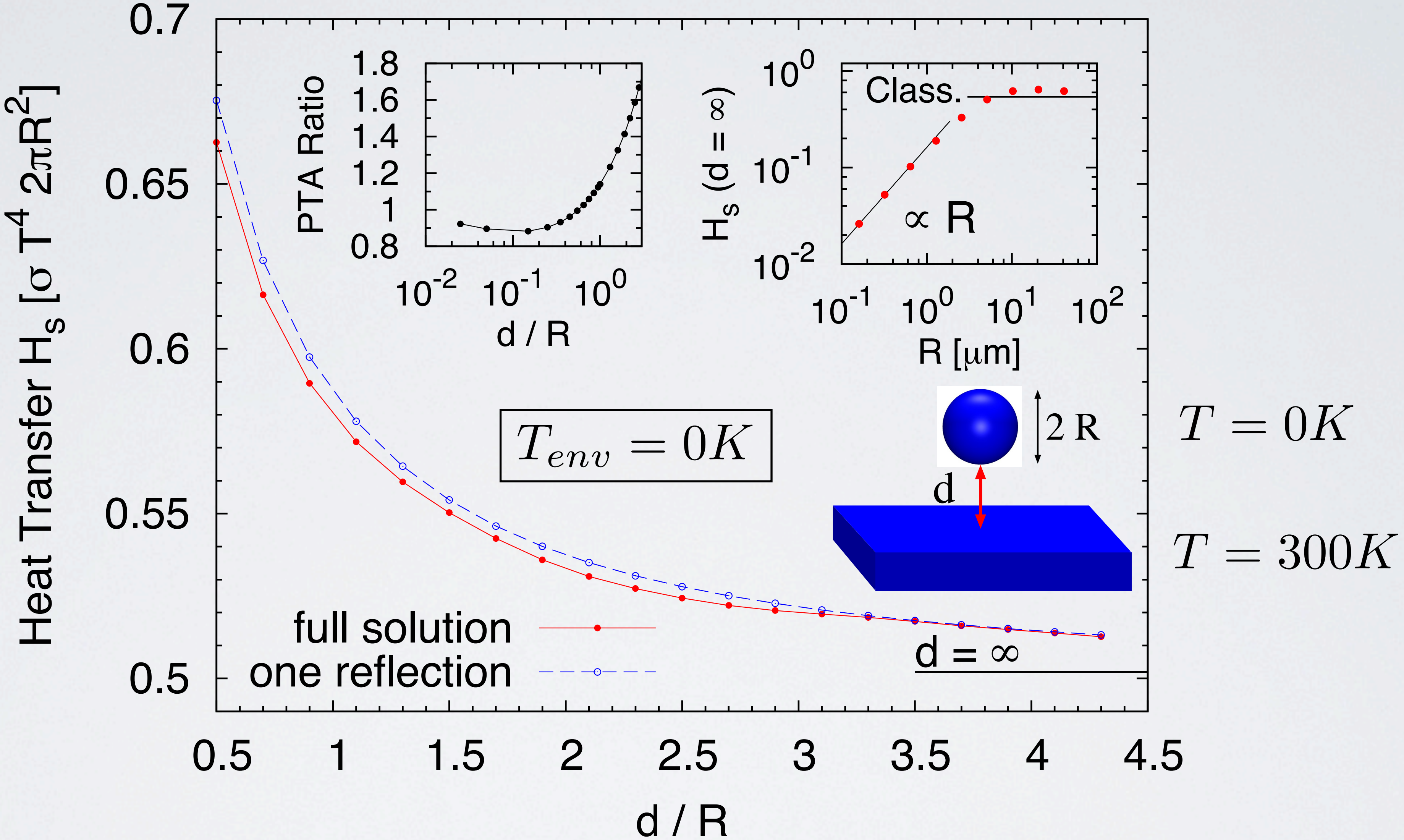
- Sphere and cylinder (SiO_2) at T=300K:



Polarization exp. observed: Y. Öhman, Nature 192, 254 (1961);
G. Bimonte et al., New J. Phys. 11, 033014 (2009).

