# Heat Transfer Between Anisotropic Objects

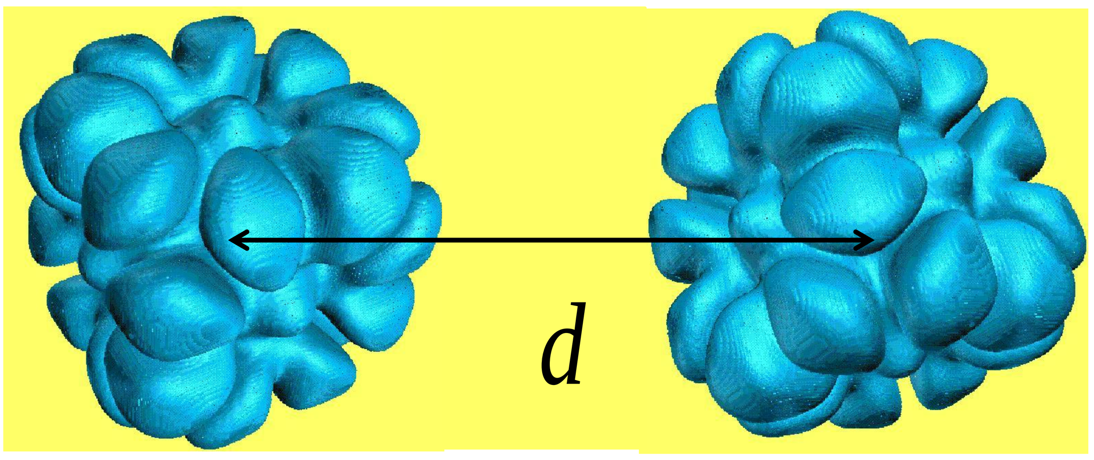
Roberta Incardone and Matthias Krüger
*Max-Planck-Institute for Intelligent Systems, Stuttgart, Germany*
*and Institut für Theoretische Physik IV, Universität Stuttgart, Germany*

MAX-PLANCK-GESELLSCHAFT

Universität Stuttgart

***We study the radiative heat transfer between anisotropic objects. In order to obtain analitically tractable expressions, we focus on the regime where these are small compared to the involved radiation wavelengths. Our computation allows to analitically understand a recent numerical finding of non-monotonicity as a function of separation between an elongated object and a perforated surface [1].***

## Scattering theory




### Heat transfer between arbitrary objects :

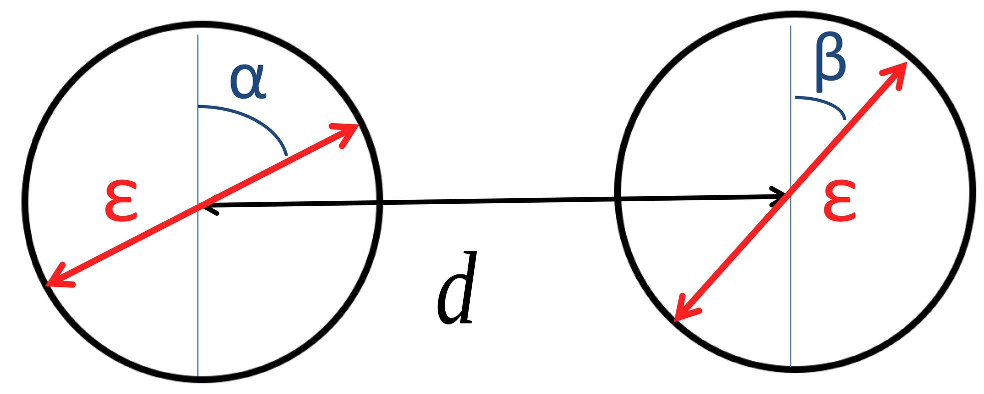
$$H=\frac{2\hbar}{\pi}\int d\omega\frac{\omega}{e^{\frac{\hbar\omega}{kT}}-1}Tr\left\{\left[\frac{T_2^\dagger+T_2}{2}+T_2^\dagger T_2\right]\frac{U}{I-UT_1UT_2}\left[\frac{T_1^\dagger+T_1}{2}+T_1^\dagger T_1\right]\frac{U^\dagger}{I-U^\dagger T_2^\dagger U^\dagger T_1^\dagger}\right\}$$

