Near-field heat transfer between anisotropic materials

Svend-Age Biehs

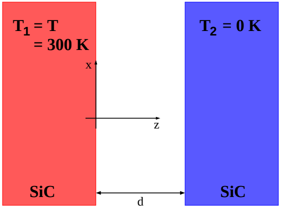
Institut für Physik, Carl von Ossietzky Universität Oldenburg

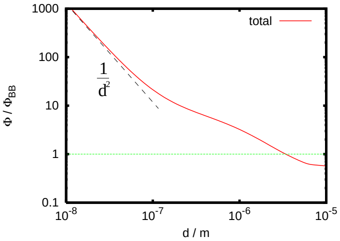
(J.-J. Greffet, P. Ben-Abdallah, K. Joulain, E. Rousseau, F.S.S. Rosa, R. Messina, M. Tschikin)

Les Houches, May 2013

heat flux expression

$$\Phi = \langle S_z \rangle = \int \frac{\mathrm{d}\,\omega}{2\pi} \frac{\hbar\omega}{e^{\hbar\omega/(k_\mathrm{B} T)} - 1} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} (\mathcal{T}_\mathrm{s} + \mathcal{T}_\mathrm{p})$$

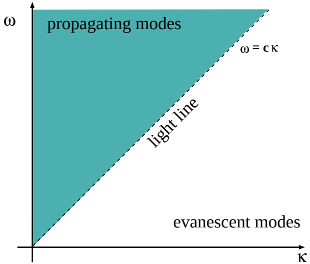


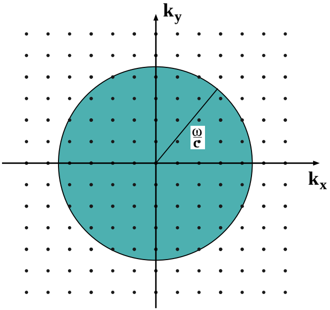


- transmission coefficient (Polder and van Hove, PRB **4**, 3303 (1971))

$$\mathcal{T}_i(\omega, \boldsymbol{\kappa}; d) = \begin{cases} \dfrac{(1-|r_i^{10}|^2)(1-|r_i^{20}|^2)}{|1-r_i^{10} r_i^{20} \exp(2\mathrm{i}k_z d)|^2}, & \kappa < \dfrac{\omega}{c} \\ \dfrac{\mathrm{Im}(r_i^{10})\mathrm{Im}(r_i^{20}) e^{-2|k_z|d}}{|1-r_i^{10} r_i^{20} \exp(2\mathrm{i}k_z d)|^2}, & \kappa > \dfrac{\omega}{c} \end{cases}$$

propagating modes





- plane wave

$$E_y = A(x, y; t)\mathrm{e}^{\mathrm{i}k_z z}$$

- prop. modes $\kappa < \frac{\omega}{c}$

$$k_z = \sqrt{\frac{\omega^2}{c^2} - \kappa^2} \in \mathbb{R}$$

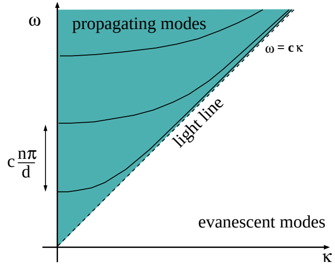
- evan. modes $\kappa > \frac{\omega}{c}$

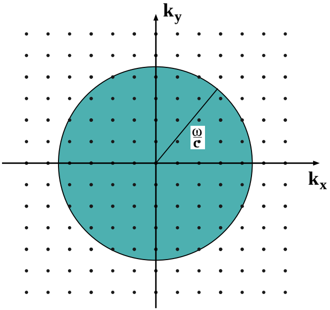
$$k_z = \mathrm{i}\sqrt{\kappa^2 - \frac{\omega^2}{c^2}} \in \mathbb{C}$$

- res. transmission

$$k_z \equiv \frac{n\pi}{d}, n \in \mathbb{N}$$

propagating modes





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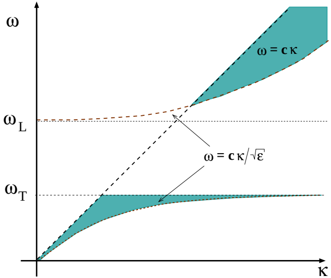
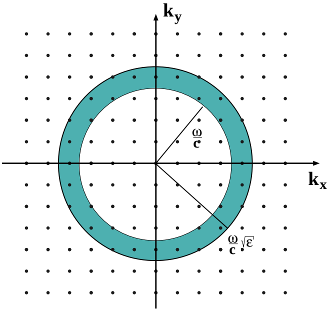
Frustrated internal reflection

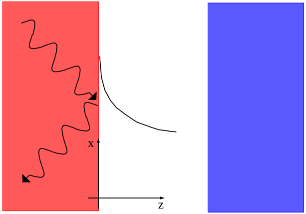




- propagating waves inside the medium

$$k_{1,z} = \sqrt{\frac{\omega^2}{c^2}\epsilon - \kappa^2} \in \mathbb{R}$$

$$\Leftrightarrow \kappa < \frac{\omega}{c}\sqrt{\epsilon}$$



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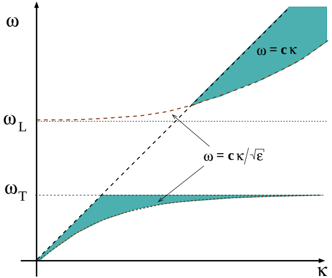

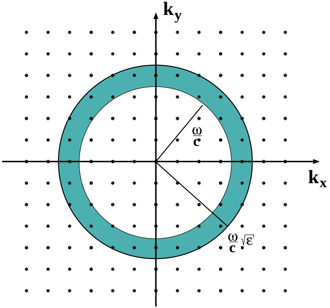

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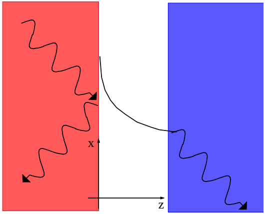
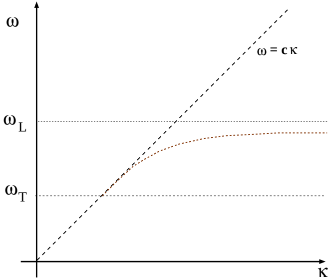
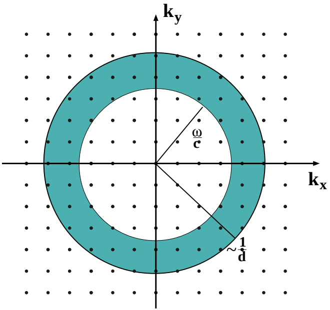