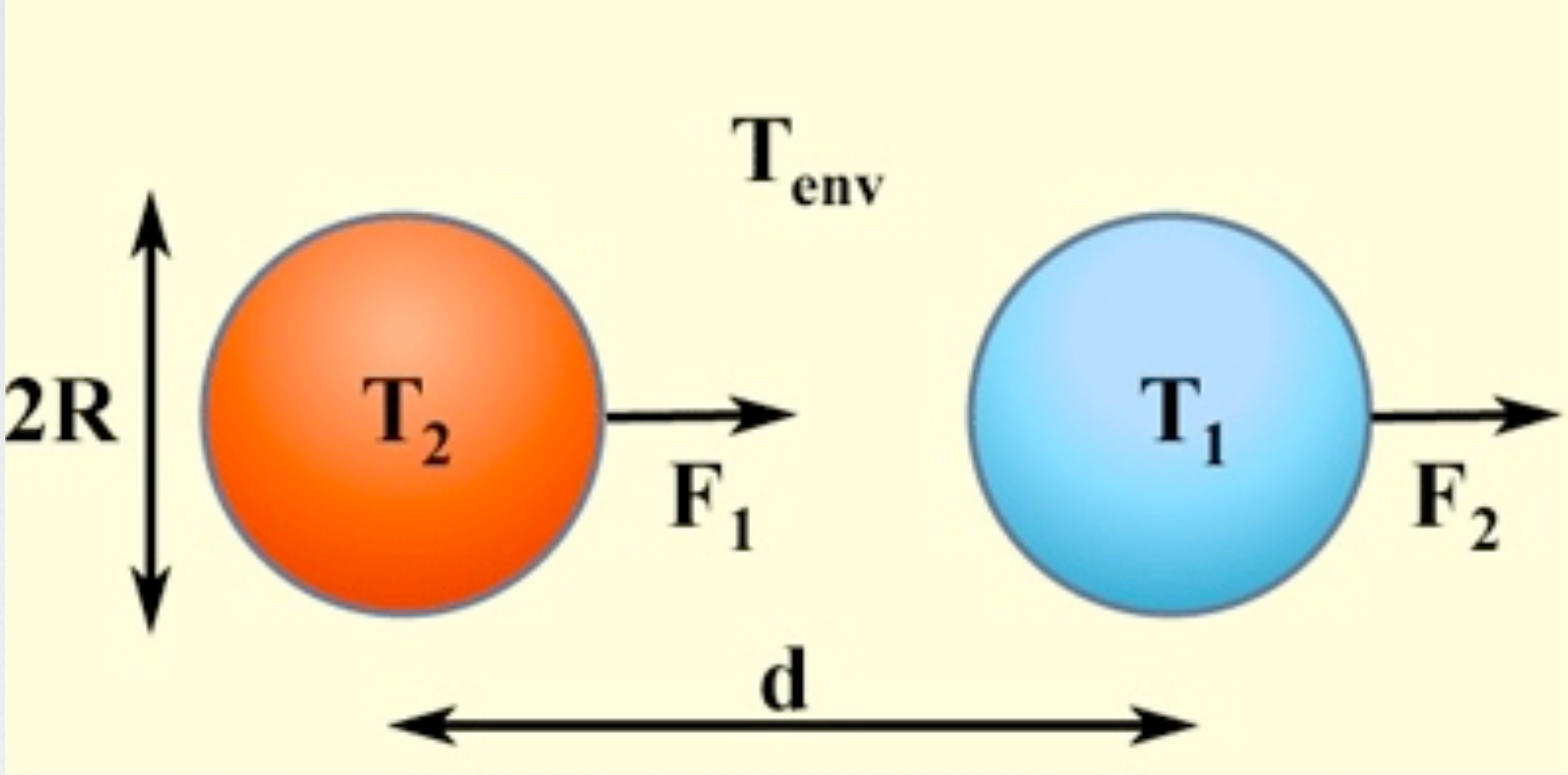
HEAT TRANSFER

- Heat transfer rate from plate to sphere (SiO_2, R=5μm)



