

Near-field heat transfer and thermal emission control with complex plasmonic systems

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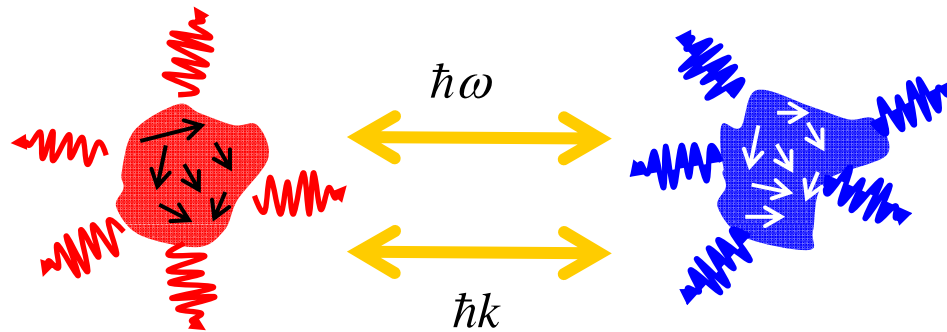


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Fluctuating and uncorrelated local sources lead to :

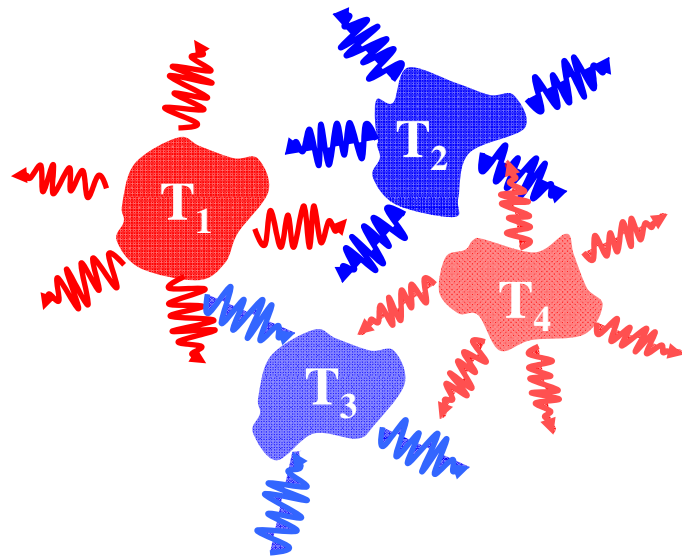
- energy and momentum exchange

These exchanges are well described by the fluctuational electrodynamic theory (Rytov)

applied to

- Radiative heat transfer (Polder and Van Hove)
- Casimir force (Lifschitz)

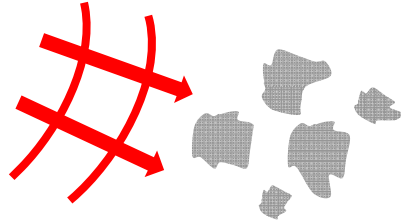
Open questions



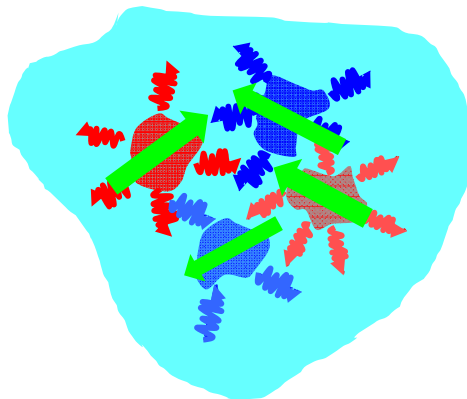
- How does the heat and momentum transport for a collection of individual objects in mutual interaction look like?
- Are there specific many body effects?
- What relation between disorder and heat transport? etc...

Needs a N-body heat and momentum transfer theory

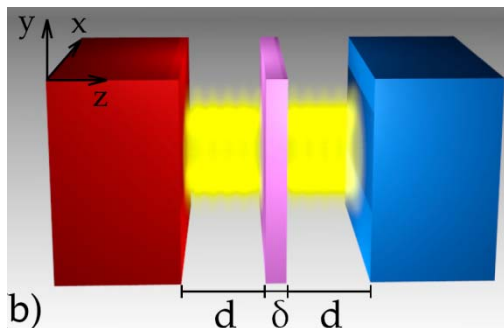
Outline



- I) Engineering the light absorption spectrum from multiple scattering interactions in dipolar systems

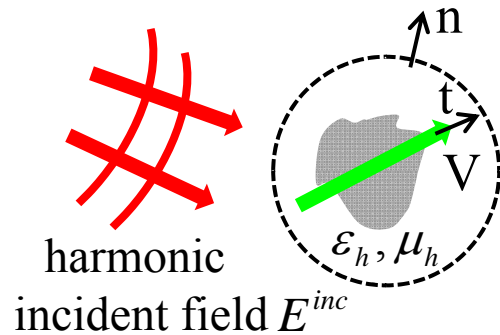


- II) Near-field heat transfer in many body systems : dipolar approximation



- III) Many body heat transfer beyond the dipolar approximation

Engineering light absorption from multiple scattering interactions : absorption by a simple particle



Rate of doing work by the em field in a volume V :

$$\wp(\omega) = \frac{1}{2} \int_V \text{Re}(j^* \cdot E) dV$$

Poynting theorem (energy conservation):

$$\frac{1}{2} \int_V j^* \cdot E dV = -\frac{1}{2} \int_V \nabla \cdot [E \times H^*] dV + i \frac{\omega}{2} \int_V (\mu_h |H|^2 - \epsilon_h^* |E|^2) dV$$

In a transparent host medium

$$\wp(\omega) = -\underbrace{\frac{1}{2} \int_S \text{Re}[E \times H^* \cdot n] dS}_{\text{Poynting flux}}$$

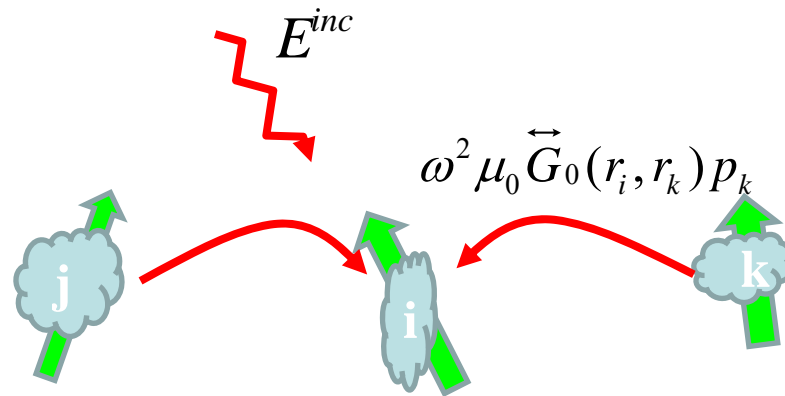
Also we have

$$E(r) = E^{ext} + i\omega\mu_0 \int_V \vec{G}_0(r, r') j(r') dr' + j(r) = -i\omega p \delta(r)$$

Thus

$$\wp(\omega) = -\frac{\omega}{2} \text{Im}[p^* E^{inc}(0)] - \frac{\omega^3 |p|^2 \mu_0}{2} t \cdot \text{Im}[\vec{G}_0(0,0)] \cdot t$$

Engineering light absorption from multiple scattering interactions : absorption by a set of particles

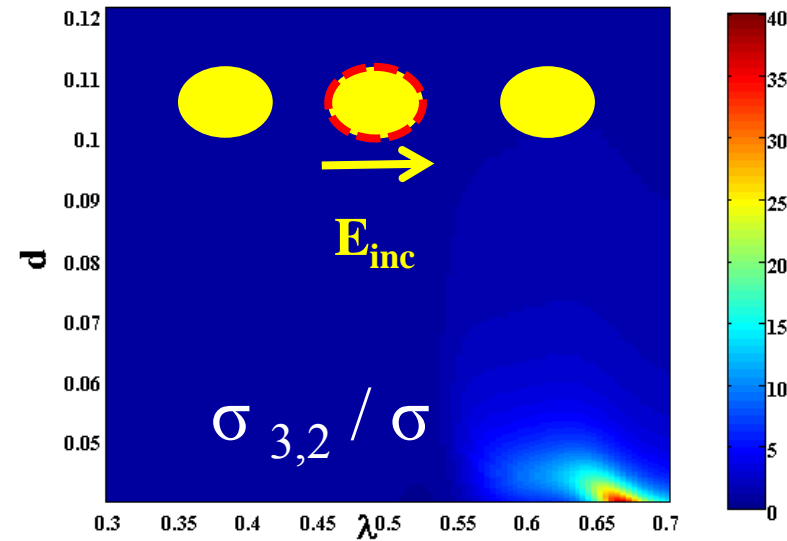
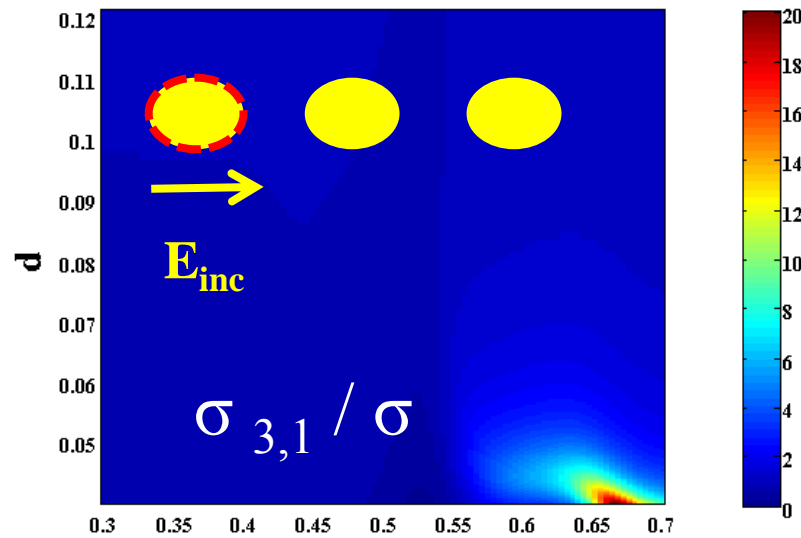
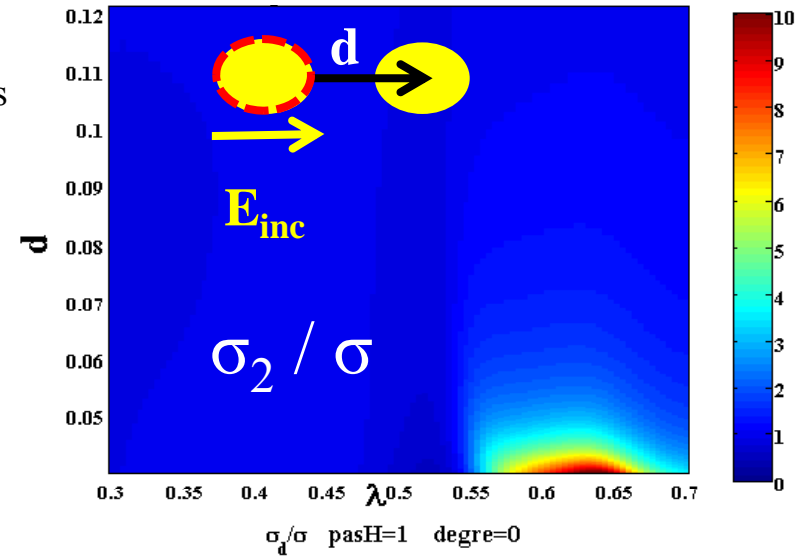
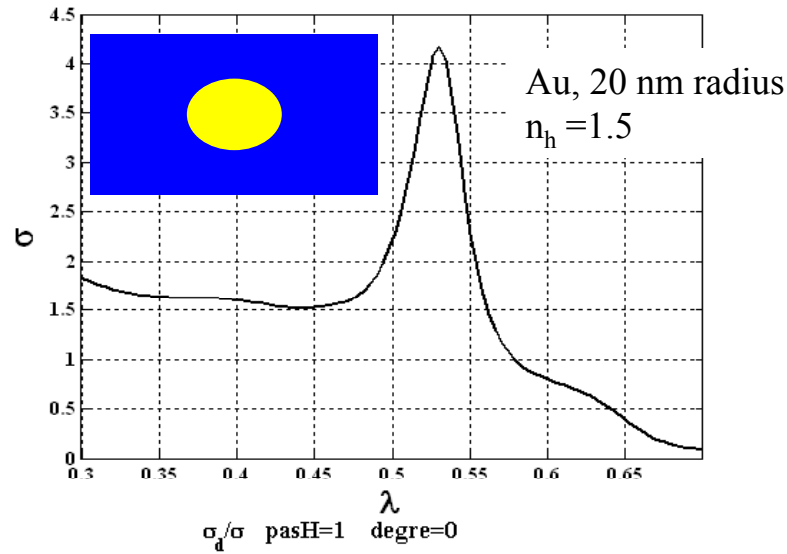