$T:$ *Scattering matrix (generalized Fresnel coefficient)*

$U:$ *Translation matrix*

**The trace involves summing over all wave indices, e.g. {l,m,P} for spherical waves.**

## Anisotropic spheres




$$(\hat{\varepsilon}-\mathrm{I})=\begin{pmatrix}((\varepsilon^R+i\varepsilon^I)-1)\cos^2\alpha & 0 & ((\varepsilon^R+i\varepsilon^I)-1)\cos\alpha\sin\alpha\\ 0 & 0 & 0\\ ((\varepsilon^R+i\varepsilon^I)-1)\cos\alpha\sin\alpha & 0 & ((\varepsilon^R+i\varepsilon^I)-1)\sin^2\alpha\end{pmatrix}$$

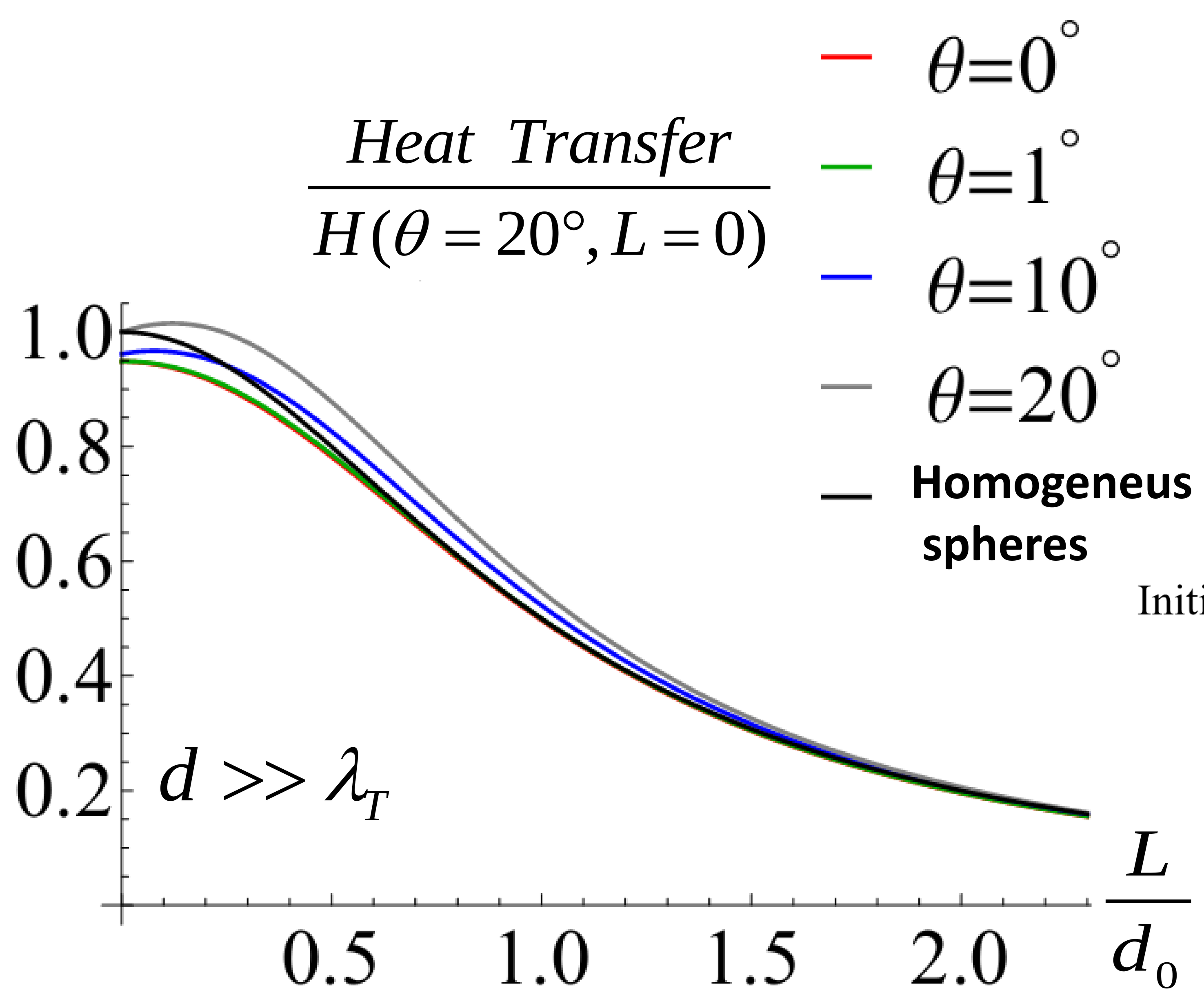
$$\frac{T_{(-1,0)}}{T_{(0,0)}}=\frac{\sqrt{2}}{3}\frac{\sin(2\alpha)}{(3+\cos(2\alpha))}\qquad\frac{T_{(-1,1)}}{T_{(0,0)}}=\frac{1}{6}\frac{(\cos(\alpha))^2}{(3+\cos(2\alpha))}\qquad\frac{T_{(0,1)}}{T_{(0,0)}}=\frac{\sqrt{2}}{3}\frac{\sin(2\alpha)}{(3+\cos(2\alpha))}$$