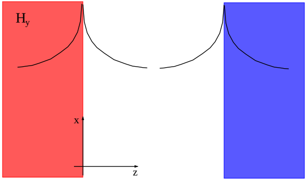
Surface modes



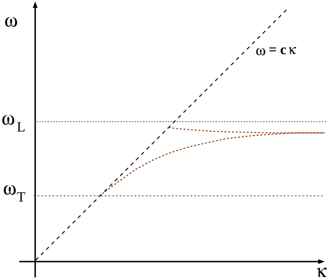


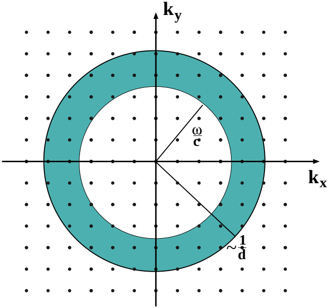
- 'bound' to the surface
- for p-polarisation only
- transmission coeff. $\kappa \gg \omega/c$

$$\mathcal{T}_{\rm p} \approx \frac{{\rm Im}(r_{\rm p}^{10}){\rm Im}(r_{\rm p}^{20})e^{-2\kappa d}}{|1 - r_{\rm p}^{10} r_{\rm p}^{20} \exp(-2\kappa d)|^2}$$



Surface modes





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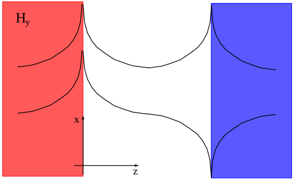
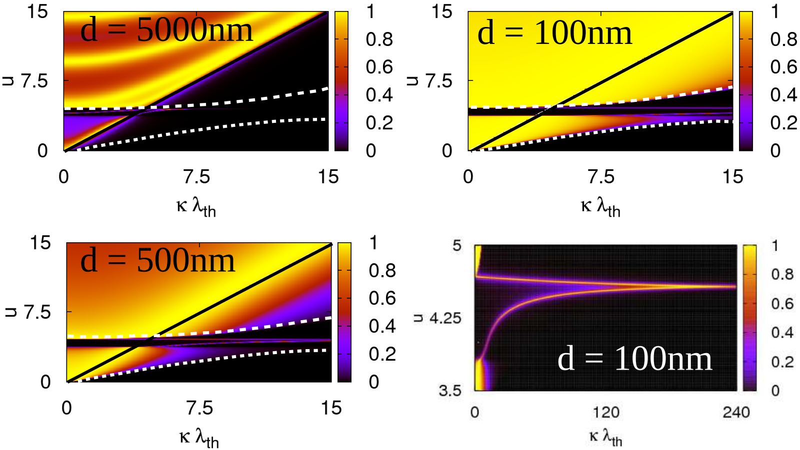
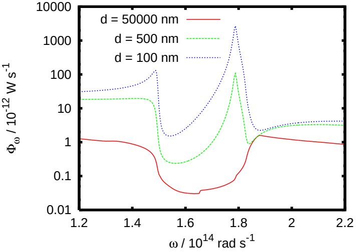
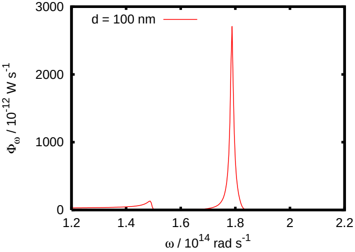


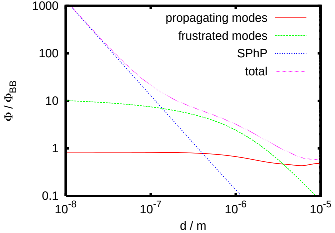
Transmission coefficient (SiC, $u = \hbar\omega / k_B T$)



heat flux (SiC, $T_1 = 300\,\mathrm{K}$, $T_2 = 0\,\mathrm{K}$)

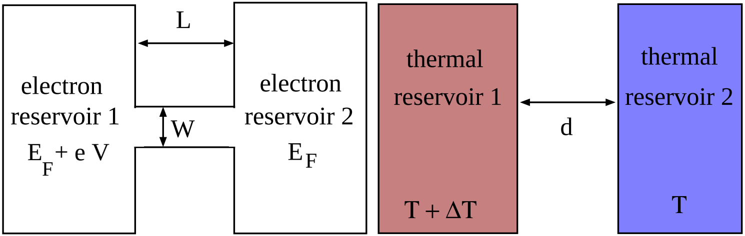




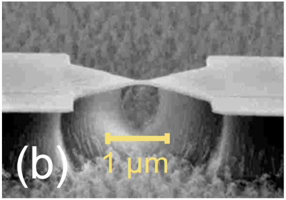


$$\Phi = \int \frac{\mathrm{d}\,\omega}{2\pi} \frac{\hbar\omega}{e^{\hbar\omega/(k_\mathrm{B}T)} - 1} \times \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \left(\mathcal{T}_\mathrm{s} + \mathcal{T}_\mathrm{p}\right)$$

Landauer-like expression for the heat flux



$$I = \Gamma V = \frac{2e^2}{h}\left[\sum_n T_n\right] V \qquad \Phi = \frac{\pi^2 k_B^2 T}{3h}\left[\sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \overline{\mathcal{T}}_i\right]\Delta T$$

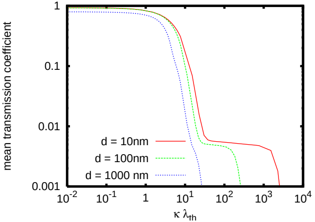


$$\overline{\mathcal{T}}_i = \frac{\int \mathrm{d}u\, u^2 f(u) \mathcal{T}_i(u,d)}{\int \mathrm{d}u\, u^2 f(u)}$$

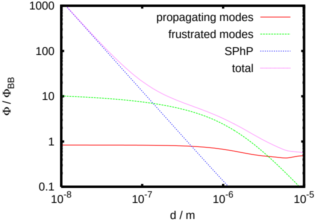
$$f(u) = \frac{u^2 \mathrm{e}^u}{(\mathrm{e}^u - 1)^2}$$

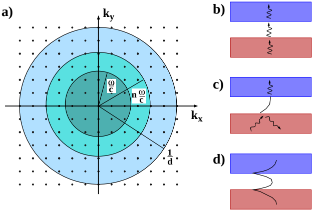
Biehs, Rousseau, Greffet, PRL **105**, 234301 (2010)

Mean transmission coefficient



$$\Phi = \frac{\pi^2 k_B^2 T}{3h} \sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \overline{\mathcal{T}}_j \Delta T$$



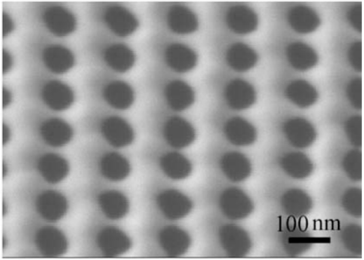


Biehs, Rousseau, Greffet, PRL **105**, 234301 (2010)

fundamental limits:Ben-Abdallah and Joulain, PRB **82**, 121419 (R)(2010)

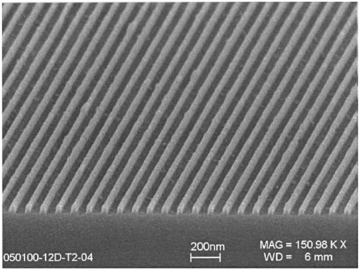
artificial media

- Nanoporous Au-film



Feng et al., APL **96**, 041112 (2010)