External field on the particle i: $E_i^{ext} = E_i^{inc} + \omega^2 \mu_0 \sum_{j \neq i} \vec{G}_0(r_i, r_j) p_j$

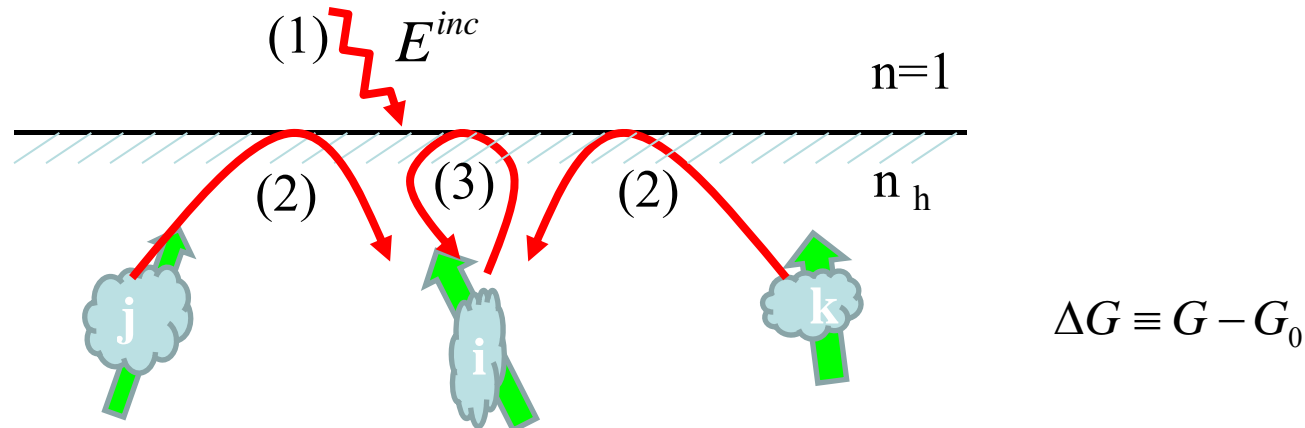
Thus

$$\rho_i(\omega) = -\frac{\omega}{2} \text{Im}[p_i^* E_i^{inc}] - \frac{\omega^3 \mu_0}{2} |p_i|^2 t_i \cdot \vec{G}_0(r_i, r_i) \cdot t_i - \underbrace{\frac{\omega^3 \mu_0}{2} \text{Im}[p_i^* \sum_{j \neq i} \vec{G}_0(r_i, r_j) \cdot p_j]}_{\text{Multiple interactions}}$$

Engineering light absorption : dressed absorption in dipolar chains



Engineering light absorption: design of an absorber



Find the optimal distribution of dipoles to get an absorption spectrum target

$$E_i^{ext} = \underbrace{E_i^{inc}}_{(1)} + \underbrace{\omega^2 \mu_0 \sum_{j \neq i} \vec{G}(r_i, r_j) p_j}_{(2)} + \underbrace{\Delta \vec{G}(r_i, r_i) p_i}_{(3)}$$

$$\wp_i(\omega) = -\frac{\omega}{2} \text{Im}[p_i^* E_i^{ext}] - \frac{\omega^3 \mu_0}{2} p_i^* \cdot \text{Im}[\vec{G}_0(r_i, r_i)] \cdot p_i$$

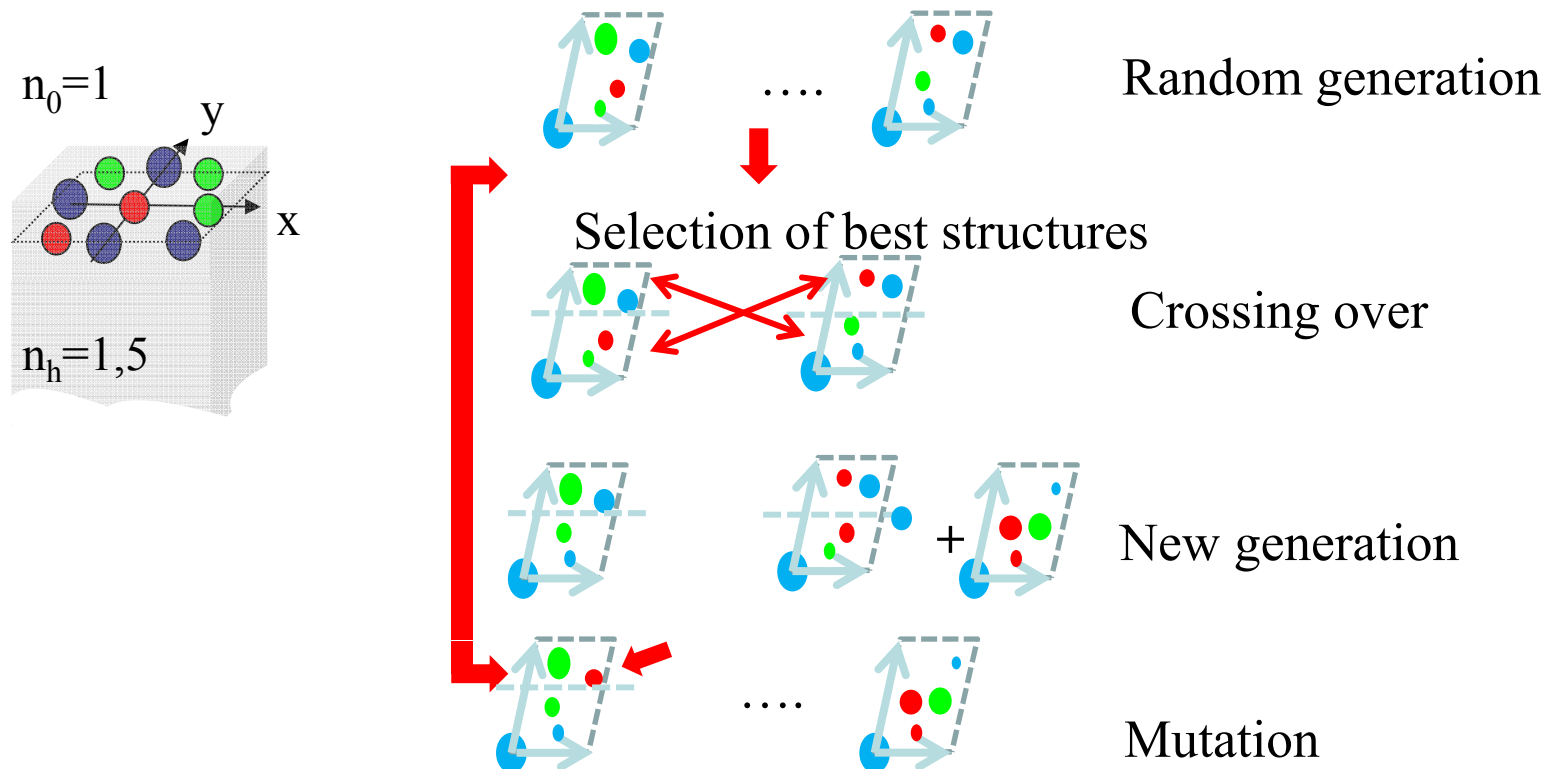
$$a = 1 - T - R \equiv \frac{\sum_i \wp_i}{A \Phi_{inc}}$$

Engineering light absorption: design of an absorber

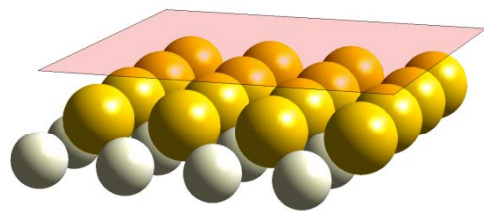
Find the position and the size of particles in the unit cell of a n-ary lattice so that:

$$\int_{\lambda_{\min}}^{\lambda_{\max}} a(\lambda) d\lambda \rightarrow \max$$

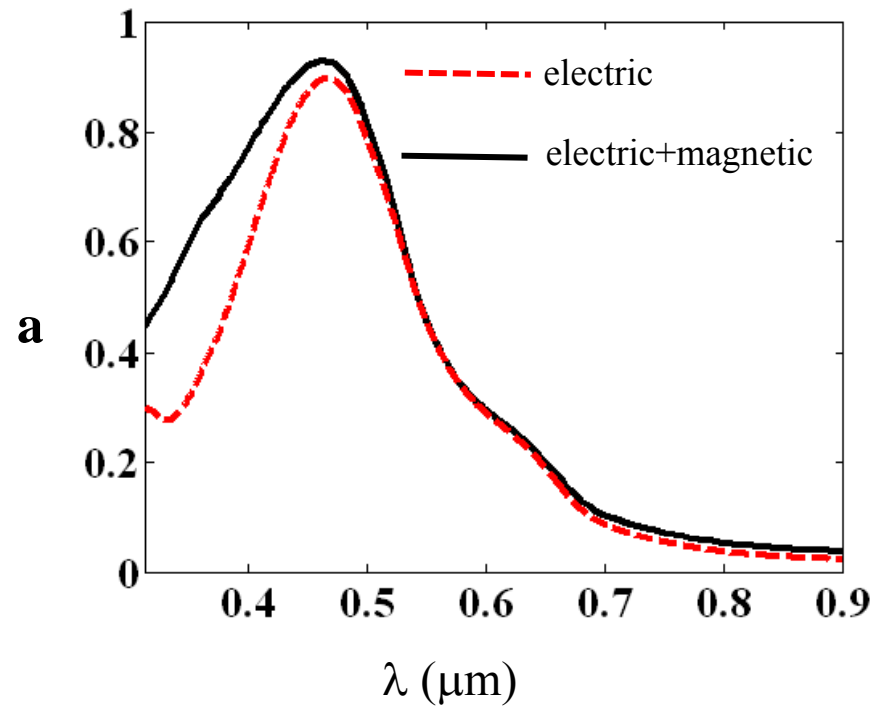
Evolutionary algorithm:



