Near-field heat transfer and thermal emission control with complex plasmonic systems

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Collaborations

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Fluctuating and uncorrelated local sources lead to:

- energy and momentum exchange

These exchanges are well described by the fluctuational electrodynamic theory (Rytov) applied to:

- Radiative heat transfer (Polder and Van Hove)
- Casimir force (Lifschitz)
Open questions

- How does the heat and momentum transport for a collection of individual objects in mutual interaction look like?
- Are there specific many body effects?
- What relation between disorder and heat transport? etc…

Needs a N-body heat and momentum transfer theory
I) Engineering the light absorption spectrum from multiple scattering interactions in dipolar systems

II) Near-field heat transfer in many body systems: dipolar approximation

III) Many body heat transfer beyond the dipolar approximation
Engineering light absorption from multiple scattering interactions: absorption by a simple particle

Rate of doing work by the em field in a volume \( V \):

\[
\varphi(\omega) = \frac{1}{2} \int V \text{Re}(j^*E) dV
\]

Poynting theorem (energy conservation):

\[
\frac{1}{2} \int_V j^*E dV = -\frac{1}{2} \int_V \nabla \cdot [E \times H^*] dV + i \frac{\omega}{2} \int_V (\mu_h |H|^2 - \varepsilon_h |E|^2) dV
\]

In a transparent host medium

\[
\varphi(\omega) = -\frac{1}{2} \int_S \text{Re}[E \times H^* \cdot n] dS
\]

Poynting flux

Also we have

\[
E(r) = E^{\text{ext}} + i \omega \mu_0 \int_V \overrightarrow{G}_0(r, r') j(r') dr' + j(r) &= -i \omega p \delta(r)
\]

Thus

\[
\varphi(\omega) = -\frac{\omega}{2} \text{Im}[p^* E^{\text{inc}}(0)] - \frac{\omega^3 |p|^2}{2} \frac{\mu_0}{t} \text{Im}[\overrightarrow{G}_0(0,0)].t
\]
Engineering light absorption from multiple scattering interactions: absorption by a set of particles

![Diagram of light absorption](image)

External field on the particle $i$: $E_{i}^{ext} = E_{i}^{inc} + \omega^2 \mu_0 \sum_{j \neq i} \vec{G}_0(r_i, r_j) p_j$

Thus

$$\omega_i(\omega) = -\frac{\omega}{2} \text{Im}[p_i^* E_{i}^{inc}] - \frac{\omega^3 \mu_0}{2} |p_i|^2 \vec{t}_i . \vec{G}_0(r_i, r_i) \vec{t}_i - \frac{\omega^3 \mu_0}{2} \text{Im}[p_i^* \sum_{j \neq i} \vec{G}_0(r_i, r_j) . p_j]$$

Multiple interactions
Engineering light absorption: dressed absorption in dipolar chains

Au, 20 nm radius

\[ n_b = 1.5 \]

\[ \frac{\sigma}{\sigma_{\text{inc}}} \]
Engineering light absorption: design of an absorber

Find the optimal distribution of dipoles to get an absorption spectrum target

\[ E_i^{\text{ext}} = E_i^{\text{inc}} + \omega^2 \mu_0 \sum_{j \neq i} \vec{G}(r_i, r_j) p_j + \Delta \vec{G}(r_i, r_i) p_i \]

\[ \phi_i(\omega) = -\frac{\omega}{2} \text{Im}[p_i^* E_i^{\text{ext}}] - \frac{\omega^3 \mu_0}{2} p_i^* \text{Im}[\vec{G}_0(r_i, r_i)] p_i \]

\[ a = 1 - T - R \equiv \frac{\sum \phi_i}{A \Phi_{\text{inc}}} \]

\[ \Delta G \equiv G - G_0 \]
Find the position and the size of particles in the unit cell of a n-ary lattice so that:

\[ \int_{\lambda_{\text{min}}}^{\lambda_{\text{max}}} a(\lambda) d\lambda \rightarrow \text{max} \]

Evolutionary algorithm:

- Random generation
- Selection of best structures
- Crossing over
- New generation
- Mutation
Engineering light absorption: design of an absorber

Au-Ag lattice

dipolar approximation

Langlais, Besbes, Hugonin, Ben-Abdallah, submitted
Engineering light absorption: design of an absorber