- Si grating



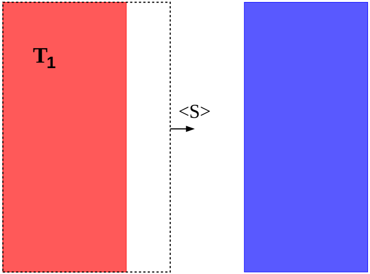
Yu et al., J. Vac. Sci. Technol. B **19**, 2816 (2001)

- $\lambda \gg$ grating constant
  $\rightarrow$ effectively homogeneous, but anisotropic

Green's function in the gap region (I)

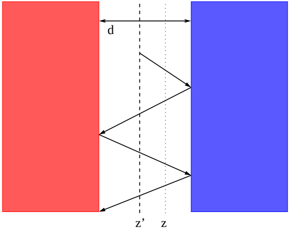
Volokitin and Persson, Rev. Mod. Phys. **79**, 1291 (2007):

$$\langle S_z \rangle = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \left[\Theta(\omega, T_1) - \Theta(\omega, T_2)\right] \times 2\,\mathrm{Re}\,\mathrm{Tr}\left[\int_A \mathrm{d}\mathbf{r}'_{\|} \left(\mathbb{G}(\mathbf{r},\mathbf{r}')\partial_z\partial'_z\mathbb{G}^\dagger(\mathbf{r},\mathbf{r}') - \partial_z\mathbb{G}^\dagger(\mathbf{r},\mathbf{r}')\partial'_z\mathbb{G}(\mathbf{r},\mathbf{r}') \right)\right].$$



$\mathbb{G}(\mathbf{r},\mathbf{r}')$ electric Green's function with $\mathbf{r}$ and $\mathbf{r}'$ inside the gap

Green's function in the gap region (II)



- Summing up multiple reflections:

$$\mathbb{G}_A(\boldsymbol{\kappa};z,z') = \frac{\mathrm{i}}{2\gamma_\mathrm{r}}\Big[\mathbb{1}\,\mathrm{e}^{\mathrm{i}\gamma_\mathrm{r}(z-z')} + \mathrm{e}^{2\mathrm{i}\gamma_\mathrm{r}d}\mathrm{e}^{-\mathrm{i}\gamma_\mathrm{r}(z+z')}\mathbb{R}_2 + \mathrm{e}^{2\mathrm{i}\gamma_\mathrm{r}d}\mathrm{e}^{\mathrm{i}\gamma_\mathrm{r}(z-z')}\mathbb{R}_1\mathbb{R}_2 + \mathrm{e}^{4\mathrm{i}\gamma_\mathrm{r}d}\mathrm{e}^{-\mathrm{i}\gamma_\mathrm{r}(z+z')}\mathbb{R}_2\mathbb{R}_1\mathbb{R}_2 + \dots\Big]$$

- complete intracavity Green's function

$$\mathbb{G}_\mathrm{intra} = \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2}\,\mathrm{e}^{\mathrm{i}\boldsymbol{\kappa}\cdot(\mathbf{x}-\mathbf{x}')}\,\frac{\mathrm{i}}{2\gamma_\mathrm{r}}\Big[\mathbb{D}^{12}\Big(\mathbb{1}\,\mathrm{e}^{\mathrm{i}\gamma_\mathrm{r}(z-z')} + \mathbb{R}_1\mathrm{e}^{\mathrm{i}\gamma_\mathrm{r}(z+z')}\Big) + \mathbb{D}^{21}\Big(\mathbb{R}_2\mathbb{R}_1\mathrm{e}^{\mathrm{i}\gamma_\mathrm{r}(z'-z)}\mathrm{e}^{2\mathrm{i}\gamma_\mathrm{r}d} + \mathbb{R}_2\mathrm{e}^{2\mathrm{i}\gamma_\mathrm{r}d}\mathrm{e}^{-\mathrm{i}\gamma_\mathrm{r}(z+z')}\Big)\Big]$$

Biehs, Rosa, Ben-Abdallah, Joulain, Greffet, Opt. Expr. **19**, A1088-A1103 (2011)

Heat flux for anisotropic materials

- heat flux ($T_1 = T$ und $T_2 = 0$)

$$\Phi = \langle S_z \rangle = \int \frac{\mathrm{d}\,\omega}{2\pi} \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/(k_\mathrm{B} T)} - 1} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \mathcal{T}(\omega, \boldsymbol{\kappa}; d)$$

- Transmission coefficient

$$\mathcal{T}(\omega, \boldsymbol{\kappa}; d) = \begin{cases} \mathrm{Tr}\left[(1 - \mathbb{R}_2^\dagger \mathbb{R}_2)\mathbb{D}^{12}(1 - \mathbb{R}_1^\dagger \mathbb{R}_1)\mathbb{D}^{12^\dagger}\right], & \kappa < \frac{\omega}{c} \\ \mathrm{Tr}\left[(\mathbb{R}_2^\dagger - \mathbb{R}_2)\mathbb{D}^{12}(\mathbb{R}_1 - \mathbb{R}_1^\dagger)\mathbb{D}^{12^\dagger}\right] e^{-2|k_z|d}, & \kappa > \frac{\omega}{c} \end{cases}$$

- Reflection matrix ($i = 1, 2$)

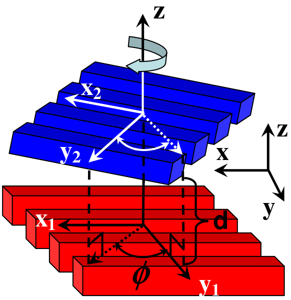
$$\mathbb{R}_i = \begin{bmatrix} r_i^{\mathrm{s,s}}(\omega, \kappa) & r_i^{\mathrm{s,p}}(\omega, \kappa) \\ r_i^{\mathrm{p,s}}(\omega, \kappa) & r_i^{\mathrm{p,p}}(\omega, \kappa) \end{bmatrix},$$

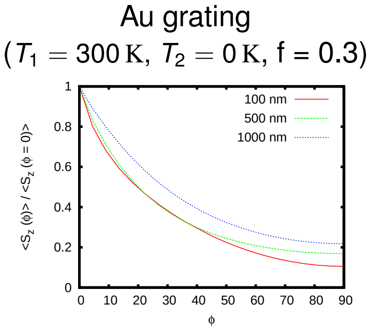
- 'Fabry-Pérot denominator'

$$\mathbb{D}^{12} = [\mathbb{1} - \mathbb{R}_1 \mathbb{R}_2 \exp(2\mathrm{i} k_z d)]^{-1}$$

Biehs, Rosa, Ben-Abdallah, Joulain, Greffet, Opt. Expr. **19**, A1088-A1103 (2011)

heat flux modulation





Biehs, Rosa, Ben-Abdallah, APL **98**, 243102 (2011)

- effective description:

$$\epsilon_{x,z} = \epsilon_{\mathrm{h}_i}(1 - f_i) + f_i$$

$$\epsilon_y = \frac{\epsilon_{\mathrm{h}_i}}{(1 - f_i) + f_i \epsilon_{\mathrm{h}_i}}$$