Engineering light absorption: design of an absorber

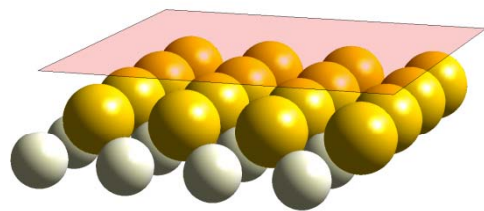


Au-Ag lattice

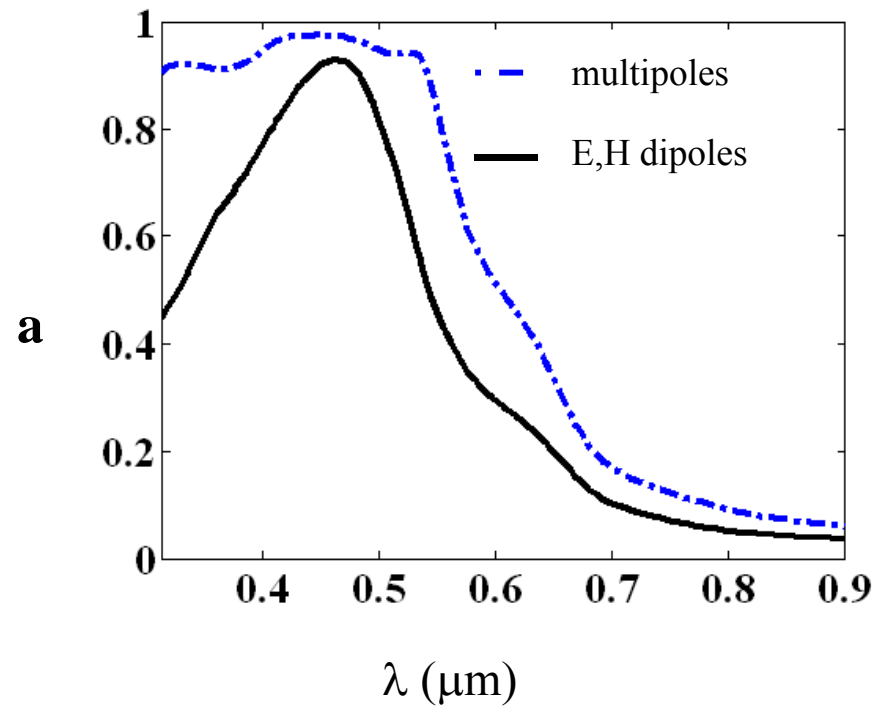


dipolar approximation

Engineering light absorption: design of an absorber

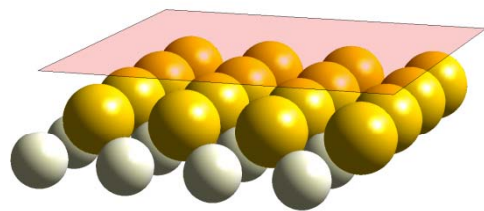


Au-Ag lattice

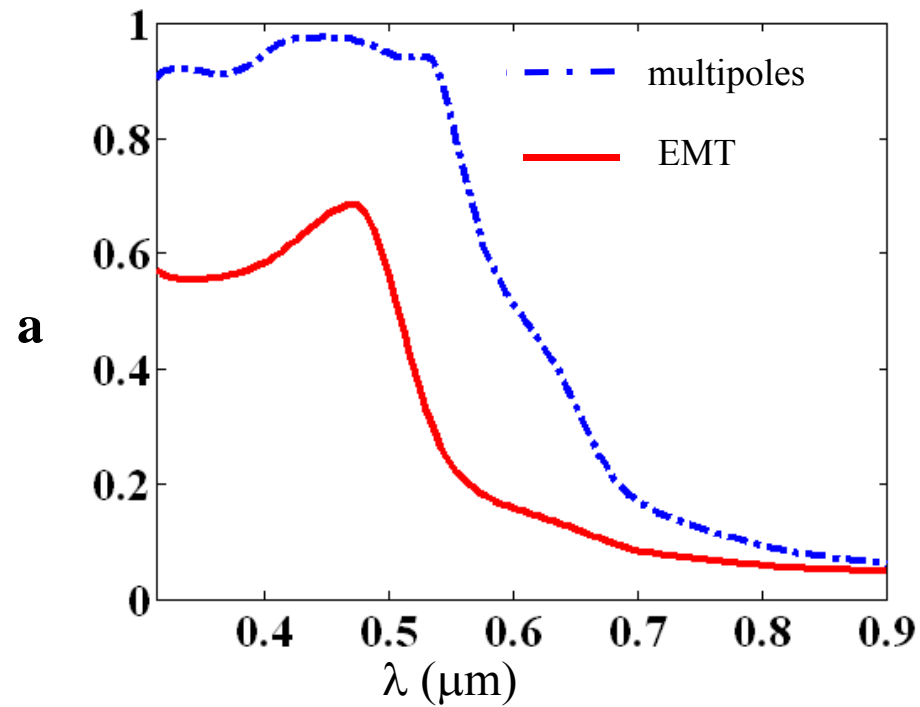


Multipolar calculation

Engineering light absorption: design of an absorber

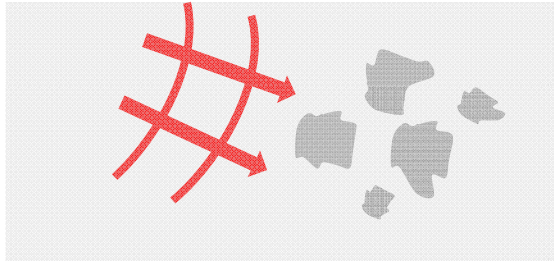


Au-Ag lattice

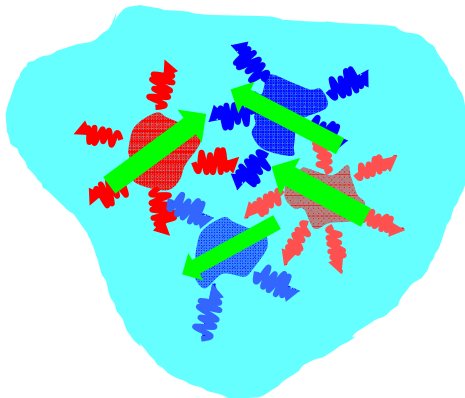


Effective medium theory

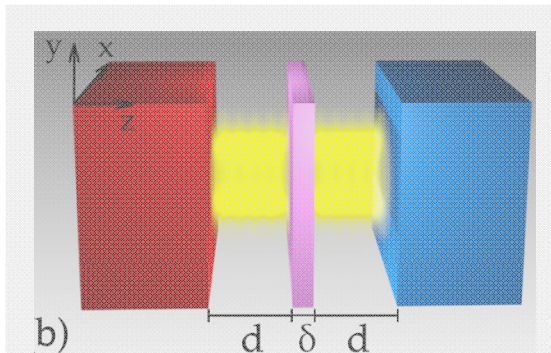
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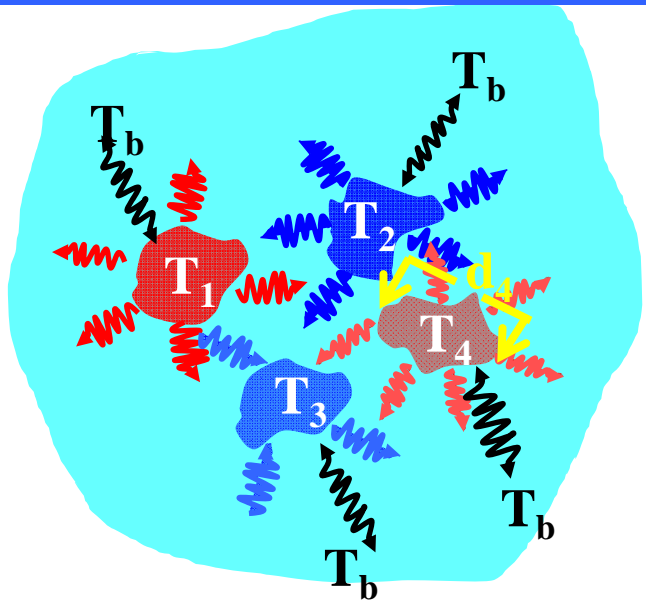


- II) Near-field heat transfer in many body systems : dipolar approximation



- III) Many body heat transfer beyond the dipolar approximation

Near-field heat transfer in many body systems: dipolar approximation



Local field

$$E(r) = E_b(r) + \omega^2 \mu_0 \sum_j \vec{G}_0(r, r_j) p_j$$

- $d_i < \min(\lambda_{T_j})$ with $\lambda_{T_j} = c\hbar / (k_B T_j)$
- Objects exchange in far field with the bath



N fluctuating dipoles in mutual interaction
inside a bosonic field

Dipole moments

$$p_i = p_i^{fluc} + p_i^{ind} \quad \text{with} \quad p_i^{ind} = \varepsilon_0 \alpha_i [E_{b,i} + \sum_{j \neq i} \vec{G}_0(r_i, r_j) p_j]$$

$$\begin{pmatrix} p_1 \\ \vdots \\ p_N \end{pmatrix} = \overline{\overline{M}} \begin{pmatrix} p_1^{fluc} \\ \vdots \\ p_N^{fluc} \end{pmatrix} + \overline{\overline{N}} \begin{pmatrix} E_1^b \\ \vdots \\ E_N^b \end{pmatrix} \quad \begin{pmatrix} E_1 \\ \vdots \\ E_N \end{pmatrix} = \overline{\overline{O}} \begin{pmatrix} p_1^{fluc} \\ \vdots \\ p_N^{fluc} \end{pmatrix} + \overline{\overline{P}} \begin{pmatrix} E_1^b \\ \vdots \\ E_N^b \end{pmatrix}$$

where $\overline{\overline{M}}, \overline{\overline{N}}, \overline{\overline{O}}, \overline{\overline{P}}$ are function of $\vec{G}_0(r_i, r_j), \alpha_1, \dots, \alpha_N$

Energy balance

Time evolution of temperatures is governed by :

$$\rho_i C_i V_i \frac{dT_i}{dt} = \wp_i(t, T_1, \dots, T_N, T_b)$$

with $\wp_i = \int_{V_i} \langle j \cdot E \rangle dV_i \approx \left\langle \frac{d\tilde{p}_i}{dt}, E \right\rangle$ (dipolar approximation)

Using the convention $\tilde{p}_i(t) = 2 \operatorname{Re} \left(\int_0^\infty p_i(\omega) \frac{e^{-i\omega t}}{2\pi} d\omega \right)$

$$\wp_i = \int_0^\infty \frac{d\omega}{2\pi} \omega \int_0^\infty \frac{d\omega'}{2\pi} \operatorname{Im} \left[\left\langle p_i(\omega) E_i^\dagger(\omega') \right\rangle e^{-i(\omega-\omega')t} \right]$$

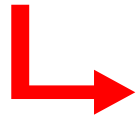
Assuming no correlation between the fluctuating dipole moments and the field of bath

$$\left\langle p_i(\omega) E_i^\dagger(\omega') \right\rangle = \sum_\alpha \sum_{jj'} \sum_{\beta\beta'} M_{ij,\alpha\beta} \left\langle p_{j,\beta}^{fluc}(\omega) p_{j'\beta'}^{fluc}(\omega') \right\rangle O_{ji,\beta'\alpha}^{\dagger} + N_{ij,\alpha\beta} \left\langle E_{j,\beta}^b(\omega) E_{j'\beta'}^b(\omega') \right\rangle P_{ji,\beta'\alpha}^{\dagger}$$