Au-Ag lattice

Multipolar calculation

Langlais, Besbes, Hugonin, Ben-Abdallah, submitted
Engineering light absorption: design of an absorber

Au-Ag lattice

Effective medium theory

Langlais, Besbes, Hugonin, Ben-Abdallah, submitted
Outline

- I) Engineering the light absorption spectrum from multiple scattering interactions in dipolar systems

- II) Near-field heat transfer in many body systems: dipolar approximation

- III) Many body heat transfer beyond the dipolar approximation
Near-field heat transfer in many body systems: dipolar approximation

- \( d_i < \min(\lambda_{T_j}) \) with \( \lambda_{T_j} = \frac{\hbar}{k_B T_j} \)
- Objects exchange in far field with the bath
- \( N \) fluctuating dipoles in mutual interaction inside a bosonic field

**Local field**

\[
E(r) = E_b(r) + \omega^2 \mu_0 \sum_j \tilde{G}_0(r,r_j) p_j
\]

**Dipole moments**

\[
p_i = p_i^{\text{fluc}} + p_i^{\text{ind}} \quad \text{with} \quad p_i^{\text{ind}} = \varepsilon_0 \alpha_i [E_{b,i} + \sum_{j \neq i} \tilde{G}_0(r_i,r_j) p_j]
\]

\[
\begin{pmatrix}
    p_1 \\
    \vdots \\
    p_N
\end{pmatrix} =
\begin{pmatrix}
    p_1^{\text{fluc}} \\
    \vdots \\
    p_N^{\text{fluc}}
\end{pmatrix} +
\begin{pmatrix}
    E_1 \\
    \vdots \\
    E_N
\end{pmatrix} =
\begin{pmatrix}
    p_1^{\text{fluc}} \\
    \vdots \\
    p_N^{\text{fluc}}
\end{pmatrix} +
\begin{pmatrix}
    E_1 \\
    \vdots \\
    E_N
\end{pmatrix}
\]

where \( M, N, O, P \) are function of \( \tilde{G}_0(r_i,r_j), \alpha_1, ..., \alpha_N \)
Energy balance

Time evolution of temperatures is governed by:

$$\rho_i C_i V_i \frac{dT_i}{dt} = \varphi_i(t, T_1, ..., T_N, T_b)$$

with

$$\varphi_i = \int \langle j_i E \rangle dV_i \approx \left\langle \frac{d\vec{p}_i}{dt}, E \right\rangle$$  (dipolar approximation)

Using the convention

$$\vec{p}_i(t) = 2 \text{Re} \left( \int_0^\infty p_i(\omega) \frac{e^{-i\omega t}}{2\pi} d\omega \right)$$

$$\varphi_i = \int_0^\infty \frac{d\omega}{2\pi} \int_0^\infty \frac{d\omega'}{2\pi} \text{Im} \left( p_i(\omega) E_{i}^\dagger (\omega') \right) e^{-i(\omega-\omega')t}$$

Assuming no correlation between the fluctuating dipole moments and the field of bath

$$\left\langle p_i(\omega) E_{i}^\dagger (\omega') \right\rangle = \sum_\alpha \sum_{jj'} \sum_{\beta\beta'} M_{ij,\alpha\beta} \left\langle p_{j,\beta}^{\text{fluc}} (\omega) p_{j',\beta'}^{\text{fluc}} (\omega') \right\rangle O_{j',i,\beta'\alpha}^\dagger + N_{ij,\alpha\beta} \left\langle E_{j,\beta}^b (\omega) E_{j',\beta'}^b (\omega') \right\rangle P_{j',i,\beta'\alpha}^\dagger$$
Energy balance

Using the FDT:

$$\langle p_{j,\beta}(\omega) p_{j',\beta'}^{\dagger}(\omega') \rangle = 2\pi \hbar \varepsilon_0 \delta_{jj'} \delta_{\beta\beta'} \chi_j \delta(\omega - \omega')(1 + 2n(\omega, T_j)) \quad \text{with} \quad \chi_j = \text{Im}(\alpha_j) - \frac{\omega^3}{6\pi c^3} |\alpha_j|^2$$

$$\langle E_{j,\beta}(\omega) E_{j',\beta'}^{\dagger}(\omega') \rangle = 2\pi \hbar \frac{\omega^2}{\varepsilon_0 c^2} \text{Im}(\tilde{G}_{0,ji',\beta\beta'}) \delta(\omega - \omega')(1 + 2n(\omega, T_b))$$

$$\langle p_i(\omega) E_i^{\dagger}(\omega') \rangle = 2\pi \delta(\omega - \omega')$$

$$\times \left[ \hbar \varepsilon_0 \sum_j \chi_j (1 + 2n(\omega, T_j)) \text{Tr}(M_{ij} O_{ji}^{\dagger}) + \frac{\hbar \omega^2}{\varepsilon_0 c^2} (1 + 2n(\omega, T_b)) \text{Tr}(N \text{Im}(\tilde{G}_0) P_{ii}^{\dagger}) \right]$$