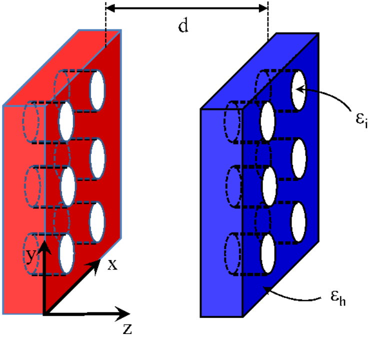
- beyond EMT:

  Rodriguez et al., Phys. Rev. Lett. 107, 114302 (2011)

  R. Guérout et al., Phys. Rev. B 85, 180301(R) (2012)

  J. Lussange et al., Phys. Rev. B 86, 085432 (2012)

Maxwell-Garnett



- Reflection matrix

$$r_{s,s}(\omega,\kappa) = \frac{k_z - k_z^s}{k_z + k_z^s}$$

$$r_{p,p}(\omega,\kappa) = \frac{\epsilon_\parallel k_z - k_z^p}{\epsilon_\parallel k_z + k_z^p}$$

$$r_{s,p} = r_{p,s} = 0$$

- wave vector components

$$k_z^s = \sqrt{\epsilon_\parallel \frac{\omega^2}{c^2} - \kappa^2}$$

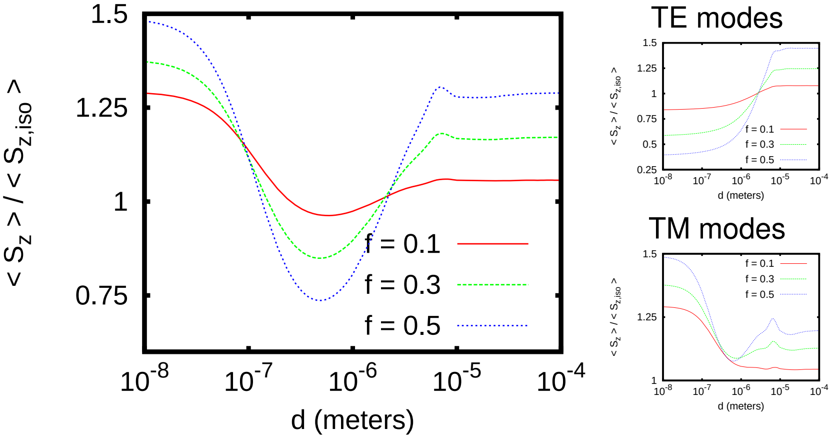
$$k_z^p = \sqrt{\epsilon_\parallel \frac{\omega^2}{c^2} - \frac{\epsilon_\parallel}{\epsilon_\perp}\kappa^2}$$

- Maxwell-Garnett

$$\epsilon_\parallel = \epsilon_h \frac{\epsilon_i(1+f) + \epsilon_h(1-f)}{\epsilon_i(1-f) + \epsilon_h(1+f)}$$

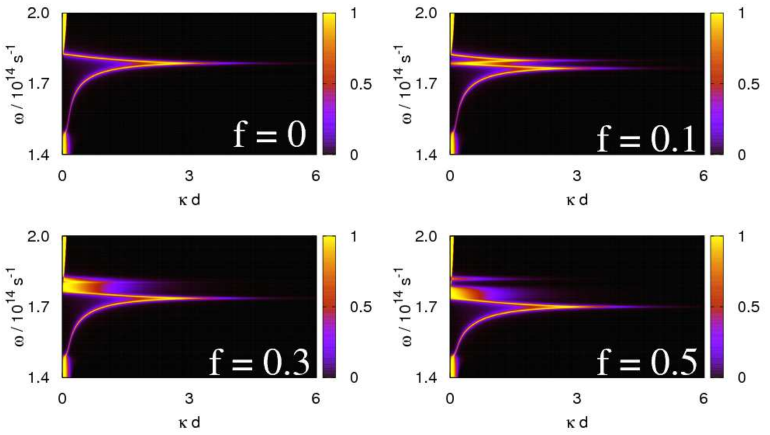
$$\epsilon_\perp = \epsilon_h(1-f) + \epsilon_i f$$

Heat flux for nanoporous SiC



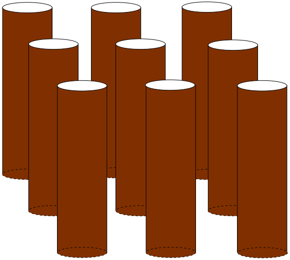
Biehs, Rosa, Ben-Abdallah, Joulain, Greffet, Opt. Expr. **19**, A1088-A1103 (2011)

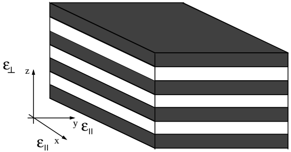
transmission coefficient $\mathcal{T}_{\rm p}(\omega, \kappa; d = 100\,{\rm nm})$



Biehs, Rosa, Ben-Abdallah, Joulain, Greffet, Opt. Expr. **19**, A1088-A1103 (2011)

uni-axial media





- reflection matrix

$$r_{\mathrm{s,s}}(\omega,\kappa) = \frac{k_z - k_z^{\mathrm{s}}}{k_z + k_z^{\mathrm{s}}}$$

$$r_{\mathrm{p,p}}(\omega,\kappa) = \frac{\epsilon_\parallel k_z - k_z^{\mathrm{p}}}{\epsilon_\parallel k_z + k_z^{\mathrm{p}}}$$

$$r_{\mathrm{s,p}} = r_{\mathrm{p,s}} = 0$$

- wave vector components

$$k_z^{\mathrm{s}} = \sqrt{\epsilon_\parallel \frac{\omega^2}{c^2} - \kappa^2}$$

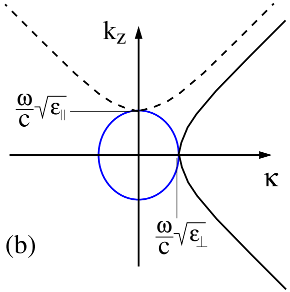
$$k_z^{\mathrm{p}} = \sqrt{\epsilon_\parallel \frac{\omega^2}{c^2} - \frac{\epsilon_\parallel}{\epsilon_\perp}\kappa^2}$$

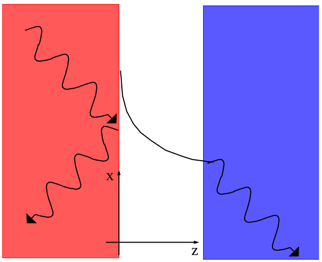
indefinite/hyperbolic Media

- dispersion relation for extra-ordinary waves:

$$\frac{\kappa^2}{\epsilon_\perp} + \frac{k_z^2}{\epsilon_\parallel} = \frac{\omega^2}{c^2}$$