Energy balance

Using the FDT:

$$\left[\begin{aligned} \langle p_{j,\beta}^{fluc}(\omega) p_{j'\beta'}^{fluc\dagger}(\omega') \rangle &= 2\pi\hbar\epsilon_0 \delta_{jj'} \delta_{\beta\beta'} \chi_j \delta(\omega - \omega') (1 + 2n(\omega, T_j)) \quad \text{with} \quad \chi_j = \text{Im}(\alpha_j) - \frac{\omega^3}{6\pi c^3} |\alpha_j|^2 \\ \langle E_{j,\beta}^b(\omega) E_{j'\beta'}^{b\dagger}(\omega') \rangle &= 2\pi\hbar \frac{\omega^2}{\epsilon_0 c^2} \text{Im}(\vec{G}_{0,jj',\beta\beta'}) \delta(\omega - \omega') (1 + 2n(\omega, T_b)) \end{aligned} \right.$$



$$\langle p_i(\omega) E_i^\dagger(\omega') \rangle = 2\pi\delta(\omega - \omega')$$

$$\times \left[\hbar\epsilon_0 \sum_j \chi_j (1 + 2n(\omega, T_j)) \text{Tr}(M_{ij} O_{ji}^\dagger) + \frac{\hbar\omega^2}{\epsilon_0 c^2} (1 + 2n(\omega, T_b)) \text{Tr}(\overline{N} \text{Im}(\vec{G}_0) \overline{P}^\dagger)_{ii} \right]$$

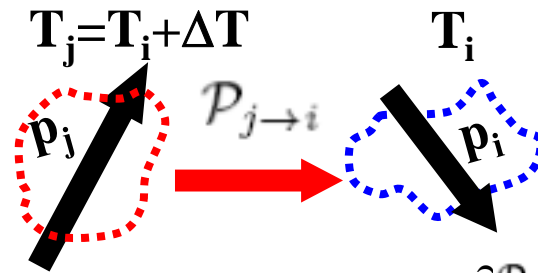
By considering exchanges with other dipoles, the thermal bath and emission :

$$\wp_i = \sum_{j \neq i} \wp_{j \rightarrow i} + \wp_{b \rightarrow i} - \wp_i^{emi}$$

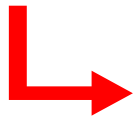
with

$$\wp_{j \rightarrow i} = 3 \int_0^\infty \frac{d\omega}{2\pi} \hbar \omega n(\omega, T_j) \tau_{ij}(\omega) \quad \tau_{ij}(\omega) = \frac{4}{3} \left(\frac{\omega}{c}\right)^4 \chi_i \chi_j \text{Tr}[\vec{G}(r_i, r_j) \vec{G}^\dagger(r_i, r_j)]$$

Landauer-like formulation of heat transport



Introducing the heat conductance between two objects $G_{ij} = \frac{\partial \mathcal{P}_{j \rightarrow i}}{\partial T_j}$ (Linearization of exchanged power)



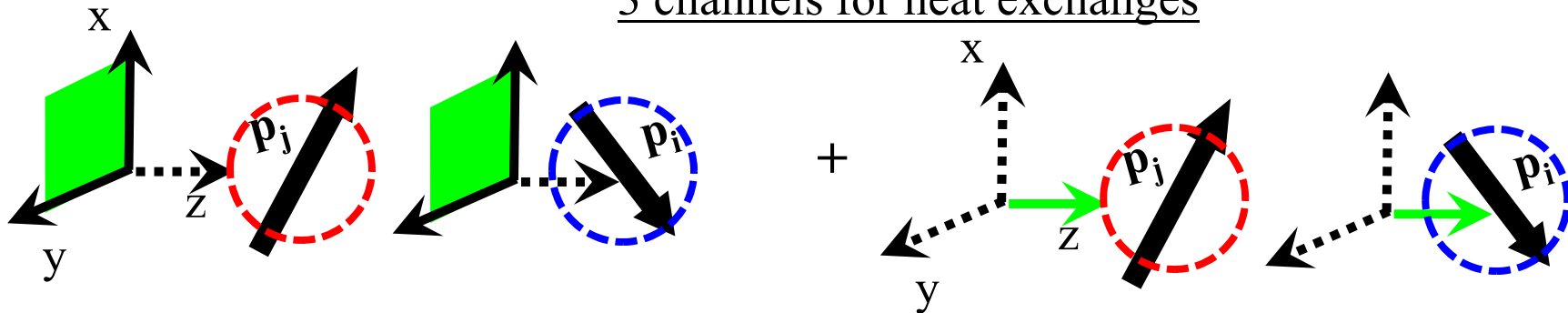
$$\mathcal{P}_{j \rightarrow i} = G_{ij} \Delta T = 3 \left(\frac{\pi^2 k_B^2 T_i}{3h} \right) \bar{\tau}_{ij} \Delta T$$

(Pendry, Math. Gen., 1983
Schwab, Nature 2000)

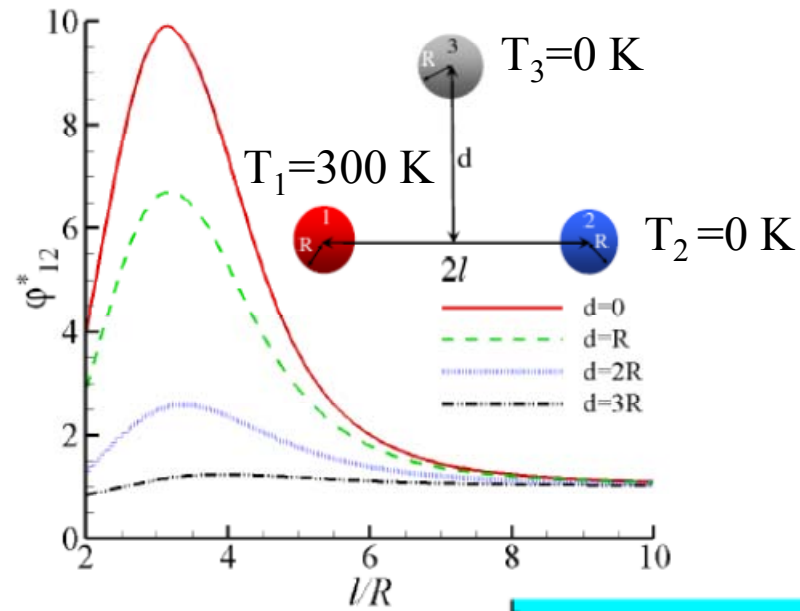
$$\bar{\tau}_{i,j} = \frac{3}{\pi^2} \int dx \frac{x^2 e^{-x}}{(e^x - 1)^2} \mathcal{T}_{i,j}$$

Mean transmission coefficient

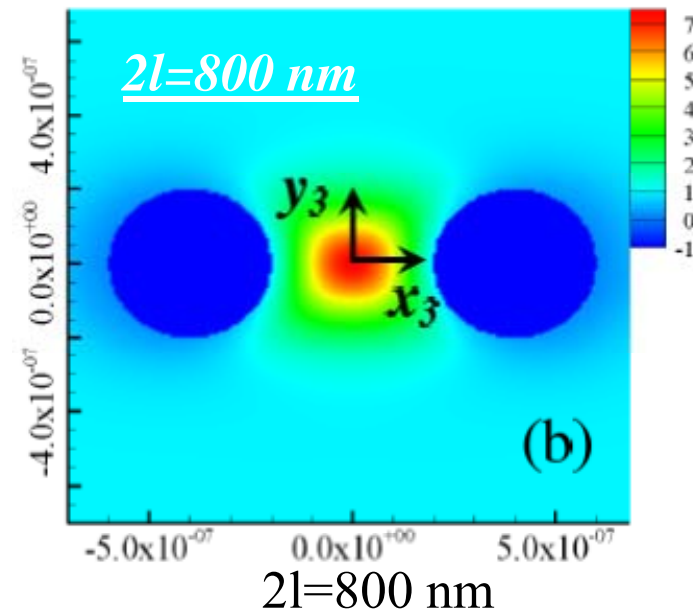
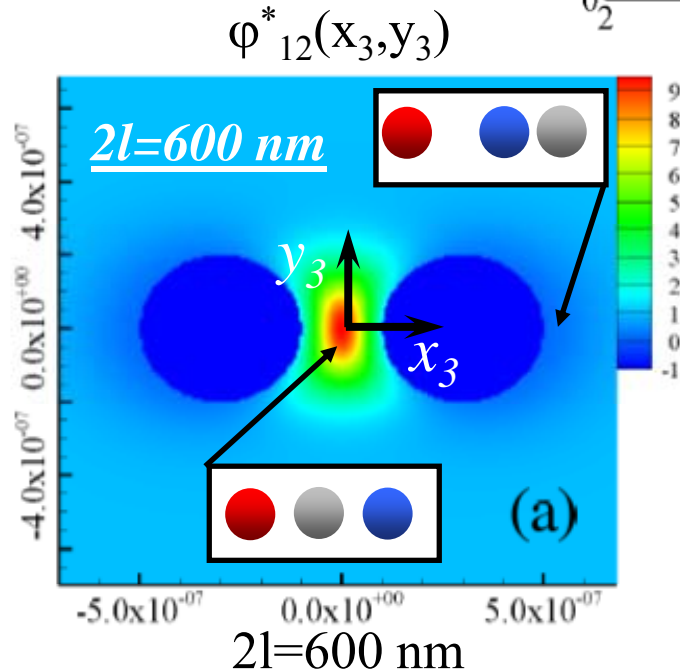
3 channels for heat exchanges



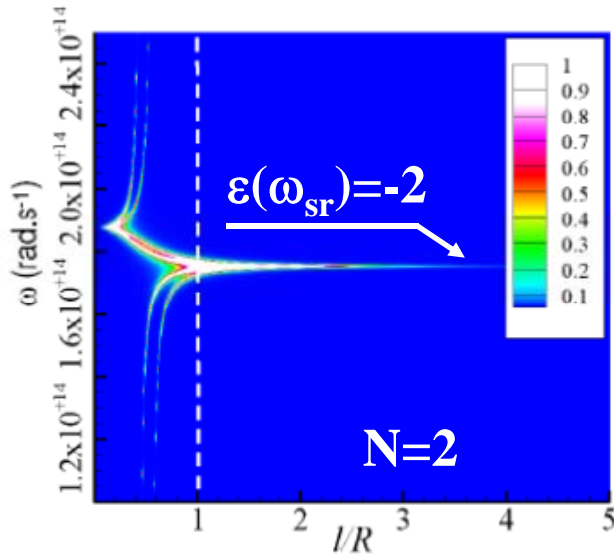
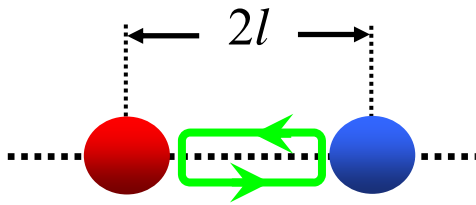
Many body effects in N body systems



SiC particles
 $R=100\text{ nm}$

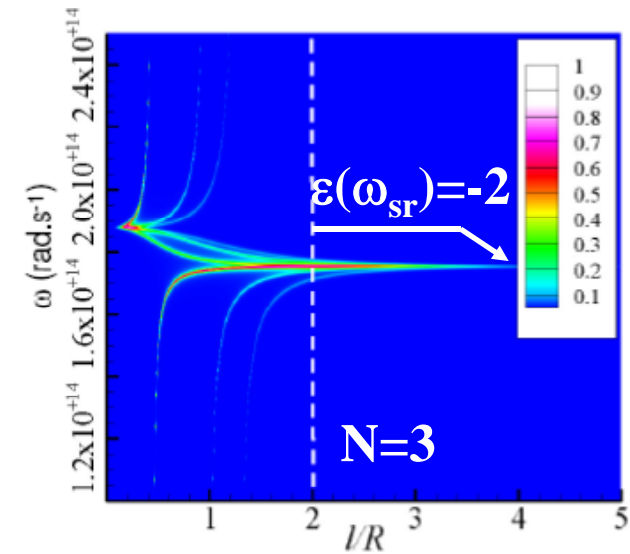
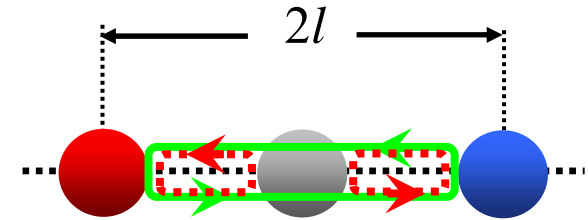