By considering exchanges with other dipoles, the thermal bath and emission:

$$\varrho_i = \sum_{j \neq i} \varrho_{j \rightarrow i} + \varrho_{b \rightarrow i} - \varrho_{i}^{\text{emi}}$$

with

$$\varrho_{j \rightarrow i} = 3 \int_0^\infty \frac{d\omega}{2\pi} \hbar \omega n(\omega, T_j) \tau_{ij}(\omega) \quad \tau_{ij}(\omega) = \frac{4}{3} \left( \frac{\omega}{c} \right)^4 \chi_i \chi_j \text{Tr}[\tilde{G}(r_i, r_j) \tilde{G}^{\dagger}(r_i, r_j)]$$

Ben-Abdallah, Biehs, Joulain, PRL, 107, 114301 (2011); Messina, Ben-Abdallah, submitted
Landauer-like formulation of heat transport

Introducing the heat conductance between two objects

\[ G_{ij} = \frac{\partial P_{j \to i}}{\partial T_j} \quad \text{(Linearization of exchanged power)} \]

\[ \phi_{j \to i} = G_{ij} \Delta T = 3 \left( \frac{\pi^2 k_B^2 T_i^4}{3h} \right) \tau_{ij} \Delta T \]

(Pendry, Math. Gen., 1983
Schwab, Nature 2000)

Mean transmission coefficient

\[ \tau_{i,j} = \frac{3}{\pi^2} \int \frac{dx}{(e^x - 1)^2} T_{i,j} \]

3 channels for heat exchanges
Many body effects in N body systems

\[ \psi_{12}(x_3, y_3) \]

\[ T_1 = 300 \text{ K} \]
\[ T_2 = 0 \text{ K} \]
\[ T_3 = 0 \text{ K} \]

SiC particles
R = 100 nm

Ben-Abdallah, Biehs, Joulain, PRL, 107, 114301 (2011)
Many body effects in N body systems

\[2l\]

\[\varepsilon(\omega_{SR}) = -2\]

\[\tau_{i,j}^{(N)}(\omega) = \frac{4}{3} k_0^2 Im \alpha tr[G_N G_N^+]\]

\[N=2\]

\[\tau_{i,j}^{(2)}(\omega_{SR}, l = 3R) = 0.12\]

\[N=3\]

\[\tau_{i,j}^{(3)}(\omega_{SR}, l = 3R) = 0.3\]

More efficient coupling at longer separation distances with a third particle
Heat transport regimes in plasmonic networks

How heat propagates throughout the network?

\[
\rho_i C_i V_i \frac{dT_i}{dt} = \sum_j G(|r_i - r_j|)(T_j - T_i) + \bar{C}_{abs,i} \sigma(T_b^4 - T_i^4) \approx \sum_j G(|r_i - r_j|)(T_j - T_i)
\]

\(\bar{C}_{abs,i}\) thermal average dressed absorption

Yannopapas, Vitanov, PRL. 110, 044302, 2013
Heat transport regimes in plasmonic networks

Remarking that \( \varphi_{j \to i} = G(|r_i - r_j|)(T_j - T_i) \) is formally an Ohm’s law

\[
\langle G(x) \rangle \propto x^{-\gamma}
\]

N=250 realizations generated with uniform distribution

\[
\langle G \rangle = \frac{1}{N} \sum_{i=1}^{N} G^{(i)}
\]

\[
\rho_i C_i V_i \frac{\partial T_i}{\partial t} = -\sum_j \frac{\Gamma_{d,\alpha}^{d+\alpha}}{|r_i - r_j|^{d+\alpha}} (T_i - T_j) \approx \beta_i \Delta^{\alpha/2} T_i \quad \text{(Fractional diffusion equation)}
\]

\[
\alpha(f = 0.2) \approx 0.6 \quad \text{superdiffusion}
\]
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Many body heat transfer: beyond the dipolar approximation