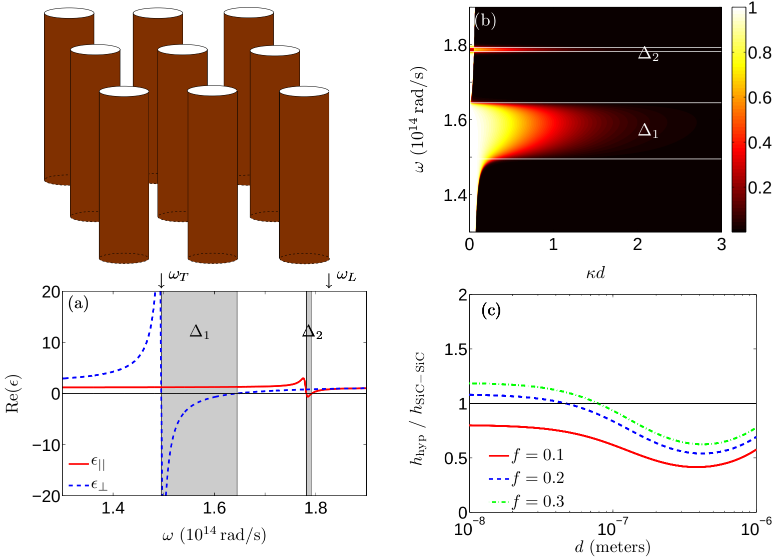
- hyperbolic, if: [$\epsilon_\perp > 0, \epsilon_\parallel < 0$] or [$\epsilon_\perp < 0, \epsilon_\parallel > 0$]





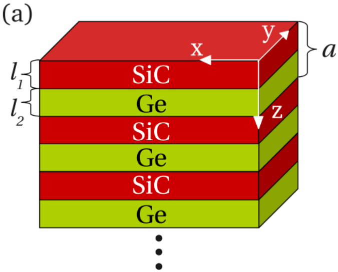
Smith und Schurig, PRL **90**, 077405 (2003)

SiC Nanowire medium (Maxwell-Garnett)



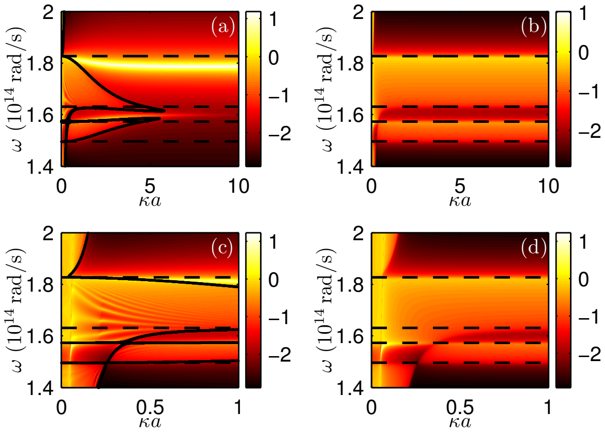
Biehs, Tschikin und Ben-Abdallah, PRL **109**, 104301 (2012)

multilayer phononic-polaritonic hyperbolic medium



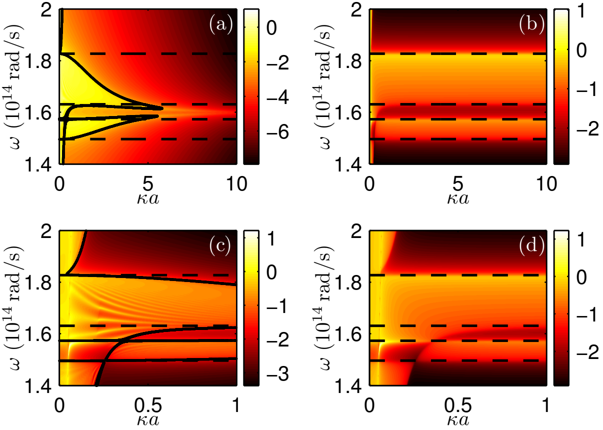
- (easier to fabricate)
- exact S-matrix caclulations
- comparison between effective and exact results

SiC/Ge multilayer hyperbolic medium (SiC on top)



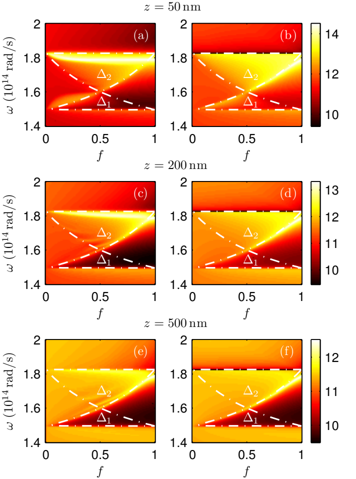
$1 - |r_p|$ for $\kappa < k_0$; $\mathrm{Im}(r_p)$ for $\kappa > k_0$; $a = 100$nm, $f = 0.4$

SiC/Ge multilayer hyperbolic medium (Ge on top)

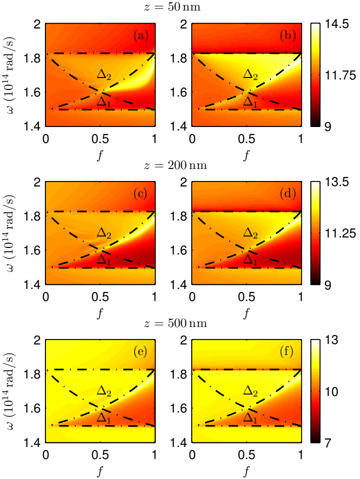


$1 - |r_p|$ for $\kappa < k_0$; $\mathrm{Im}(r_p)$ for $\kappa > k_0$; $a = 100$nm, $f = 0.4$

spectral heat transfer coefficient (SiC/Ge multilayer)