Many body effects in N body systems



$$\tau_{i,j}^{(2)}(\omega_{SR}, l = 3R) = 0.12$$

$$\mathcal{T}_{i,j}^{(N)}(\omega) = \frac{4}{3} k_0^2 \text{Im}^2 \alpha \text{tr}[G_N G_N^+]$$

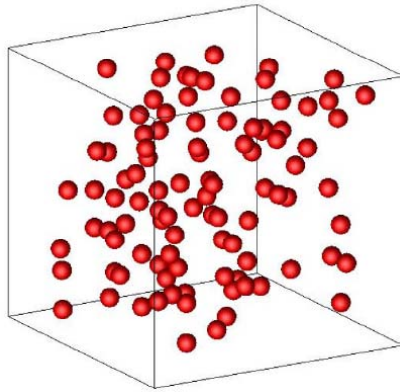


$$\tau_{i,j}^{(3)}(\omega_{SR}, l = 3R) = 0.3$$

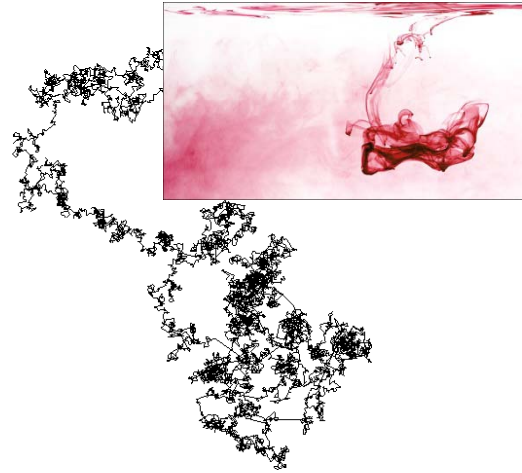
➔ More efficient coupling at longer separation distances with a third particle

Heat transport regimes in plasmonic networks

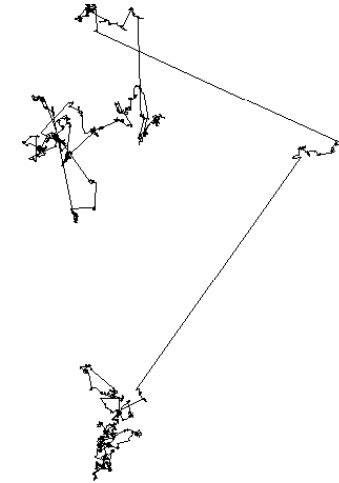
How heat propagates throughout the network ?



100 SiC nanoparticle



Gaussian
(diffusive)



Anomalous
(sub/superdiffusive)

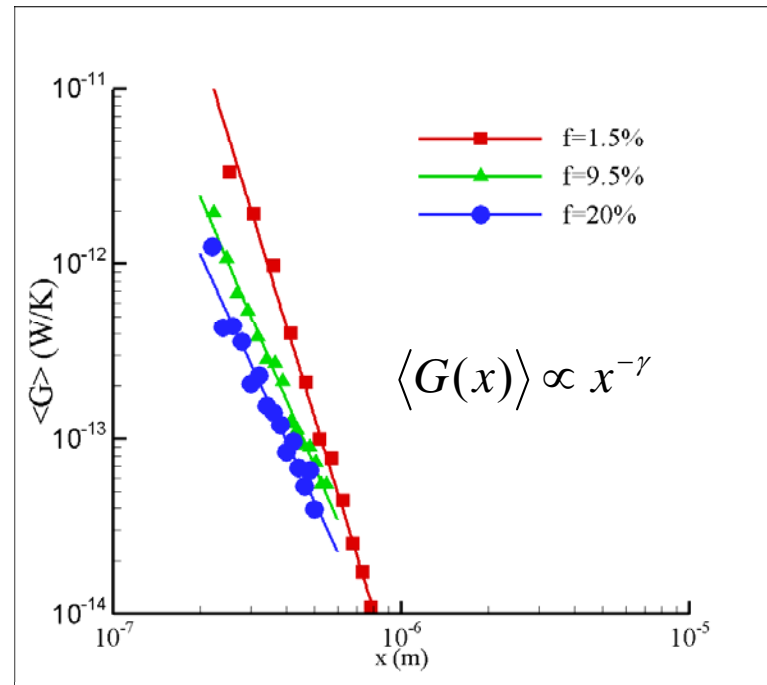
$$\rho_i C_i V_i \frac{dT_i}{dt} = \sum_j G(|r_i - r_j|)(T_j - T_i) + \bar{C}_{abs,i} \sigma (T_b^4 - T_i^4) \approx \sum_j G(|r_i - r_j|)(T_j - T_i)$$

$\bar{C}_{abs,i}$ thermal average dressed absorption

Yannopapas, Vitanov, PRL. **110**, 044302, 2013

Heat transport regimes in plasmonic networks

Remarking that $\varrho_{j \rightarrow i} = G(|r_i - r_j|)(T_j - T_i)$ is formally an Ohm's law



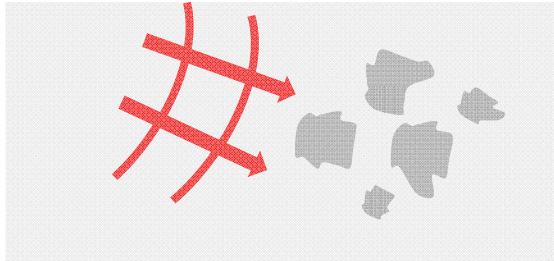
N=250 realizations generated with uniform distribution

$$\langle G \rangle = \frac{1}{N} \sum_{i=1}^N G^{(i)}$$

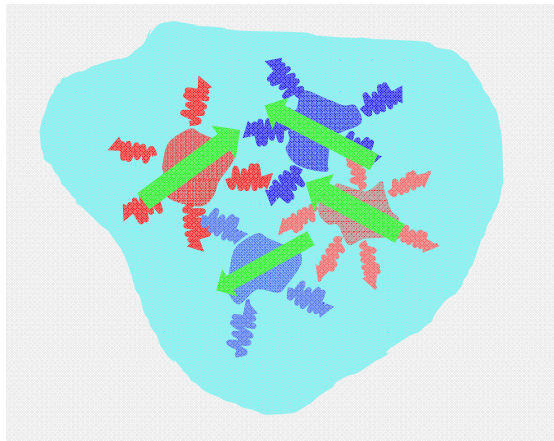
$$\rho_i C_i V_i \frac{\partial T_i}{\partial t} = - \sum_j \frac{\Gamma_{d;\alpha}}{|r_i - r_j|^{d+\alpha}} (T_i - T_j) \approx \beta_i \Delta^{\alpha/2} T|_i \quad (\text{Fractional diffusion equation})$$

$\alpha(f = 0.2) \approx 0,6 \rightarrow$ superdiffusion

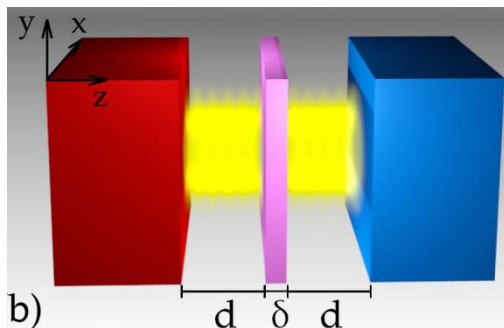
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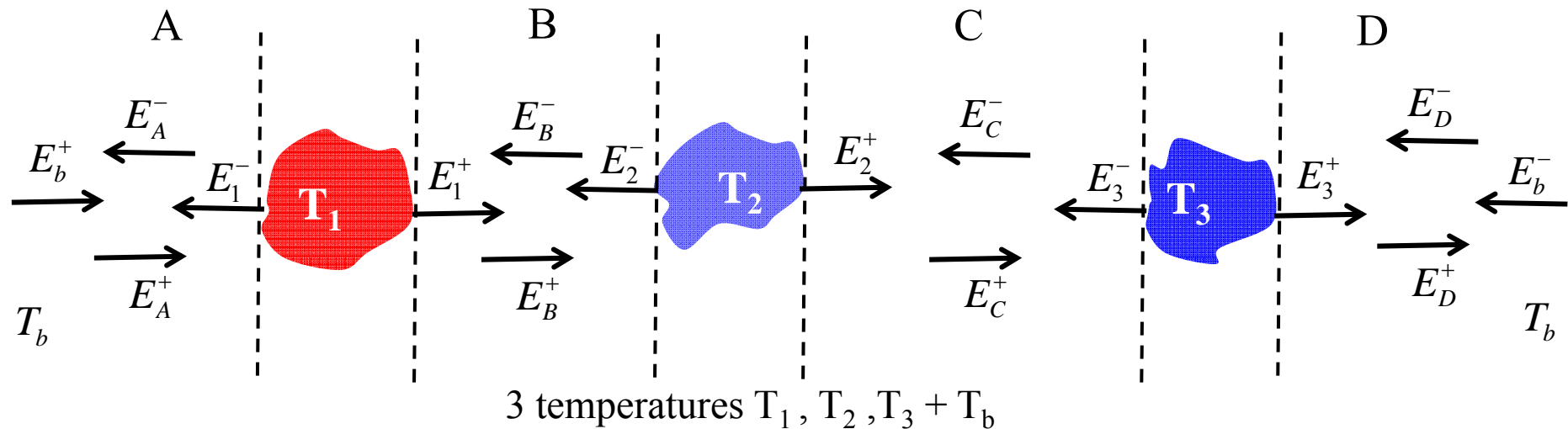


- II) Near-field heat transfer in many body systems : dipolar approximation



- III) Many body heat transfer beyond the dipolar approximation

Many body heat transfer: beyond the dipolar approximation



From the scattering matrix theory:

$$E_B^+ = E_1^+ + t_1 E_b^+ + r_1 E_B^-$$

$$E_B^- = E_2^- + t_2 E_b^- + r_2 E_B^+$$

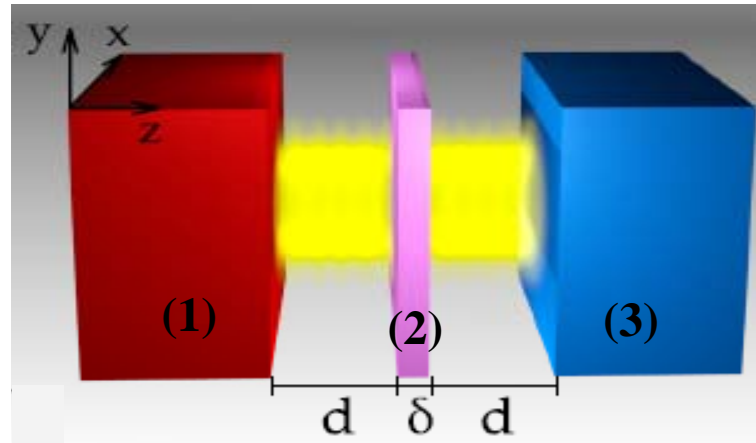
etc...