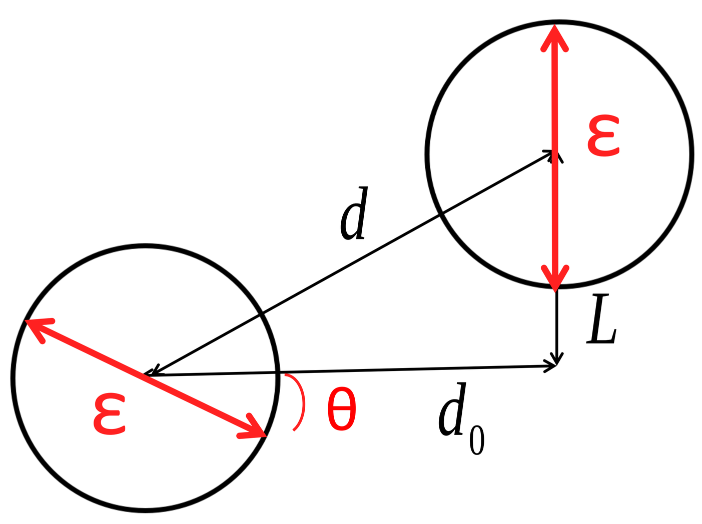
$T_{(m,m_p)}$ **: Incoming wave with index m, scattered wave with index $m_p$.**

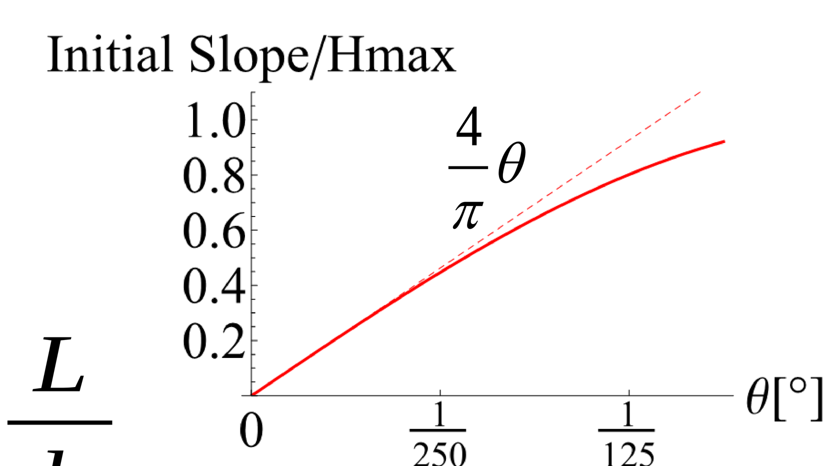
**The heat transfer in lowest order of $(R/\lambda_T)$ and $(\varepsilon-1)$ is:**