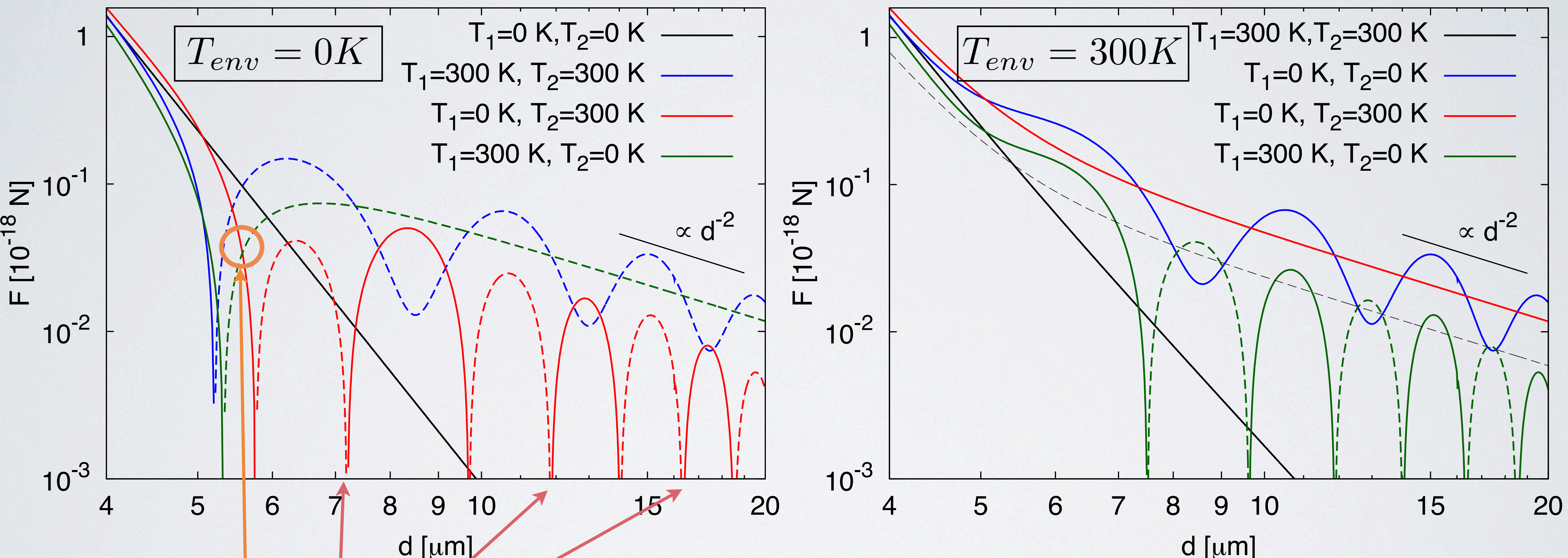
- Increased heat transfer at small d due to tunneling of evanesc. waves.
- At small d proximity transfer approximation (PTA) is valid: $H_s \sim 1/d$
- Volume-to-surface crossover around $R \approx \lambda_T$

TWO SPHERES AT DIFFERENT TEMPERATURES



FORCE BETWEEN TWO SPHERES (SiO_2)

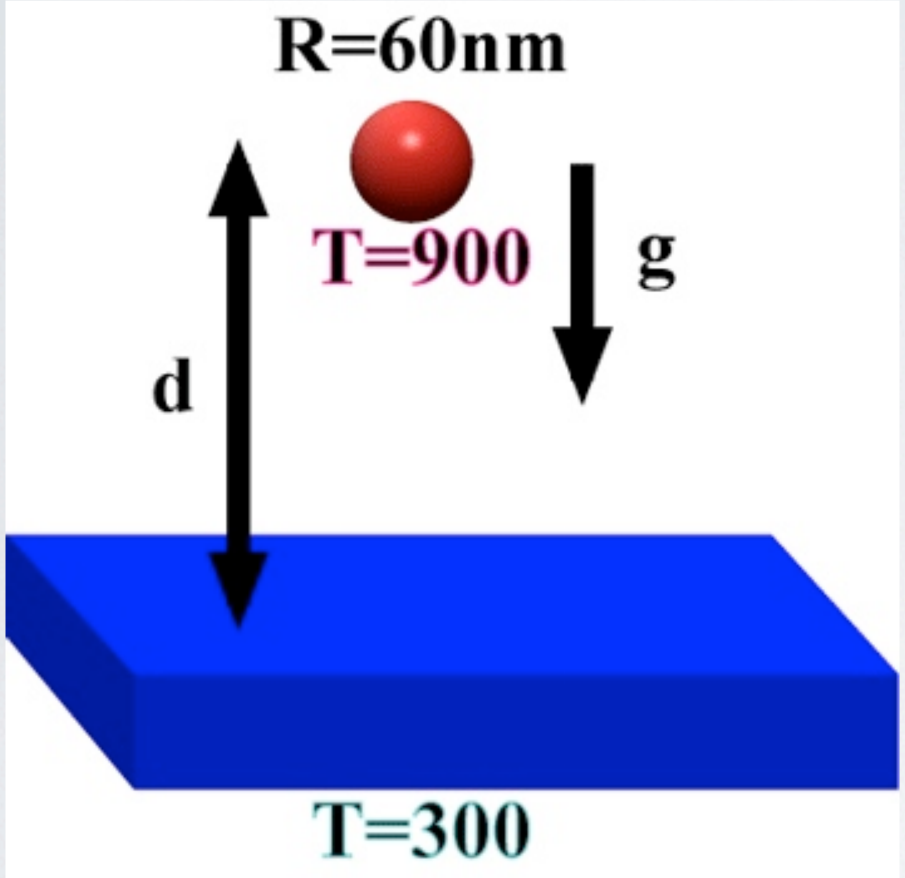
- Dipole approximation, one reflection: assume radius $R \ll d, \lambda_T = \frac{\hbar c}{K_B T}$
- Force on sphere 2: attraction (solid lines) and repulsion (dashed lines)