t_i, r_i partial transmission and reflection

Normal component of Poynting vector:

$$\langle S \rangle \cdot e_z = \langle E \times H \rangle \cdot e_z = \sum_p \int \frac{d^2 k}{(2\pi)^2} \int_0^\infty \frac{d\omega}{2\pi} F_p(k, \omega) \langle E_p(k, \omega), E_p(k, \omega) \rangle$$

Heat transfer in a three slab system



Monochromatic heat flux on semi-infinite medium (3) decomposes into:

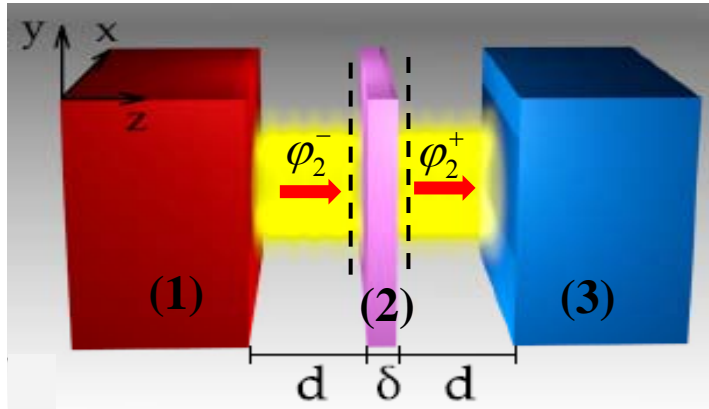
$$\varphi_3(\omega, d, \delta) = \varphi_3^{(12)}(\omega, d, \delta) + \varphi_3^{(23)}(\omega, d, \delta)$$

where $\varphi_3^{(12)} = \hbar\omega \sum_p \int \frac{d^2k}{(2\pi)^2} [n(\omega, T_1) - n(\omega, T_2)] \tau_p^{(12)}(\omega, k, d, \delta)$

$$\varphi_3^{(23)} = \hbar\omega \sum_p \int \frac{d^2k}{(2\pi)^2} [n(\omega, T_2) - n(\omega, T_3)] \tau_p^{(23)}(\omega, k, d, \delta)$$

$$\tau_p^{(23)} = \frac{4 \operatorname{Im}(\rho_{12,p}(\delta)) \operatorname{Im}(\rho_{3,p}) e^{-2\operatorname{Im}(k_z)d}}{|1 - \rho_{12,p}(\delta) \rho_{3,p} e^{-2\operatorname{Im}(k_z)d}|^2} \quad \tau_p^{(12)} = \frac{4 |t_{2,p}(\delta)|^2 \operatorname{Im}(\rho_{1,p}) \operatorname{Im}(\rho_{3,p}) e^{-4\operatorname{Im}(k_z)d}}{|1 - \rho_{12,p}(\delta) \rho_{3,p} e^{-2\operatorname{Im}(k_z)d}|^2 |1 - \rho_{1,p} \rho_{2,p}(\delta) e^{-2\operatorname{Im}(k_z)d}|^2}$$

Temperature of the intermediate layer



At thermal steady state: $\varphi_1 + \varphi_2 + \varphi_3 = 0$

$$\varphi_2 = -(\varphi_1 + \varphi_3) = -\hbar\omega \sum_p \int \frac{d^2k}{(2\pi)^2} \{ [n_{12} + n_{32}] \tau_p^{12} + [n_{23} + n_{21}] \tau_p^{23} \}$$

Symmetrical geometric configuration \rightarrow

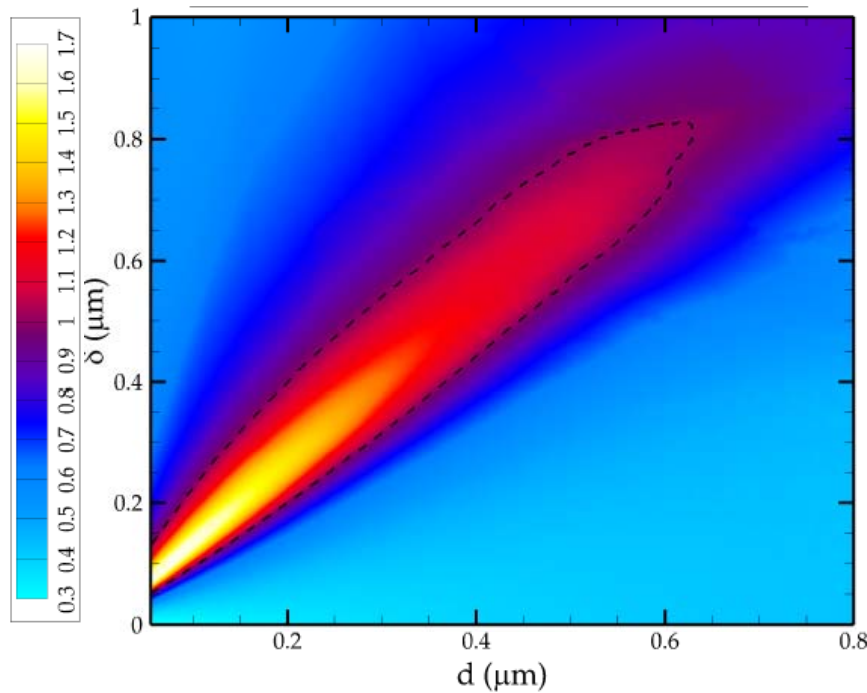
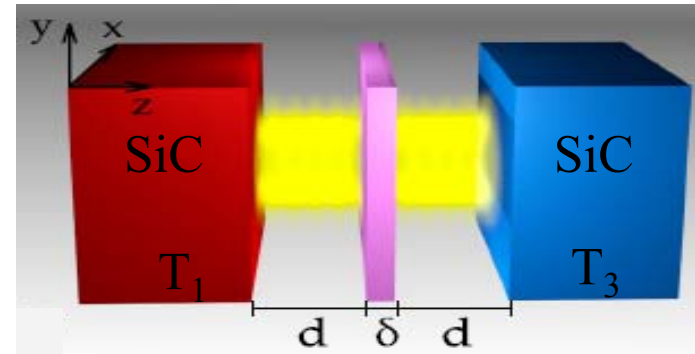
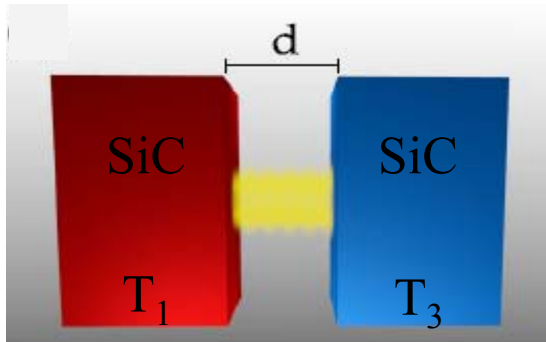
$$= \hbar\omega [2n(T_2) - n(T_1) - n(T_3)] \sum_p \int \frac{d^2k}{(2\pi)^2} (\tau_p^{12} - \tau_p^{23})$$

For a quasi-monochromatic heat flux spectrum around $\omega = \omega^*$

$$\int \varphi_2 d\omega \approx \hbar\omega^* \Delta\omega [2n(\omega^*, T_2) - n(\omega^*, T_1) - n(\omega^*, T_3)] \sum_p \int \frac{d^2k}{(2\pi)^2} (\tau_p^{12} - \tau_p^{23}) = 0$$

$$2n(\omega^*, T_2) - n(\omega^*, T_1) - n(\omega^*, T_3) = 0$$

Heat flux amplification



$$\epsilon_2 = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \quad (\text{Drude})$$

Surface plasmon at :

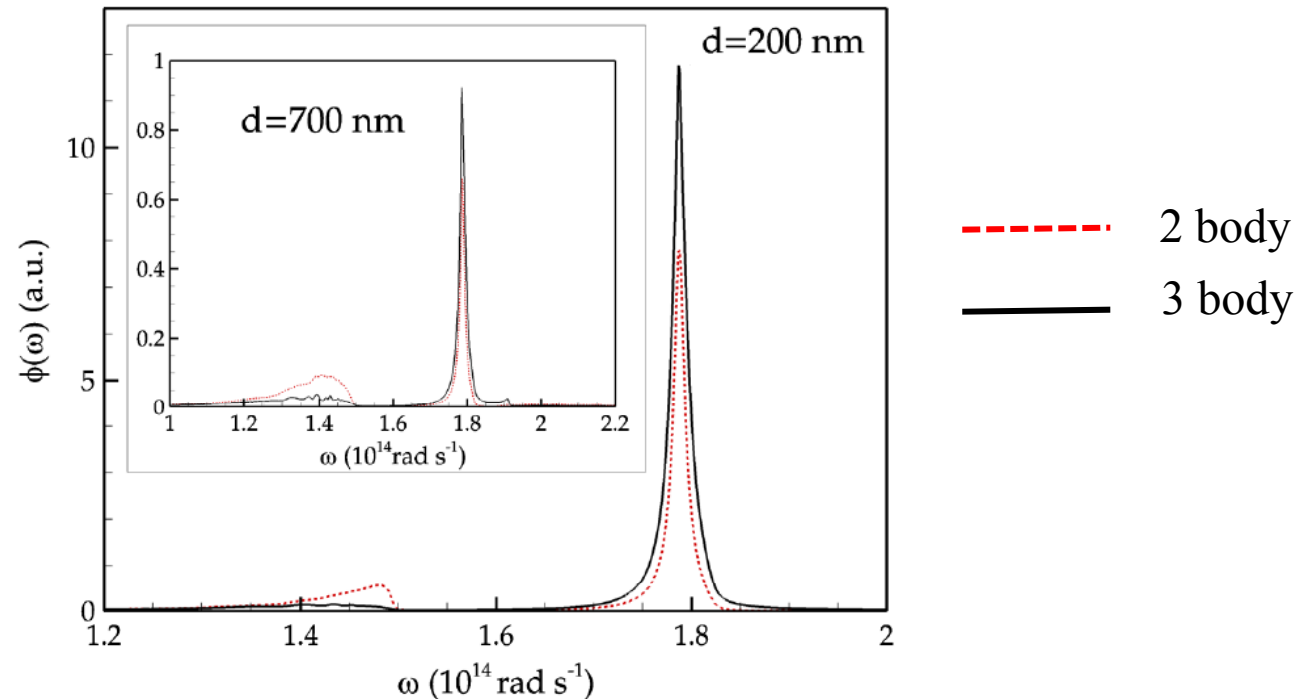
$$\omega^* \approx \omega_p / \sqrt{2} \quad \text{with} \quad \omega_p = \sqrt{2} \omega_{SPP_SiC}$$

$$\omega_{SPP_SiC} = 1.787 \times 10^{14} \text{ rad.s}^{-1}$$

$$T_1 = 400\text{K}; T_3 = 300\text{K}$$

Origin of amplification mechanism

Flux spectrum for different values of d at the optimal value of δ :



- ➔ 3 body heat flux is enhanced at ω_{SPP}
- ➔ The presence of intermediate slab does not enhance flux at smaller frequencies (quasi-monochromatic enhancement)