From the scattering matrix theory:

\[ E_B^+ = E_1^+ + t_1 E_b^+ + r_1 E_B^- \]
\[ E_B^- = E_2^- + t_2 E_b^- + r_2 E_B^+ \]

\( t_i, r_i \) partial transmission and reflection etc…

Normal component of Poynting vector:

\[ \langle S \rangle . e_z = \langle E \times H \rangle . e_z = \sum_p \int \frac{d^2 k}{(2\pi)^2} \int_0^{\infty} \frac{d\omega}{2\pi} F_p(k, \omega) \langle E_p(k, \omega), E_p(k, \omega) \rangle \]
Heat transfer in a three slab system

Monochromatic heat flux on semi-infinite medium (3) decomposes into:

\[ \varphi_3(\omega, d, \delta) = \varphi_3^{(12)}(\omega, d, \delta) + \varphi_3^{(23)}(\omega, d, \delta) \]

where

\[ \varphi_3^{(12)} = \hbar \omega \sum_p \int \frac{d^2k}{(2\pi)^2} \left[ n(\omega, T_1) - n(\omega, T_2) \right] \tau_p^{(12)}(\omega, k, d, \delta) \]

\[ \varphi_3^{(23)} = \hbar \omega \sum_p \int \frac{d^2k}{(2\pi)^2} \left[ n(\omega, T_2) - n(\omega, T_3) \right] \tau_p^{(23)}(\omega, k, d, \delta) \]

\[ \tau_p^{(23)} = \frac{4 \text{Im}(\rho_{12,p}(\delta)) \text{Im}(\rho_{3,p}) e^{-2\text{Im}(k_z)d}}{\left| 1 - \rho_{12,p}(\delta) \rho_{3,p} e^{-2\text{Im}(k_z)d} \right|^2} \]

\[ \tau_p^{(12)} = \frac{4 |\sigma_{2,p}(\delta)|^2 \text{Im}(\rho_{1,p}) \text{Im}(\rho_{3,p}) e^{-4\text{Im}(k_z)d}}{\left| 1 - \rho_{12,p}(\delta) \rho_{3,p} e^{-2\text{Im}(k_z)d} \right|^2 \left| 1 - \rho_{1,p} \rho_{2,p}(\delta) e^{-2\text{Im}(k_z)d} \right|^2} \]

Messina, Antezza, Ben-Abdallah, PRL, 109, 244302 (2012)
Temperature of the intermediate layer

At thermal steady state: \( \varphi_1 + \varphi_2 + \varphi_3 = 0 \)

\[ \varphi_2 = -(\varphi_1 + \varphi_3) = -\hbar \omega \sum_p \int \frac{d^2k}{(2\pi)^2} \left\{ [n_{12} + n_{32}] \tau_p^{12} + [n_{23} + n_{21}] \tau_p^{23} \right\} \]

Symmetrical geometric configuration

\[ = \hbar \omega [2n(T_2) - n(T_1) - n(T_3)] \sum_p \int \frac{d^2k}{(2\pi)^2} (\tau_p^{12} - \tau_p^{23}) \]

For a quasi-monochromatic heat flux spectrum around \( \omega = \omega^* \)

\[ \int \varphi_2 d\omega \approx \hbar \omega^* \Delta \omega [2n(\omega^*, T_2) - n(\omega^*, T_1) - n(\omega^*, T_3)] \sum_p \int \frac{d^2k}{(2\pi)^2} (\tau_p^{12} - \tau_p^{23}) = 0 \]

\[ 2n(\omega^*, T_2) - n(\omega^*, T_1) - n(\omega^*, T_3) = 0 \]

Messina, Antezza, Ben-Abdallah, PRL, 109, 244302 (2012)
Heat flux amplification

\[ \varepsilon_2 = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \quad \text{(Drude)} \]

Surface plasmon at:

\[ \omega^* \approx \omega_p / \sqrt{2} \quad \text{with} \quad \omega_p = \sqrt{2} \omega_{SPP_{SiC}} \]

\[ \omega_{SPP_{SiC}} = 1.787 \times 10^{14} \text{ rad.s}^{-1} \]

\[ T_1 = 400\text{K}; \quad T_3 = 300\text{K} \]