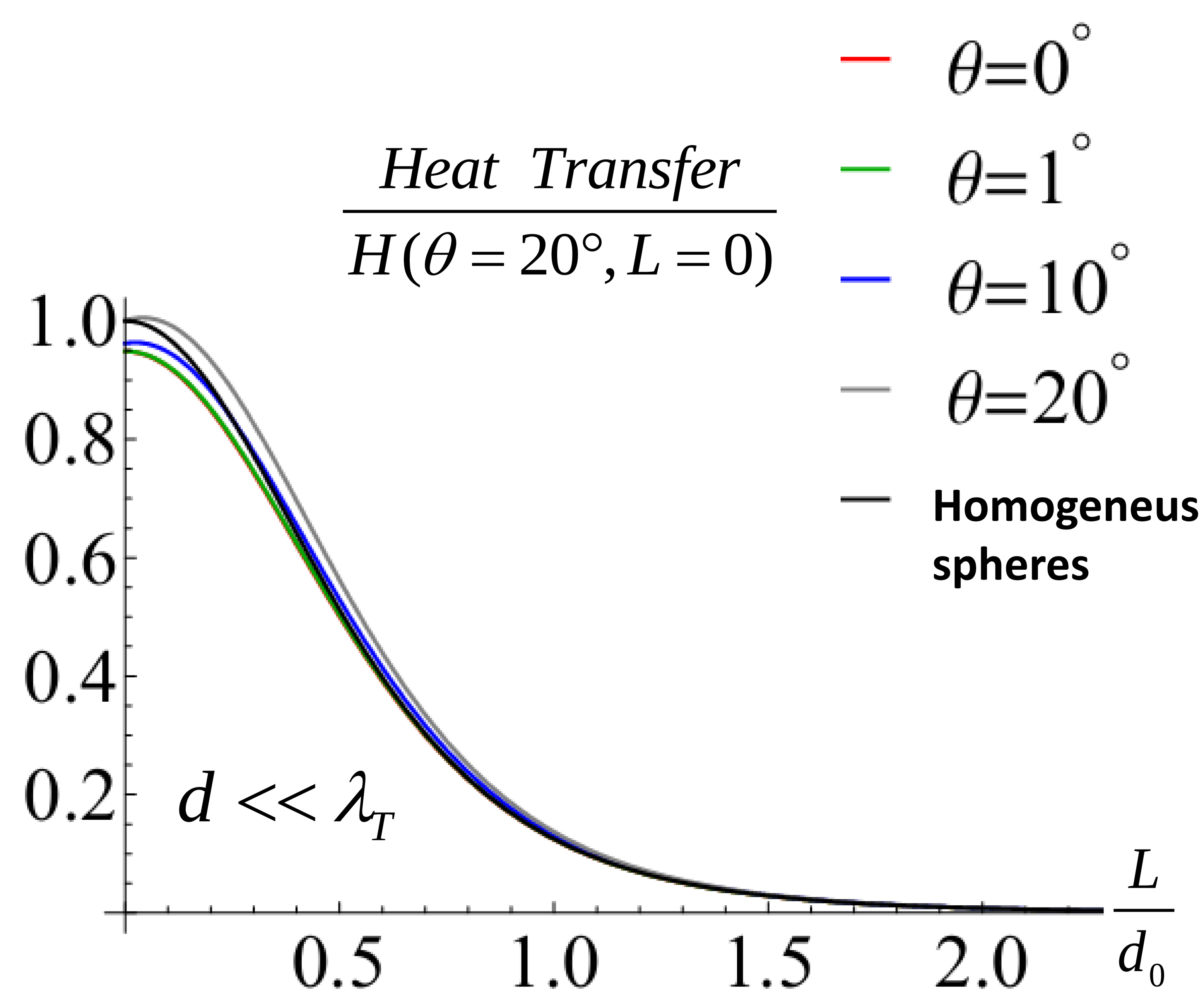
$$H=\frac{(512)\hbar}{\pi}\int d\omega\left(\frac{(\varepsilon^I)^2\omega^7R^6}{e^{\frac{\hbar\omega}{kT}}-1}\right)\left(\frac{1}{9}\frac{c^2}{\omega^2d^2}+\frac{1}{3}\frac{c^4}{\omega^4d^4}+\frac{c^6}{\omega^6d^6}\right)f(\alpha,\beta)$$