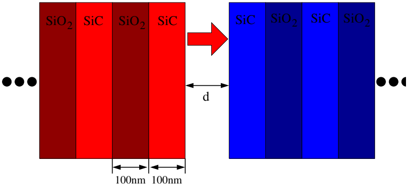


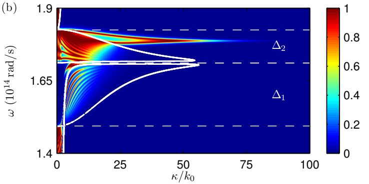
SiC on top

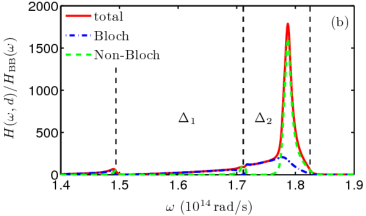


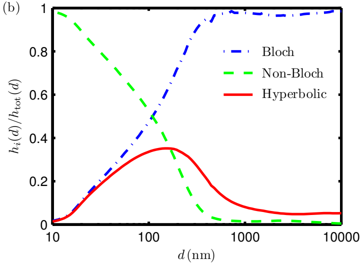
Ge on top

SiC/SiO_2 multilayer with SiC on top





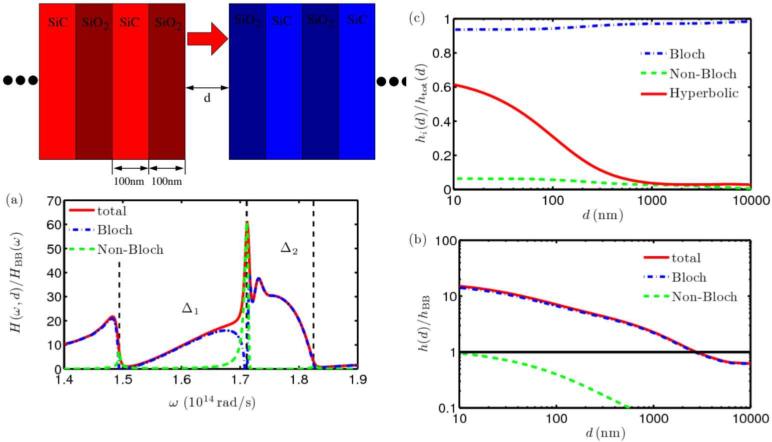




Y. Guo, C. L. Cortes, S. Molesky, and Z. Jacob, APL **101**, 131106 (2012)

Biehs, Tschikin, Messina, und Ben-Abdallah, Appl. Phys. Lett. **102**, 131106 (2013)

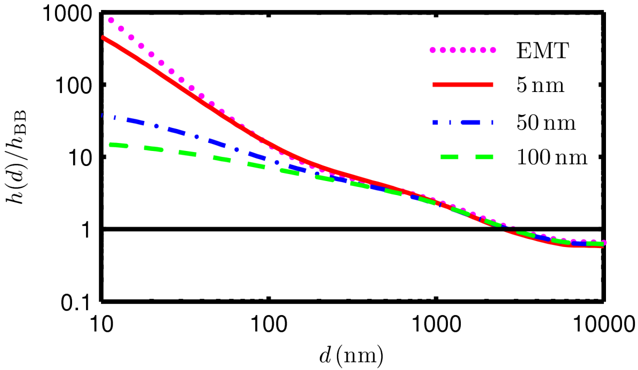
SiC/SiO$_2$ with SiO$_2$ on top



Y. Guo, C. L. Cortes, S. Molesky, and Z. Jacob, APL **101**, 131106 (2012)

Biehs, Tschikin, Messina, und Ben-Abdallah, Appl. Phys. Lett. **102**, 131106 (2013)

SiC/SiO_2 multilayer with SiO_2 on top



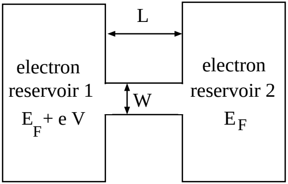
Biehs, Tschikin, Messina, und Ben-Abdallah, Appl. Phys. Lett. **102**, 131106 (2013)

Summary

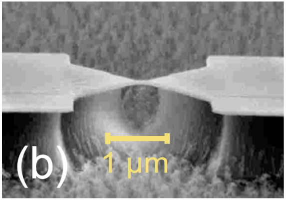
- Super-Planckian thermal radiation
  - frustrated modes
  - surface modes
- Landauer-form
- anisotropic metamaterials
  - control/tayloring thermal radiation
- hyperbolic near-field emitters
  - broad-band thermal radiation
  - $\frac{1}{d^2}$-like frustrated mode contribution

Thank you very much for your attention!!!

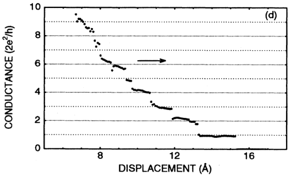
Ladungstransport in mesoskopischen Systemen



- makroskopisch, $I = \Gamma V$:
  - $\Gamma \propto \frac{W}{L}$
- mesoskopisch, $L < L_m, L_\varphi$:
  - $\Gamma \to \Gamma_C$ für $L \to 0$
  - $\Gamma \to 0$ stufenweise für $W \to 0$

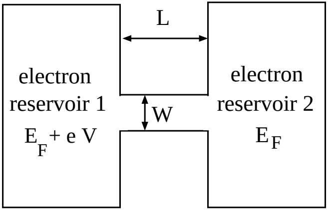


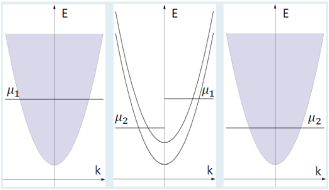
Wu et al., PRB **78**, 235421 (2008)



Brandbyge et al., PRB **52**, 8499 (1995)

Landauer Formel



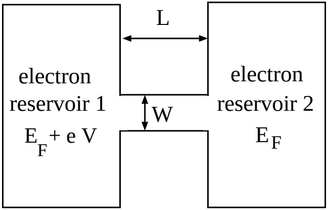


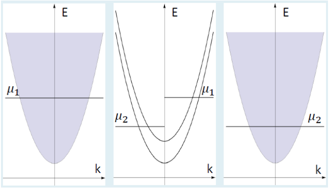
$$E_R = \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2 k_\perp^2}{2m}$$
$$E_L = \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2 \pi^2 n^2}{2mW^2}$$

- Ladungstransport im mesoskopischen Bereich (Landauer)

$$I = \Gamma V = \frac{2e^2}{h} MV$$

Landauer Formel





$$E_R = \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2 k_\perp^2}{2m}$$
$$E_L = \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2 \pi^2 n^2}{2mW^2}$$

- Ladungstransport im mesoskopischen Bereich (Landauer)

$$I = \Gamma V = \frac{2e^2}{h}\left[\sum_n T_n\right] V$$

Landauer-like expression for the heat flux

- final result

$$\Phi = \frac{\pi^2 k_{\rm B}^2 T}{3h}\left[\sum_{i={\rm s,p}} \int \frac{{\rm d}^2\kappa}{(2\pi)^2} \overline{\mathcal{T}}_i\right] \Delta T \qquad \leftrightarrow \qquad I = \Gamma V = \frac{2e^2}{h}\left[\sum_n T_n\right] V$$

- sum over all modes per area L^2

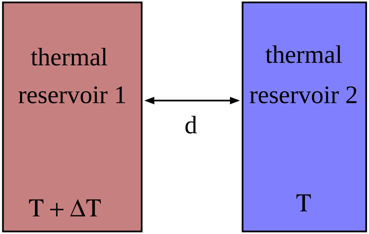
$$\frac{1}{L^2}\int \frac{{\rm d}^2\kappa}{\left(\frac{2\pi}{L}\right)^2} \leftrightarrow \frac{1}{L^2}\sum_{n_x,n_y}$$

- quantum of thermal conductance

$$\frac{\pi^2 k_{\rm B}^2 T}{3h} = \frac{\pi k_{\rm B}^2 T}{6\hbar} = 0.95 \cdot 10^{-12}\, T\, \frac{\rm W}{\rm K}$$

Pendry, J. Phys. A **16**, 2161 (1983)