Messina, Antezza, Ben-Abdallah, PRL, 109, 244302 (2012)
Origin of amplification mechanism

Flux spectrum for different values of $d$ at the optimal value of $\delta$:

3 body heat flux is enhanced at $\omega_{\text{SPP}}$

The presence of intermediate slab does not enhance flux at smaller frequencies (quasi-monochromatic enhancement)

Messina, Antezza, Ben-Abdallah, PRL, 109, 244302 (2012)
Transmission probability in a SiC-Drude-SiC system

Quasimonochromaticity of transfer \( n_{12}(\omega_{SPP}) = n_{23}(\omega_{SPP}) = \frac{n_{13}(\omega_{SPP})}{2} \)

Thus

\[
\phi_3(\omega_{SPP}) = n_{12}\hbar\omega \sum_p \int \frac{d^2k}{(2\pi)^2} [\tau_{3,p}^{(12)} + \tau_{3,p}^{(23)}]
\]

\[
\phi_2(\omega_{SPP}) = n_{13}\hbar\omega \sum_p \int \frac{d^2k}{(2\pi)^2} \tau_{2,p} = 2n_{12}\hbar\omega \sum_p \int \frac{d^2k}{(2\pi)^2} \tau_{2,p}
\]

To compare the coupling efficiency between modes in 2 and 3 body configurations we must compare \( \tau_2 \) with \( \tau_3 = (\tau_{3}^{(12)} + \tau_{3}^{(23)}) / 2 \)
Transmission probability: asymptotic regimes

Compared to 2 body, the shift of the cut off wavevector $k_c$ comes from the difference of distance from $d$ to $2d$.
Transmission probability: asymptotic regimes

\[ k_c \propto \frac{1}{\sqrt{\text{Im}\varepsilon_{\text{Drude}} \text{Im}\varepsilon_{\text{SiC}}}} \quad \text{and} \quad \text{Im}\varepsilon_{\text{Drude}} \prec \text{Im}\varepsilon_{\text{SiC}} \]

Compared to 2 body SiC-SiC, the cut off wavenumber \( k_c \) is shifted toward larger values.

\( \delta \gg d \) 

(\( \tau_3 \) does not depend on \( \delta \))

\[ \varphi_3 < \varphi_2 \quad \text{because} \quad T_2 < T_1 \]
Transmission probability: three body regime

3 body interactions take place

III) $\delta \sim d$
Transmission probability: three body regime

\[ \tau_p^{(23)} = \frac{4 \text{Im}(\rho_{12,p}(\delta)) \text{Im}(\rho_{3,p}) e^{-2 \text{Im}(k_z)d}}{|1 - \rho_{12,p}(\delta) \rho_{3,p} e^{-2 \text{Im}(k_z)d}|^2} \]

\[ \tau_p^{(12)} = \frac{4|t_{2,p}(\delta)|^2 \text{Im}(\rho_{1,p}) \text{Im}(\rho_{3,p}) e^{-4 \text{Im}(k_z)d}}{|1 - \rho_{12,p}(\delta) \rho_{3,p} e^{-2 \text{Im}(k_z)d}|^2 |1 - \rho_{1,p} \rho_{2,p}(\delta) e^{-2 \text{Im}(k_z)d}|^2} \]

2 body exchange between the couple (1,2) at temperature T_2 and (3)

purely 3 body effect

Messina, Antezza, Ben-Abdallah, PRL, 109, 244302 (2012)
The 3 body amplification of photon tunneling result from a shifting of the cutoff wavector toward higher values thanks to the presence passive relay

Messina, Antezza, Ben-Abdallah, PRL, 109, 244302 (2012)
Application: graphene-based PV cell for near-field energy conversion

Concluding remarks

- Dipolar (multipolar) interactions in N-body systems can be used to enhance and to tailor the absorption spectrum

- Revisited the fluctuationnal electrodynamics in multiple dipolar systems

- Highlighted N-body near-field properties
  - enhancing or inhibiting heat exchanges
  - export the near-field effects at longer separation distances
  - existence of anomalous heat transport regimes

- Applications: NTPV, thermal management at nanoscale

- Still open questions:
  - role of localized modes in the heat transport in disordered plasmonic systems
  - transport in N-body systems at mesoscopic scale
  - dynamic of cooling/heating

...