## Non-monotonic Heat Transfer

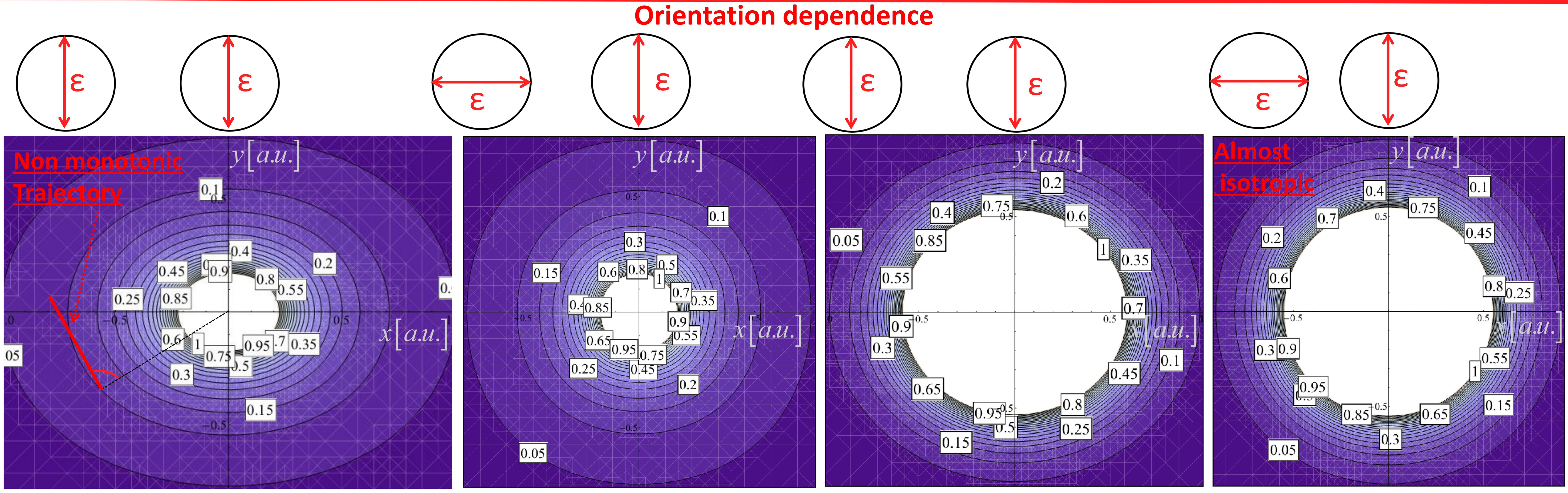



## Orientation dependence




***Reference:***
[1] *A.W. Rodirguez, M.T.H. Reid, J. Valera, J.D. Joannopoulos, F. Capasso, S. Johnson. Physical Review Letters,* **110, 014301 (2012).**
[2] *M. Krüger, G. Bimonte, T. Emig, M. Kardar. . Physical Review B,* **86, 115423 (2012)**