Wärmestrom



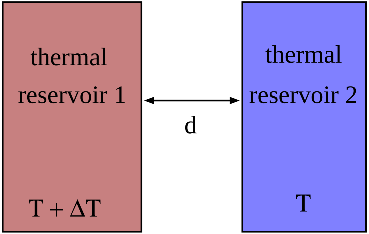
$$b(T) = \frac{1}{\mathrm{e}^{(\hbar\omega)/(k_\mathrm{B} T)} - 1}$$

- resultierender Wärmestrom

$$\Phi = \int \frac{\mathrm{d}\,\omega}{2\pi} \hbar\omega \left[b(T + \Delta T) - b(T) \right] \sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \mathcal{T}_i$$

$$\Delta T \ll T$$

Wärmestrom



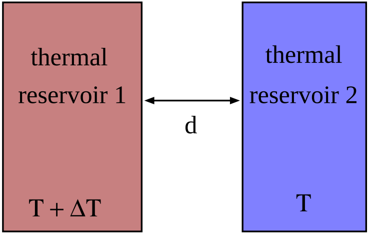
$$b(T) = \frac{1}{e^{(\hbar\omega)/(k_B T)} - 1}$$

- resultierender Wärmestrom

$$\Phi = \int \frac{d\,\omega}{2\pi} \hbar\omega \left(\frac{\partial}{\partial T} b(T) \right) \sum_{i=s,p} \int \frac{d^2\kappa}{(2\pi)^2} \mathcal{T}_i \Delta T$$

$$\Delta T \ll T$$

Wärmestrom



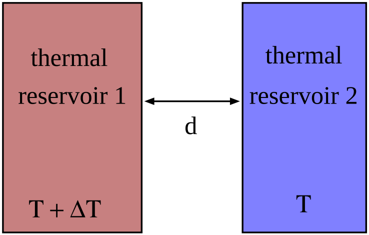
$$b(T) = \frac{1}{e^{(\hbar\omega)/(k_B T)} - 1}$$

- resultierender Wärmestrom

$$\Phi = \frac{k_B^2 T}{h} \int \mathrm{d}u\, u^2 \left(-\frac{\partial}{\partial u} b(u) \right) \sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2 \kappa}{(2\pi)^2} \mathcal{T}_i \Delta T$$

Substitution $u = \hbar\omega/(k_B T)$

Wärmestrom



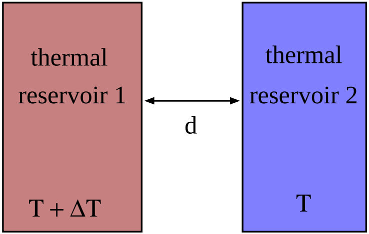
$$b(T) = \frac{1}{e^{(\hbar\omega)/(k_B T)} - 1}$$

- resultierender Wärmestrom

$$\Phi = \frac{k_B^2 T}{h} \sum_{i=s,p} \int \frac{d^2\kappa}{(2\pi)^2} \left[\int du\, u^2 \left(-\frac{\partial}{\partial u} b(u) \right) \mathcal{T}_i \right] \Delta T$$

Vertauschen der Integrale

Wärmestrom



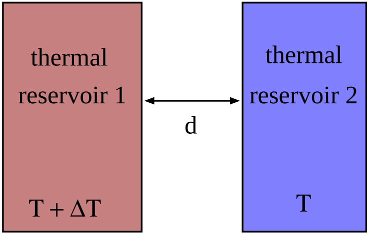
$$b(T) = \frac{1}{\mathrm{e}^{(\hbar\omega)/(k_\mathrm{B} T)} - 1}$$

- resultierender Wärmestrom

$$\Phi = \frac{k_\mathrm{B}^2 T}{h} \sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \left[\frac{\int \mathrm{d}u\, u^2 \left(-\frac{\partial}{\partial u} b(u) \right) \mathcal{T}_i}{\int \mathrm{d}u\, u^2 \left(-\frac{\partial}{\partial u} b(u) \right)} \int \mathrm{d}u\, u^2 \left(-\frac{\partial}{\partial u} b(u) \right) \right] \Delta T$$

Eins einfügen $\int \mathrm{d}u\, u^2 \left(-\frac{\partial}{\partial u} b(u) \right) = \pi^2/3$

Wärmestrom



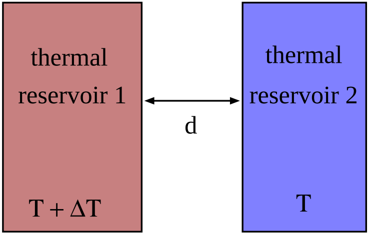
$$b(T) = \frac{1}{e^{(\hbar\omega)/(k_B T)} - 1}$$

- resultierender Wärmestrom

$$\Phi = \frac{k_B^2 T}{h} \sum_{i=s,p} \int \frac{d^2\kappa}{(2\pi)^2} \left[\frac{\int du\, u^2 \left(-\frac{\partial}{\partial u} b(u)\right) \mathcal{T}_i}{\int du\, u^2 \left(-\frac{\partial}{\partial u} b(u)\right)} \frac{\pi^2}{3} \right] \Delta T$$

Eins einfügen $\int du\, u^2 \left(-\frac{\partial}{\partial u} b(u)\right) = \pi^2/3$

Wärmestrom



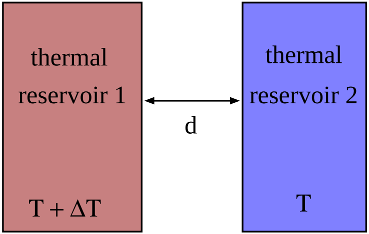
$$b(T) = \frac{1}{\mathrm{e}^{(\hbar\omega)/(k_\mathrm{B}T)} - 1}$$

- resultierender Wärmestrom

$$\Phi = \frac{\pi^2 k_\mathrm{B}^2 T}{3h} \sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \left[\frac{\int \mathrm{d}u\, u^2 \left(-\frac{\partial}{\partial u} b(u)\right) \mathcal{T}_i}{\int \mathrm{d}u\, u^2 \left(-\frac{\partial}{\partial u} b(u)\right)} \right] \Delta T$$

$$\text{definiere } \overline{\mathcal{T}}_i \equiv \frac{\int \mathrm{d}u\, u^2 \left(\frac{\partial}{\partial u} b(u)\right) \mathcal{T}_i}{\int \mathrm{d}u\, u^2 \left(\frac{\partial}{\partial u} b(u)\right)}$$

Wärmestrom



$$b(T) = \frac{1}{e^{(\hbar\omega)/(k_\mathrm{B} T)} - 1}$$

- resultierender Wärmestrom

$$\Phi = \frac{\pi^2 k_\mathrm{B}^2 T}{3h} \left[\sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \overline{\mathcal{T}}_i \right] \Delta T$$

$$\text{definiere } \overline{\mathcal{T}}_i \equiv \frac{\int \mathrm{d}u\, u^2 \left(\frac{\partial}{\partial u} b(u) \right) \mathcal{T}_i}{\int \mathrm{d}u\, u^2 \left(\frac{\partial}{\partial u} b(u) \right)}$$