Transmission probability in a SiC-Drude-SiC system

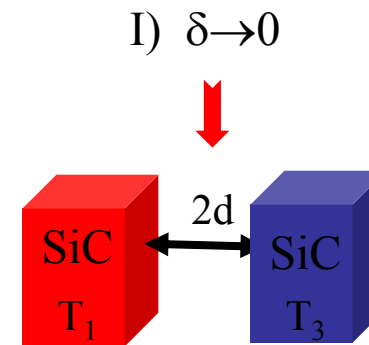
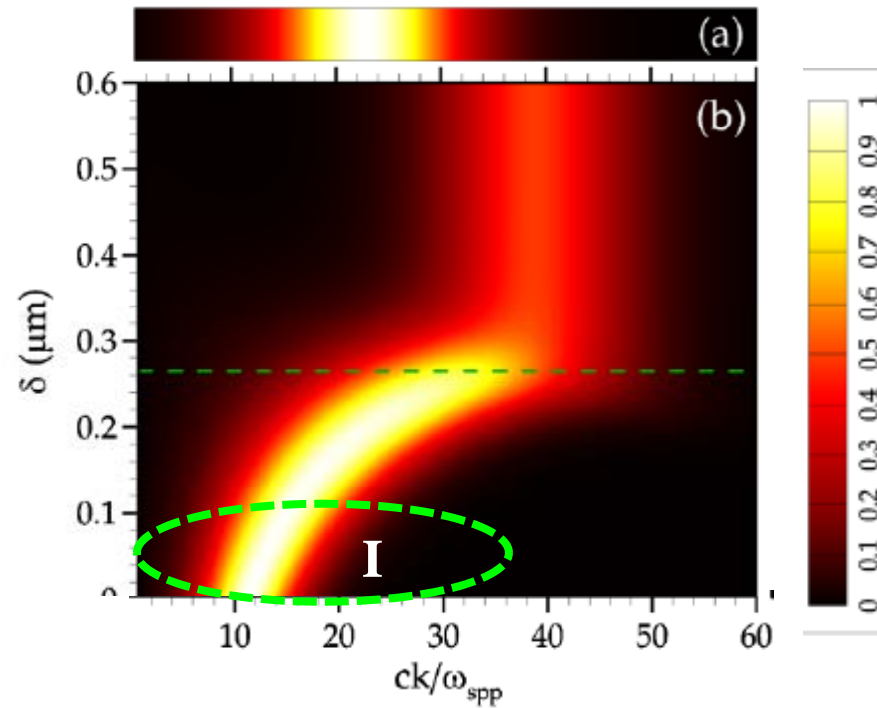
Quasimonochromaticity of transfer $\rightarrow n_{12}(\omega_{SPP}) = n_{23}(\omega_{SPP}) = \frac{n_{13}(\omega_{SPP})}{2}$

Thus

$$\left[\begin{array}{l} \varphi_3(\omega_{SPP}) = n_{12} \hbar \omega \sum_p \int \frac{d^2 k}{(2\pi)^2} [\tau_{3,p}^{(12)} + \tau_{3,p}^{(23)}] \\ \varphi_2(\omega_{SPP}) = n_{13} \hbar \omega \sum_p \int \frac{d^2 k}{(2\pi)^2} \tau_{2,p} = 2n_{12} \hbar \omega \sum_p \int \frac{d^2 k}{(2\pi)^2} \tau_{2,p} \end{array} \right.$$

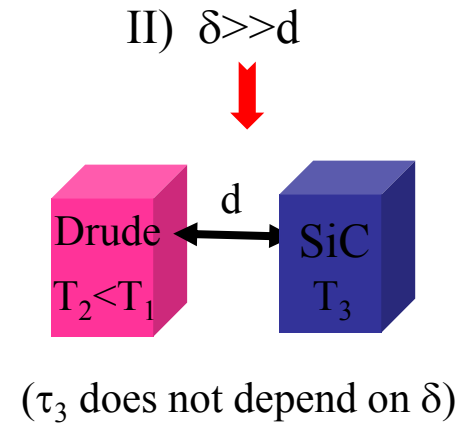
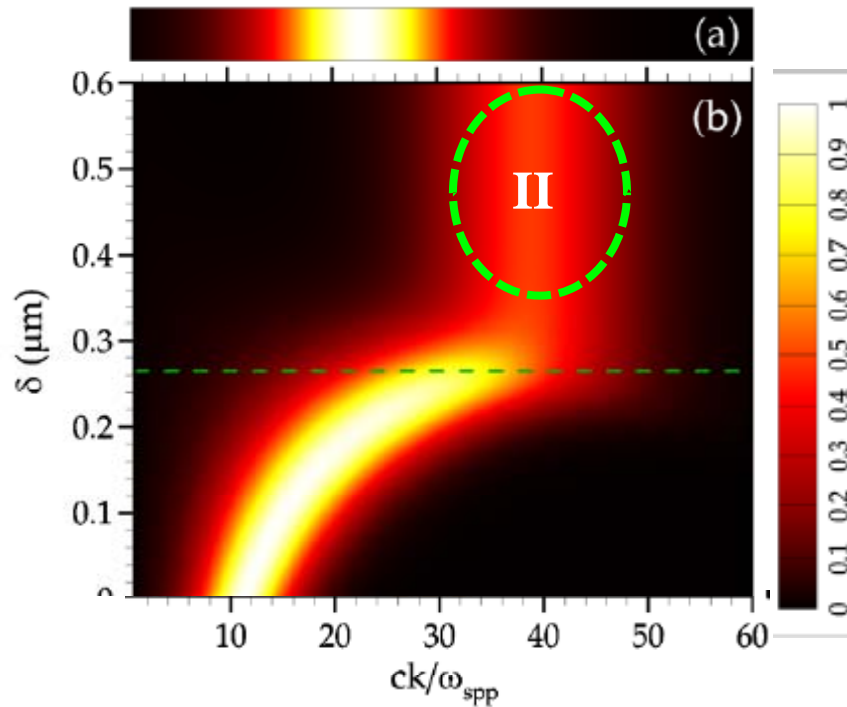
\rightarrow To compare the coupling efficiency between modes in 2 and 3 body configurations we must compare τ_2 with $\tau_3 = (\tau_3^{(12)} + \tau_3^{(23)}) / 2$

Transmission probability : asymptotic regimes



Compared to 2 body, the shift of the cut off wavenumber k_c comes from the difference of distance from d to $2d$

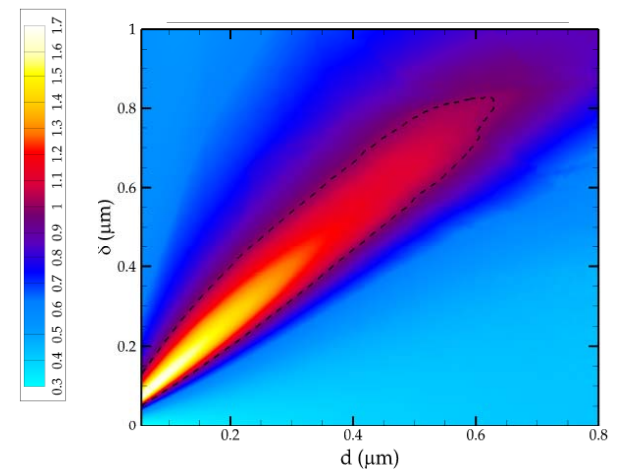
Transmission probability : asymptotic regimes



$$k_c \propto \frac{1}{\sqrt{\text{Im } \epsilon_{\text{Drude}} \text{Im } \epsilon_{\text{SiC}}}} \text{ and } \text{Im } \epsilon_{\text{Drude}} < \text{Im } \epsilon_{\text{SiC}}$$

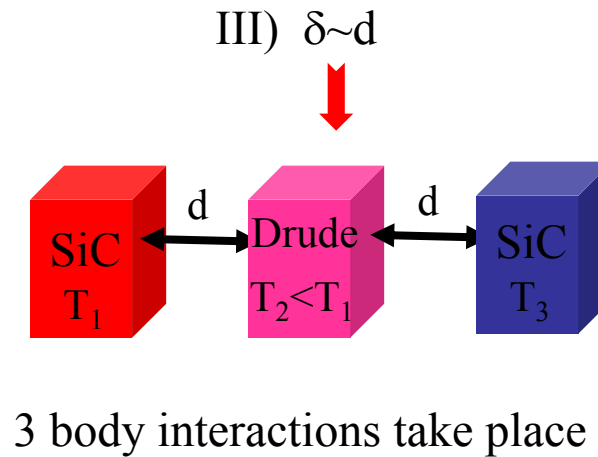
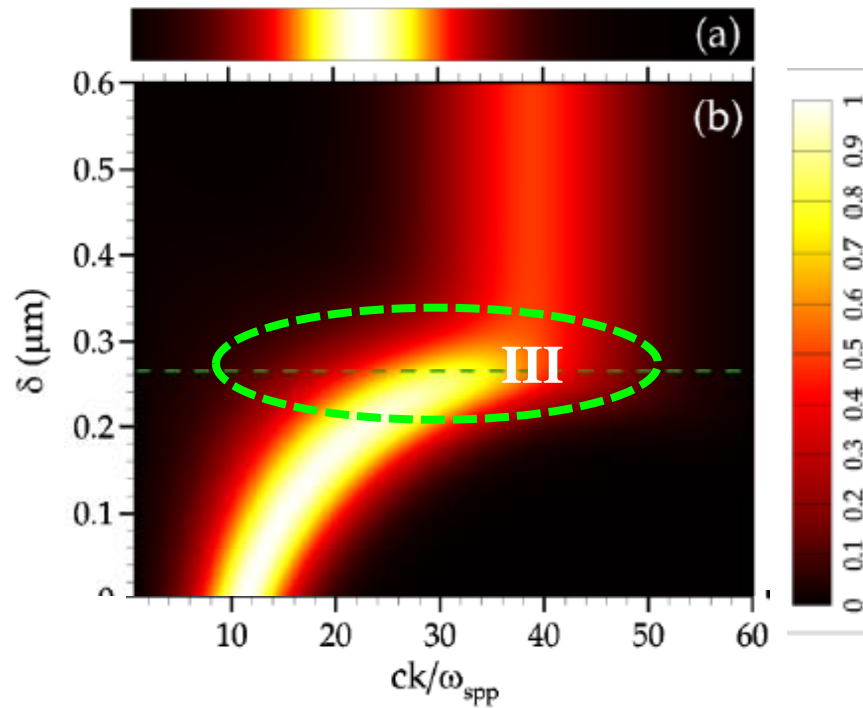


Compared to 2 body SiC-SiC, the cut off wavenumber k_c is shifted toward larger values