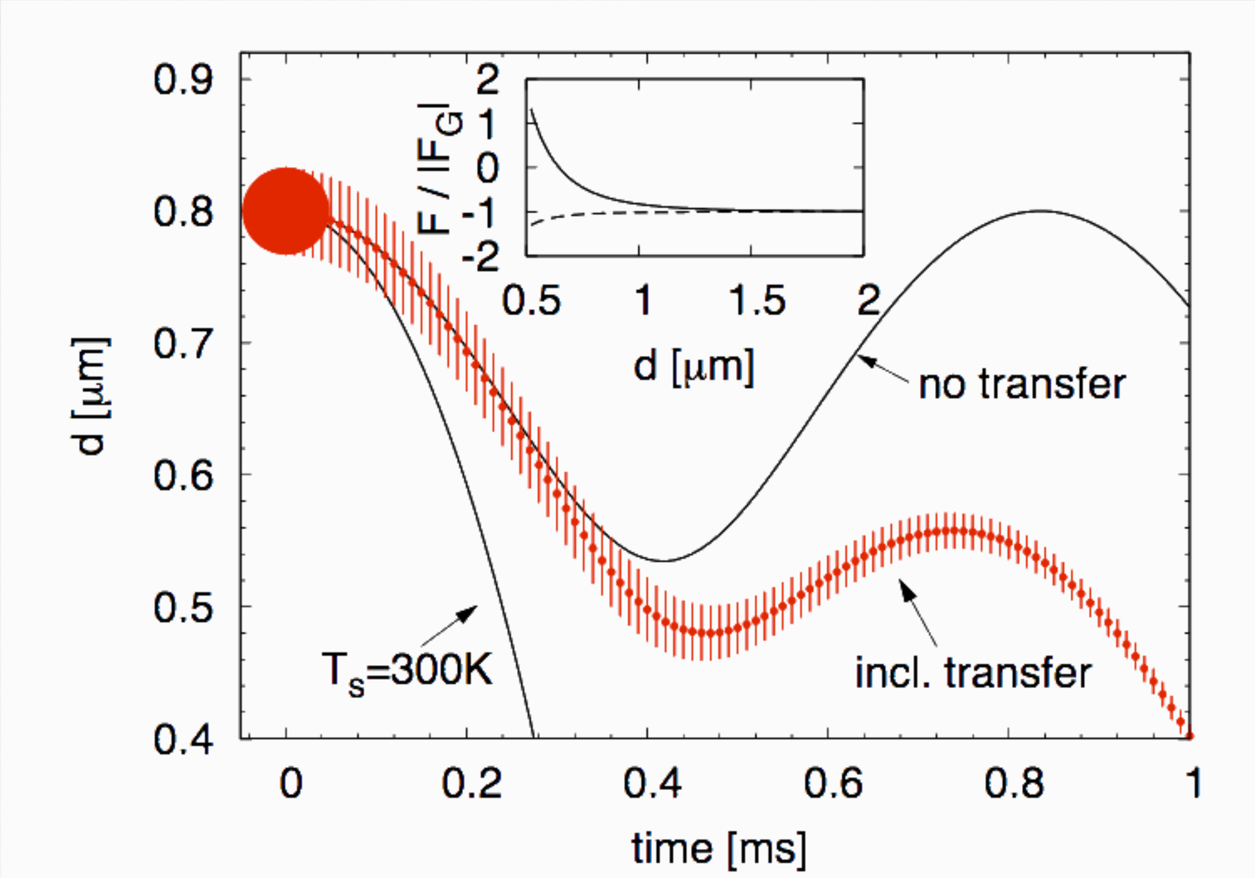


- Oscillations from $\mathbf{F}^{\alpha}_{\alpha}$ due to interference of reflected and non-reflected radiation. Set by material resonances.
- Stable equilibrium positions.
- Self-propelled pairs: equal acceleration in the same direction.

CASIMIR LEVITATION

- Non-equilibrium situation:





- Hot microsphere levitates above a cold dielectric plate
- If sphere cools down (including heat transfer) it will fall down

OUTLOOK / NEW DIRECTIONS

- Radiation/Transfer: effect of shape? e.g. non-parallel cylinders, disorder (roughness)?
- Fluctuation of forces / radiation / transfer? distribution functions? Related to friction (Einstein relation): Quantum friction?
- Relation to random matrices?
- Dynamic effects: Radiation due to motion
- ...