Landauerartiger Ausdruck für Wärmestrahlung

- Endresultat

$$\Phi = \frac{\pi^2 k_\mathrm{B}^2 T}{3h}\left[\sum_{i=\mathrm{s,p}} \int \frac{\mathrm{d}^2\kappa}{(2\pi)^2} \overline{\mathcal{T}}_i\right] \Delta T \qquad \leftrightarrow \qquad I = \Gamma V = \frac{2e^2}{h}\left[\sum_n T_n\right] V$$

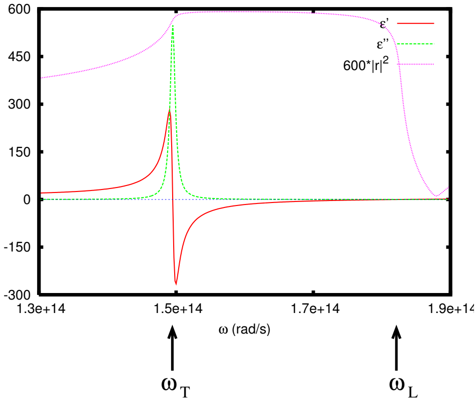
- Summe über alle Moden pro Fläche L^2

$$\frac{1}{L^2} \int \frac{\mathrm{d}^2\kappa}{\left(\frac{2\pi}{L}\right)^2} \leftrightarrow \frac{1}{L^2} \sum_{n_x, n_y}$$

- mittlerer Transmissionskoeffizient

$$\overline{\mathcal{T}}_i = \frac{\int \mathrm{d}u\, u^2 \left(\frac{\partial}{\partial u} b(u)\right) \mathcal{T}_i}{\int \mathrm{d}u\, u^2 \left(\frac{\partial}{\partial u} b(u)\right)}$$

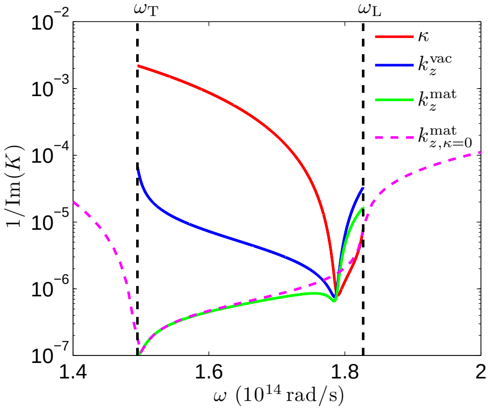
Permittivität von SiC



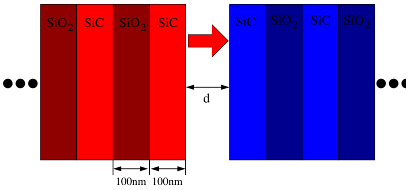
- Reststrahlenformel: $\epsilon(\omega) = \epsilon_\infty \left(\frac{\omega_L^2 - \omega^2 - i\omega\gamma}{\omega_T^2 - \omega^2 - i\omega\gamma} \right)$

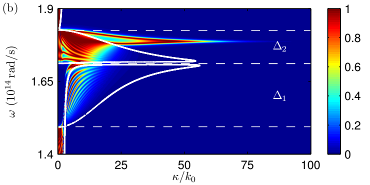
  (parameters: $\epsilon_\infty = 6.7, \omega_T = 1.495 \cdot 10^{14} \mathrm{rad/s}, \omega_L = 1.827 \cdot 10^{14} \mathrm{rad/s}, \gamma = 0.9 \cdot 10^{12}$)

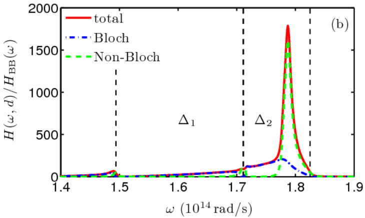
Eindringtiefe/Propagationslänge SPhP (SiC)

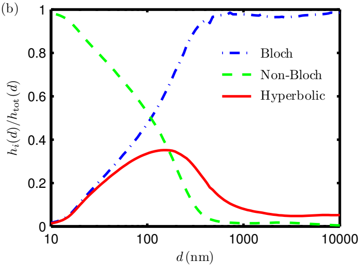


SiC/SiO_2 multilayer with SiC on top



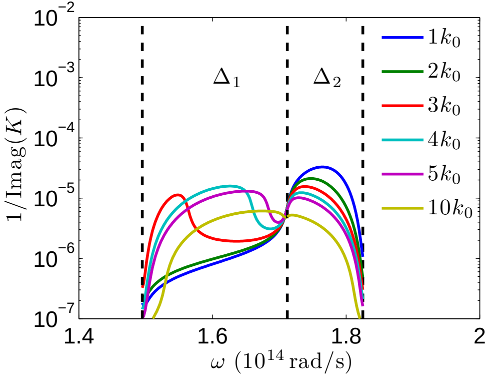




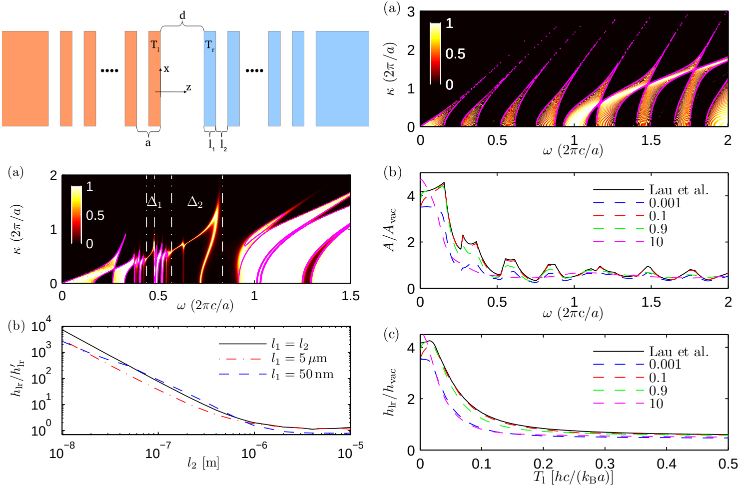


Biehs, Tschikin, Messina, und Ben-Abdallah, to be published in APL (2013)

Propagationslänge hyperbolische Moden (SiC)



Photonic Crystal



Tschikin, Biehs, Ben-Abdallah, Phys. Lett. A **376**, 3462 (2012).

- Clausius-Mosotti relation

$$\frac{\epsilon_{\text{eff}} - 1}{\epsilon_{\text{eff}} + 2} = \frac{4\pi}{3}\frac{N}{V}\alpha$$

- Polarizability (spherical inclusions with radius R)

$$\alpha = R^3 \frac{\epsilon_{\text{i}} - 1}{\epsilon_{\text{i}} + 2}$$

- Zusammen:

$$\frac{\epsilon_{\text{eff}} - 1}{\epsilon_{\text{eff}} + 2} = \frac{N\frac{4\pi R^3}{3}}{V}\frac{\epsilon_{\text{i}} - 1}{\epsilon_{\text{i}} + 2}$$
$$= f\frac{\epsilon_{\text{i}} - 1}{\epsilon_{\text{i}} + 2}$$