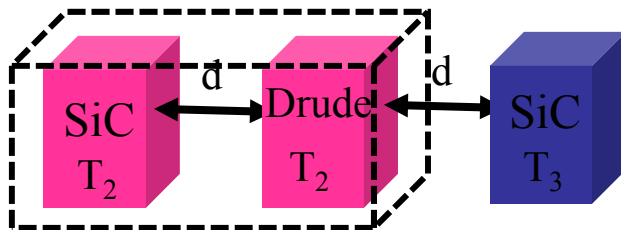
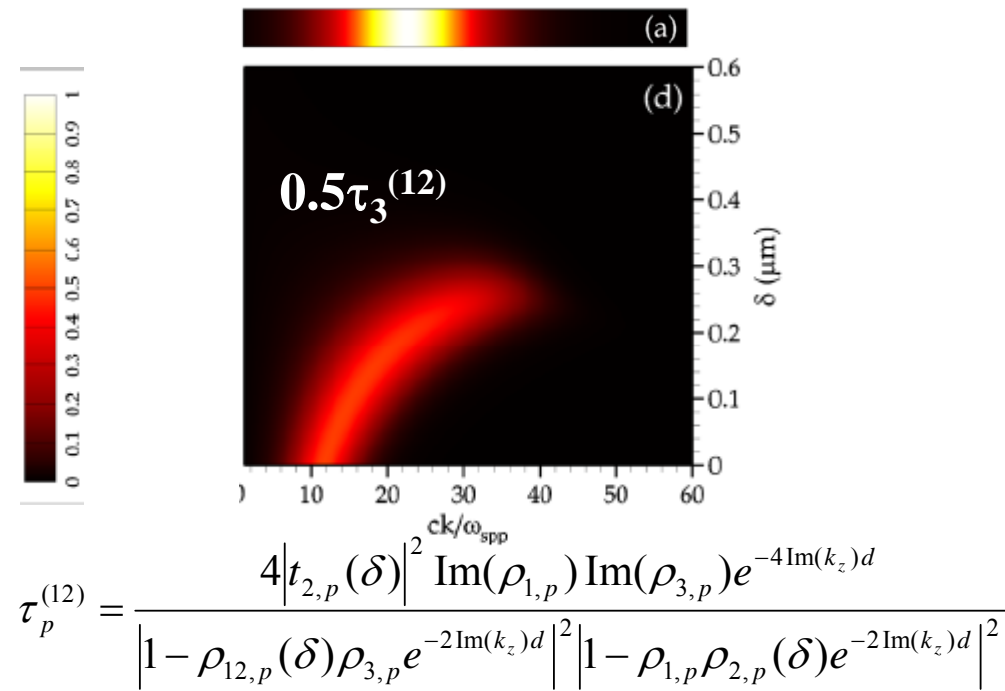
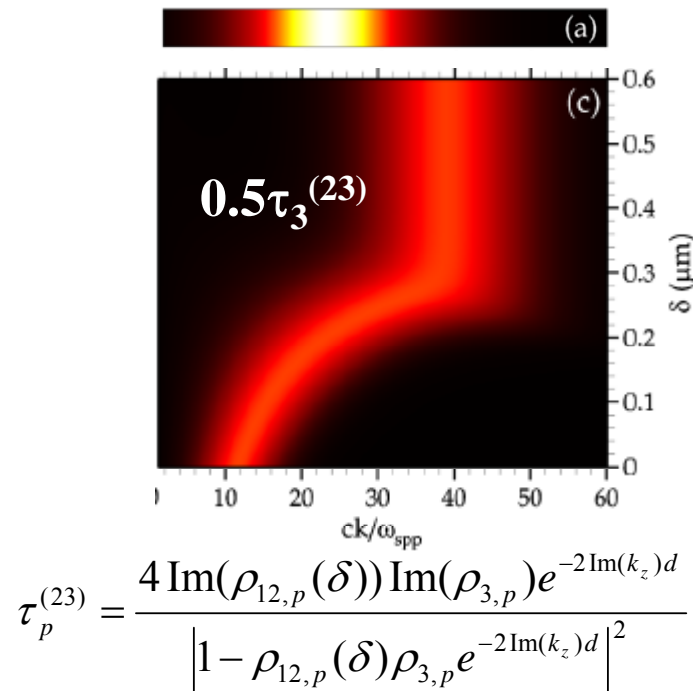


$\varphi_3 < \varphi_2$ because $T_2 < T_1$

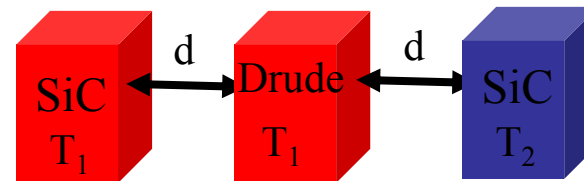
Transmission probability : three body regime



Transmission probability : three body regime

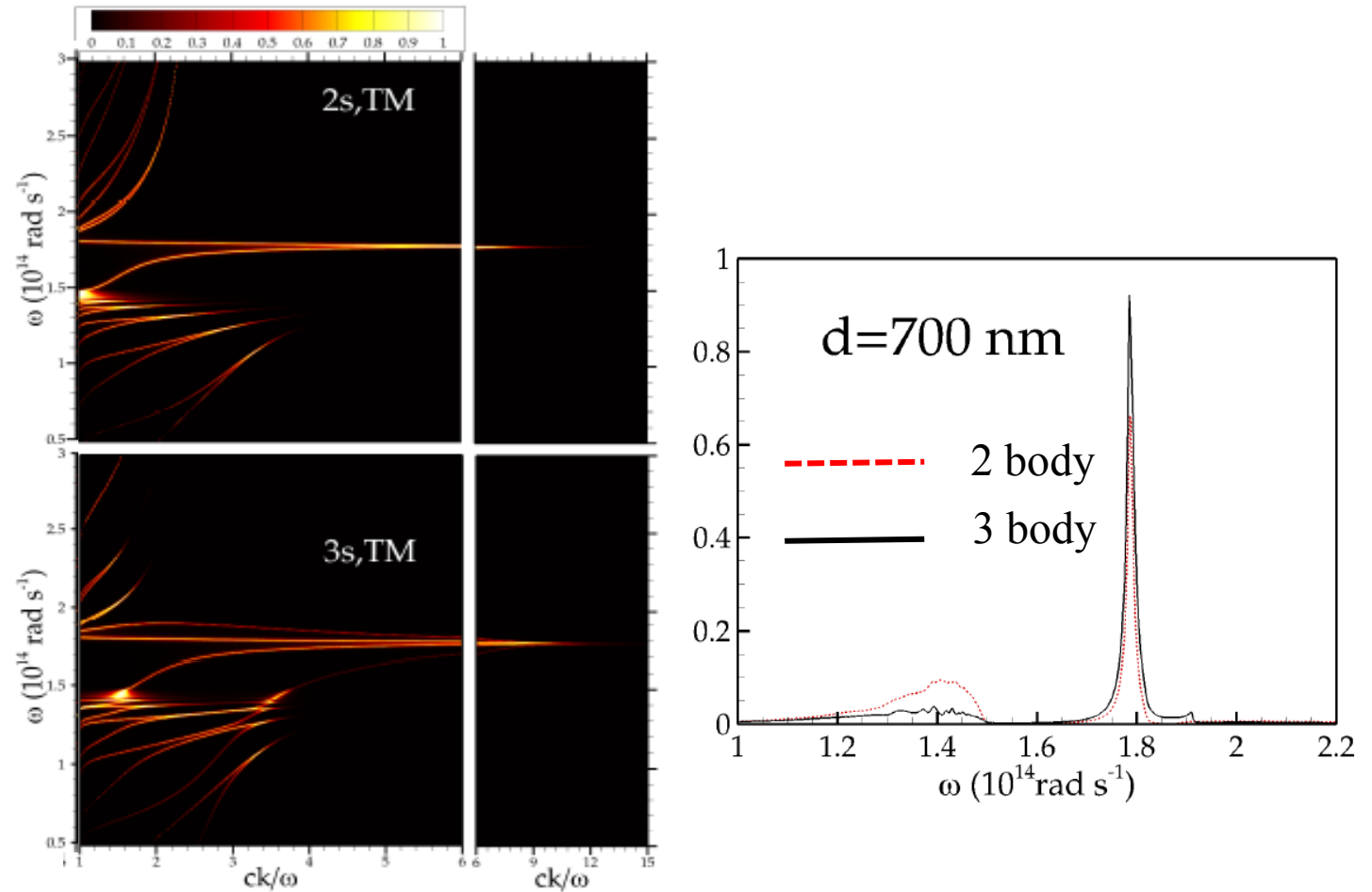


2 body exchange between
the couple (1,2) at temperature T_2 and (3)



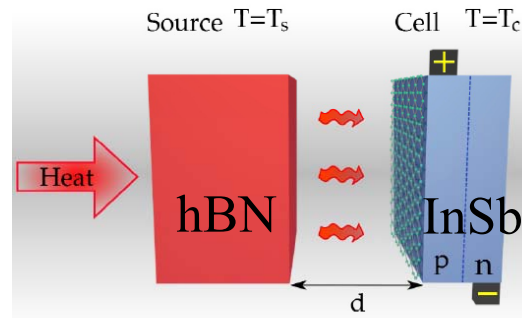
purely 3 body effect

Transmission probability : three body regime

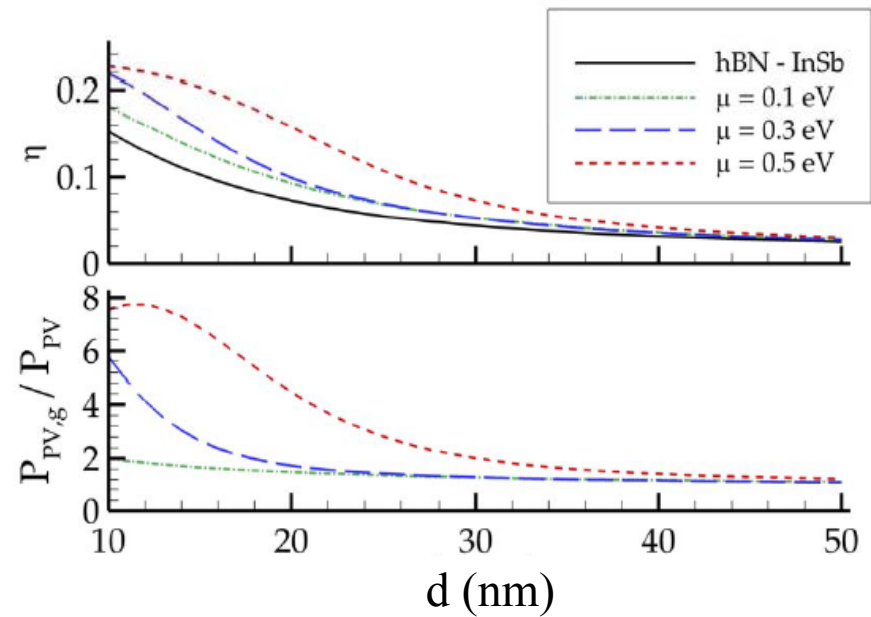
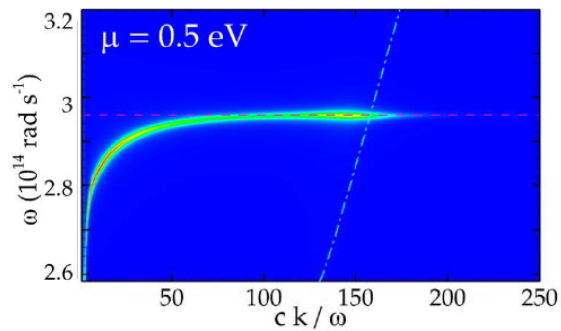
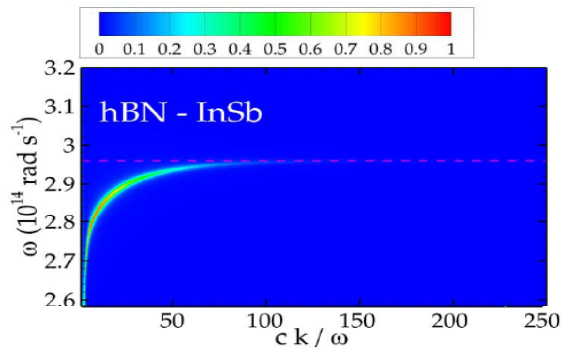


The 3 body amplification of photon tunneling result from a shifting of the cutoff wavenumber toward higher values thanks to the presence passive relay

Application: graphene-based PV cell for near-field energy conversion



Transmission



Concluding remarks

- Dipolar (multipolar) interactions in N-body systems can be used to enhance and to tailor the absorption spectrum
- Revisited the fluctuational electrodynamics in multiple dipolar systems
- Highlighted N-body near-field properties
 - enhancing or inhibiting heat exchanges
 - export the near-field effects at longer separation distances
 - existence of anomalous heat transport regimes
- Applications : NTPV, thermal management at nanoscale
- Still open questions:
 - role of localized modes in the heat transport in disordered plasmonic systems
 - transport in N-body systems at mesoscopic scale
 - dynamic of cooling/heating
 - ...